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By T. Chakraborty and P. Pietiläinen

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The Fractional Quantum Hall Effect

Properties of an Incompressible Quantum Fluid

With 85 Figures

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Foreword

The experimental discovery of the fractional quantum Hall effect (FQHE) at the end of 1981 by Tsui, Störmer and Gossard was absolutely unexpected since, at this time, no theoretical work existed that could predict new structures in the magnetotransport coefficients under conditions representing the extreme quantum limit. It is more than thirty years since investigations of bulk semiconductors in very strong magnetic fields were begun. Under these conditions, only the lowest Landau level is occupied and the theory predicted a monotonic variation of the resistivity with increasing magnetic field, depending sensitively on the scattering mechanism. However, the experimental data could not be analyzed accurately since magnetic freeze-out effects and the transitions from a degenerate to a nondegenerate system complicated the interpretation of the data. For a two-dimensional electron gas, where the positive background charge is well separated from the two-dimensional system, magnetic freeze-out effects are barely visible and an analysis of the data in the extreme quantum limit seems to be easier. First measurements in this magnetic field region on silicon field-effect transistors were not successful because the disorder in these devices was so large that all electrons in the lowest Landau level were localized. Consequently, models of a spin glass and finally of a Wigner solid were developed and much effort was put into developing the technology for improving the quality of semiconductor materials and devices, especially in the field of two-dimensional electron systems.

The formation of a Wigner lattice has been observed for the two-dimensional electron gas at the helium surface with the consequence that all sorts of unexpected results on two-dimensional systems in semiconductors were assigned to some kind of charge-density-wave or Wigner crystallization. First attempts to explain the FQHE were therefore guided by the picture of a Wigner solid with triangular crystal symmetry. However, a critical analysis of the data demonstrated that the idea of the formation of an incompressible quantum fluid introduced by Laughlin seems to be the most likely explanation.

The theoretical work collected in this book demonstrates that the Laughlin wave function forms a very good basis for a discussion of the FQHE. Even

though many questions in the field of FQHE remain unanswered, this book offers a valuable source of information and is the first general review of the work of different groups in this field. The intense activity in the field of high-T_c superconductivity also calls for a book about the FQHE since certain similarities seem to be emerging in the theoretical treatment of the quantum Hall effect and that of high-T_c superconductivity.

I hope that this book will inspire scientists to new ideas.

Stuttgart, June 1988 Klaus von Klitzing

Preface

In the field of the fractional quantum Hall effect, we have witnessed tremendous theoretical and experimental developments in recent years. Our intention here is to present a general survey of most of the theoretical work in this area. In doing so, we have also tried to provide the details of formal steps, which, in many cases, are avoided in the literature. Our effort is motivated by the hope that the present compilation of theoretical work will encourage a nonexpert to explore this fascinating field, and at the same time, that it will provide guidelines for further study in this field, in particular on many of the open problems highlighted in this review. Although the focus is on the theoretical investigations, to see these in their right perspective, a brief review of the experimental results on the excitation gap is also presented. This review is of course, by no means complete; the field continues to present new surprises, and more theoretical work is still emerging. However, we hope that the compilation in its present form will to some extent satisfy the need of the experts, nonexperts and the curious.

Stuttgart, Oulu,
January, 1988

Tapash Chakraborty
Pekka Pietiläinen

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