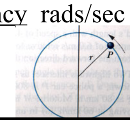
16.107 Review

Simple Harmonic Motion

$x(t) = x_m \cos(\omega t + \phi)$

- x_m is the amplitude (maximum value)
- ϕ is the phase angle (determines x(0))"phi"
- ω is ? "omega"
- $x(t + T) = x(t)$ ------> $\omega(t+T)+ \phi = \omega t + \phi$
- $\omega(t+T) = \omega t + 2\pi$ ----> $\omega T = 2\pi$
- $\omega = 2\pi/T = 2\pi f$ angular frequency rads/sec
- recall circular motion $\omega = \Delta\theta/\Delta t$



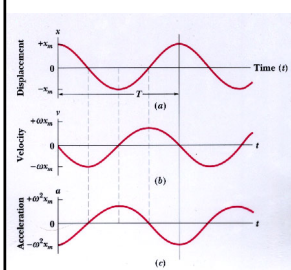
SHM

$x(t) = x_m \cos(\omega t + \phi)$

- x_m is a constant
- ϕ is a constant "phi"
- ω is a constant "omega"

- $\omega t + \phi$ varies with time t
- x(t) varies with time t

$x(t) = x_m \cos(\omega t + \phi)$



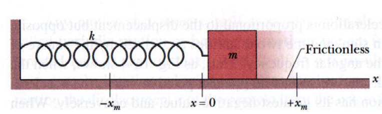
- $x(t)=x_m\cos(\omega t+\phi)$
- $v(t)=-\omega x_m \sin(\omega t+\phi)$
- $v_m = \omega x_m$ 'amplitude'
- shifted by T/4 (90^0)
- $a(t)=-\omega^2 x_m \cos(\omega t+\phi)$
- $a_m = \omega^2 x_m$ 'amplitude'
- shifted by 2T/4 (180^0)
- $d^2x/dt^2 = -\omega^2 x$

$$\ddot{x} = -\omega^2 x$$

Force Law for SHM

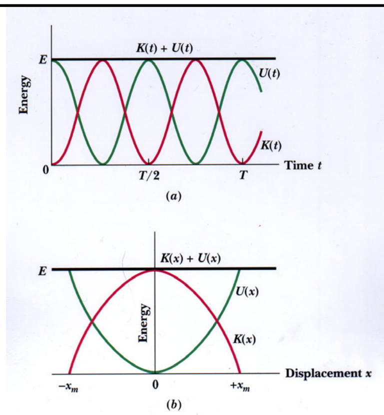
- Newton's second law F = m a
- 'a' non-zero ==> there is a force
- $F = ma = m(-\omega^2 x) = -m\omega^2 x = -kx$
- force ∝ displacement (in opposite direction)
- Hooke's law for springs with $k = m\omega^2$
- SHM $d^2x/dt^2 = -\omega^2 x$
- or $F = -kx$

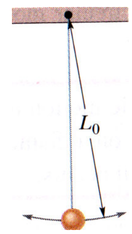
Simple harmonic motion is the motion executed by a particle of mass *m* subject to a force that is proportional to the displacement of the particle but opposite in sign.



$a = -\omega^2 x$ plus $F=ma$ ==> $F = -kx$ where $k=m\omega^2$

$\omega = (k/m)^{1/2}$ $T = 2\pi/\omega = 2\pi(m/k)^{1/2}$





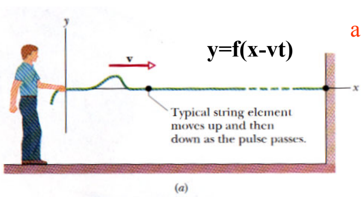
$T = 2\pi (L_0/g)^{1/2}$

Waves

a pulse

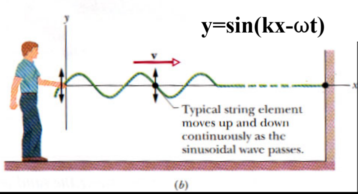
$y=f(x-vt)$

fixed shape moves to right



a travelling wave

$y=\sin(kx-\omega t)$

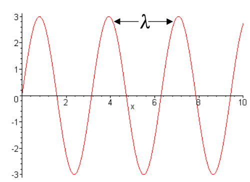


t=0 $y=y_m \sin(kx)$

$y(x+\lambda)= y_m \sin(kx+k\lambda)$
$= y(x)$

Hence $k\lambda = 2\pi$

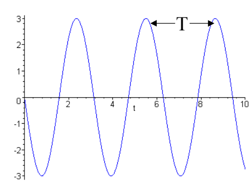
Example $y(x,t)=y_m \sin(kx-\omega t) = 3\sin(2x-2t)$



Snapshot at t=0

$y(x,0) = 3\sin(2x)$

Repeats in space with period λ
wavelength



Focus on point x=0

$y(0,t) = 3\sin(-2t)$

Repeats in time with period T

Summary

- $\omega = 2\pi f = 2\pi/T$ $k = 2\pi/\lambda$
- $v = \omega/k = f\lambda = \lambda/T$
- wave speed= one wavelength per period
- $y(x,t)=y_m \sin(kx-\omega t)$ describes a wave moving right at constant speed $v=\omega/k$
- $kx-\omega t$ = const labels a point on the wave
  $x = (\omega/k)\,t + \text{const}$
- $y(x,t)=y_m \sin(kx+\omega t)$ is a wave moving left
- $kx+\omega t$ = const labels a point on the wave
  $x = -(\omega/k)\,t + \text{const}$

Wave speed of a stretched string

- $v = C\,(F/\mu)^{1/2}$ $(MLT^{-2}/ML^{-1})^{1/2} = L/T$
- detailed calculation using 2nd law yields C=1

$v = (F/\mu)^{1/2}$

- speed depends only on characteristics of string
- independent of the frequency of the wave
  - f due to source that produced it
- once f is determined by the generator, then
- $\lambda = v/f = vT$

Interference

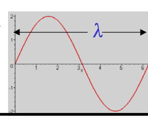
- $y(x,t) = y_1(x,t) + y_2(x,t)$
- $y(x,t) = y_m [\sin(kx-\omega t) + \sin(kx-\omega t-\phi)]$
- $\sin A + \sin B = 2 \sin[(A+B)/2] \cos[(A-B)/2]$
- $y(x,t) = 2 y_m [\sin(kx-\omega t- \phi/2)] \cos[\phi/2]$
- $y(x,t) = [2 y_m \cos(\phi/2)] [\sin(kx-\omega t- \phi/2)]$
- result is a sinusoidal wave travelling in same direction with
  - 'amplitude' $2 y_m \cos(\phi/2)$
  - 'phase' $(kx-\omega t- \phi/2)$

Standing Waves

- Consider two sinusoidal waves moving in opposite directions
- $y(x,t) = y_1(x,t) + y_2(x,t)$
- $y(x,t) = y_m [\sin(kx-\omega t) + \sin(kx+\omega t)]$
- at t=0, the waves are in phase $y=2y_m \sin(kx)$
- at $t \neq 0$, the waves are out of phase
- phase difference = $(kx+\omega t) - (kx-\omega t) = 2\omega t$
- interfere constructively when $2\omega t = m2\pi$
- hence $t = m2\pi/2\omega = mT/2$ (same as t=0)

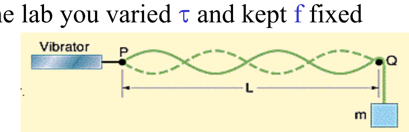
Standing Waves

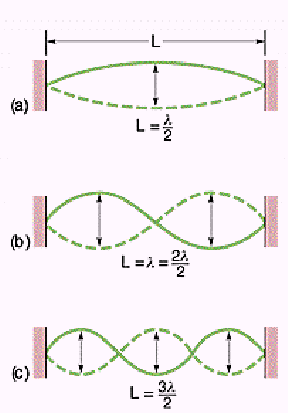
- $y(x,t) = y_m [\sin(kx-\omega t) + \sin(kx+\omega t)]$
- $\sin A + \sin B = 2 \sin[(A+B)/2] \cos[(A-B)/2]$
- $y(x,t) = [2 y_m \sin(kx)] \cos[-\omega t]$
- amplitude depends on position x
- not of the form f(x-vt) but rather g(x)h(t)
- not a travelling wave!
- Amplitude is zero when $kx=n\pi$, n=0,1,2,…
- but $k= 2\pi/\lambda$ => 'nodes' at $x=n \lambda/2$
- separated by $\lambda/2$



Resonance

- send a sinusoidal wave down a string with the far end fixed
- frequency f is determined by the source
- wave reflects and interferes with incident wave
- for given tension F and mass density μ the speed is determined by the medium $v=(F/\mu)^{1/2}$
- since $v=\lambda f$, then λ is determined
- in the lab you varied τ and kept f fixed





Need to adjust f so that λ=2L

$f=v/\lambda = v/2L$
'fundamental mode'

$v^2 = \tau/\mu$ => $\tau = \mu 4L^2f^2$

λ=L for next stable pattern
second harmonic

$\lambda=2L/n$ in general
n is the number of loops

$f_n = n\, v/2L = n f_1$ 'harmonics'

natural frequencies of the wire

Natural Frequencies

- Any object or structure has a set of natural frequencies
- if we shake it at this frequency, then a large amplitude vibration occurs
- important factor in engineering design
- atoms and molecules have 'natural' frequencies as well

Musical Sounds

- Consider a hollow pipe open at both ends
- a wave reflects even if the end is open =>free 'end' =>anti-node

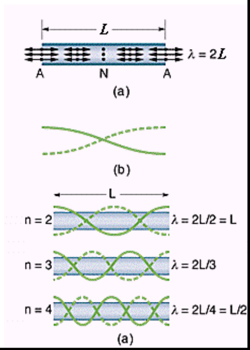
Fundamental or first harmonic

$f_1 = v/\lambda = v/2L$

for L=.4m, v=343m/s, f_1=429Hz

In general, λ_n=2L/n n=1,2,3,…

$f_n = v/\lambda_n$ =nv/2L

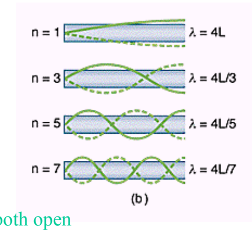


Musical Sounds

- Consider a pipe with one end closed
- waves reflect at both ends but there is a node at the closed end and an anti-node at the open end

Fundamental has $\lambda/4$ = L

$f_1 = v/\lambda = v/4L$ Lower than both open



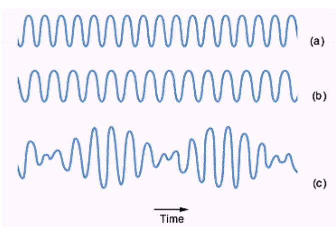
In general, λ_n = 4L/n but n is odd!

Lower frequency as L increases

$f_n = v/\lambda_n$ = nv/4L n=1,3,5,...

Beats

Consider the pressure variations Δp of two sound waves when detected separately and when detected simultaneously - can you hear the difference?



Resultant has the average frequency and the sound intensity varies at a frequency equal to the difference of the two waves

Beats

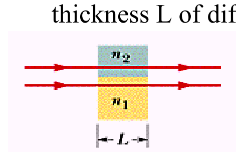
- Displacements due to the two waves at a particular location are
- $s_1=s_m \cos\omega_1 t$ and $s_2=s_m \cos\omega_2 t$
- resultant $s(t) = s_m [\cos\omega_1 t + \cos\omega_2 t]$
  $= s_m [2 \cos\{(\omega_1 t-\omega_2 t)/2\} \cos \{(\omega_1 t+\omega_2 t)/2\}]$
- let $\omega' = (\omega_1-\omega_2)/2$ $\omega = (\omega_1+\omega_2)/2$
- $s(t) = [2s_m \cos\omega' t] \cos\omega t$
- "amplitude modulated" "carrier" at average $\omega=2\pi f$
- maximum intensity when $\cos\omega' t = \pm 1$
- $\omega_{beat} = 2\omega' = 2 (\omega_1-\omega_2)/2 = 2\pi(f_1 - f_2) = 2\pi f_{beat}$
- $f_{beat} = f_1 - f_2$

Doppler Effect

- For moving source and fixed observer
- $f' = f [v/(v \mp v_S)]$
- if both are moving $f' = f [(v \pm v_O) /(v \mp v_S)]$
- upper sign if approaching
- lower sign if separating

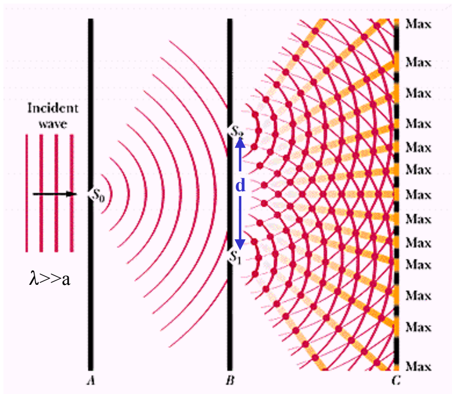
Refraction

- In general $v = \lambda f$ and λ changes if v does
- in vacuum $c = \lambda f$
- in a medium $c/n = \lambda_n f$
- hence $\lambda_n = \lambda/n$ which is less than λ
- consider two light waves which are in phase in air (n=1) and each passes through a thickness L of different material



upper wave has $\lambda_2 = \lambda/n_2$

lower wave has $\lambda_1 = \lambda/n_1$



Points of same phase add constructively   Vertical screen

Double Slit

- Bright fringe: $\Delta L = m\,\lambda$
  $d \sin\theta = m\,\lambda$, $m=0,1,2,\ldots$
- Dark fringe: $\Delta L = (m+1/2)\,\lambda$
  $d \sin\theta = (m+1/2)\,\lambda$, $m=0,1,2,\ldots$
- use 'm' to label the bright fringes
- $m=0$ is the bright fringe at $\theta=0$
  "central maximum"

Double Slit Interference

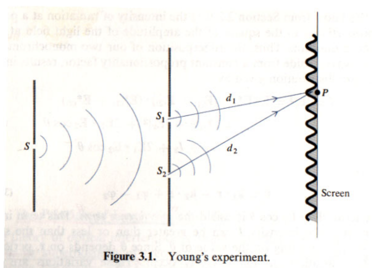


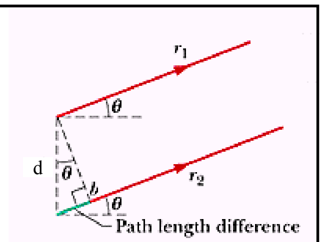
Figure 3.1. Young's experiment.

Intensity of Interference Pattern

- net effect at point P is the superposition of two waves
- $E(r,t) = E_m \sin(kr_1-\omega t) + E_m \sin(kr_2-\omega t)$
  =>same wavelength and frequency but travel different distances r_1 and $r_2 = r_1 + \Delta L$
- $E(r,t) = E_m \sin(kr_1-\omega t) + E_m \sin[k(r_1+\Delta L)-\omega t]$
- $= E_m[\sin(kr_1-\omega t) + \sin(kr_1-\omega t+\Delta\phi)]$
- where $\Delta\phi = k\,\Delta L$
- $\sin(A)+\sin(B) = 2\cos[(A-B)/2]\sin[(A+B)/2]$
- $E(r,t) = 2E_m \cos(\Delta\phi/2)\sin(kr_1-\omega t+\Delta\phi/2)$

$\Delta\phi = k\,\Delta L$

$\Delta L = d \sin\theta$



$E(r,t) = [2E_m \cos(\Delta\phi/2)]\sin(kr_1-\omega t+\Delta\phi/2)$

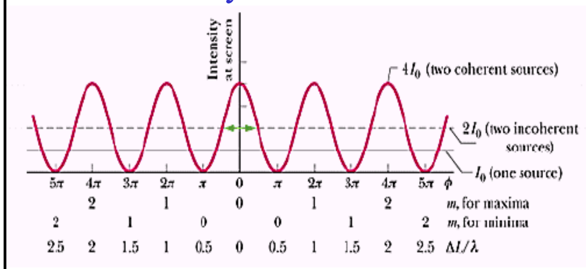
Amplitude= $2E_m \cos(\Delta\phi/2)$

Intensity= (amplitude)^2

Intensity of Interference Pattern

- Amplitude of electric field at P is
  $2E_m\cos(\Delta\phi/2) = 2E_m\cos(kd\sin\theta/2)$
- $= 2E_m\cos(\pi d \sin\theta/\lambda)$
- intensity of field is the amplitude squared
- $I = I_0 \cos^2(\pi d \sin\theta/\lambda)$ where $I_0 = 4(E_m)^2$
- $I = I_0$ when $\pi d \sin\theta/\lambda = m\pi$ => $d \sin\theta = m\lambda$
- $I = 0$ when $\pi d \sin\theta/\lambda = (m+1/2)\pi$
  => $d \sin\theta = (m+1/2)\lambda$
- $I = I_0 \cos^2(\beta)$ with $\beta = \pi d \sin\theta/\lambda$

Intensity of Double Slit

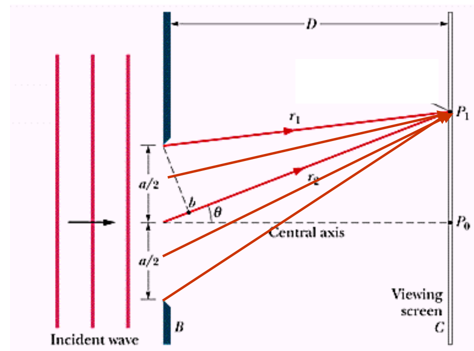


$E = E_1 + E_2$

$I = E^2 = E_1^2 + E_2^2 + 2\,E_1\,E_2$

$= I_1 + I_2$ + "interference" <== vanishes if incoherent

Single Slit Diffraction



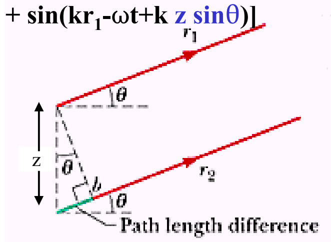
Assume screen is far enough away that red rays are parallel

Path difference between neighbouring rays is

$z \sin\theta$

Total electric field due to r_1 and r_2 is

$E(r,t)=E_m[\sin(kr_1-\omega t) + \sin(kr_1-\omega t+k\,z\sin\theta)]$



Single Slit

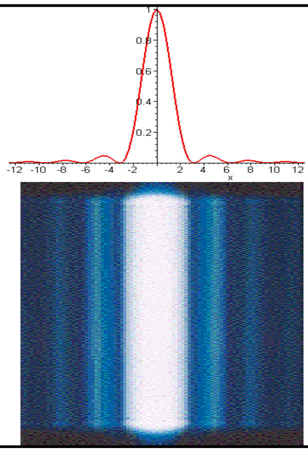
- $E(r,t)=E_m[\sin(kr_1-\omega t) + \sin(kr_1-\omega t+k\,z\sin\theta)]$
- where range on z is $0 \le z \le a$
- $E(r,t) = E_m \int_0^a \sin(kr_1-\omega t+k\,z\sin\theta)\,dz$
- $= -(E_m/k\sin\theta)\cos(kr_1-\omega t+k\,z\sin\theta)\Big|_0^a$
- $= (E_m/k\sin\theta)\,[\cos(kr_1-\omega t)-\cos(kr_1-\omega t+k\,a\sin\theta)]$
- $=[2(E_m/k\sin\theta)\sin(ka\sin\theta/2)]\sin(kr_1-\omega t+ka\sin\theta/2)$
- ↑ amplitude

Single Slit

- **Amplitude** $=[2(E_m/k\sin\theta)\sin(ka\sin\theta/2)]$
  $=[2(E_m a/ka\sin\theta)\sin(ka\sin\theta/2)]$
- $= (E_m a)\sin(\alpha)/\alpha$
- **where** $\alpha = ka\sin\theta/2 = \pi a\sin\theta/\lambda$
- $I = I_0\,(\sin(\alpha)/\alpha)^2$
- **where** $I_0 = (E_m a)^2$ **is the maximum intensity**
- **note: lim** $\sin(\alpha)/\alpha \Rightarrow 1$
  $\alpha \Rightarrow 0$
- **intensity is maximum at** $\theta = 0$

Single Slit

- $I = I_0\,(\sin(\alpha)/\alpha)^2 = 0$ **when** $\alpha = m\pi = \pi a\sin\theta/\lambda$
- $a\sin\theta = m\lambda$ **for a dark fringe** ($m \neq 0$)
- **<u>note</u>: $m=0$ is a maximum!**
- **where are the other maxima?**
- **maximize** $\sin(\alpha)/\alpha$ **with respect to** α
- $(d/d\alpha)\,[\sin(\alpha)/\alpha] = \cos(\alpha)/\alpha - \sin(\alpha)/\alpha^2 = 0$
- **or** $\alpha = \tan(\alpha)$ $\Rightarrow$ $\alpha = 0$ **is a solution**
- **plot α and $\tan(\alpha)$ versus α and look for intersections**



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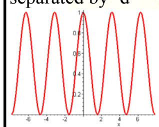
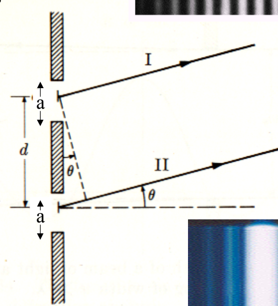
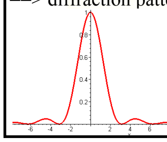
Double-Slit Diffraction

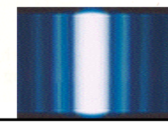
Young's slits a<<λ
=>interference pattern
from two point sources
separated by 'd'





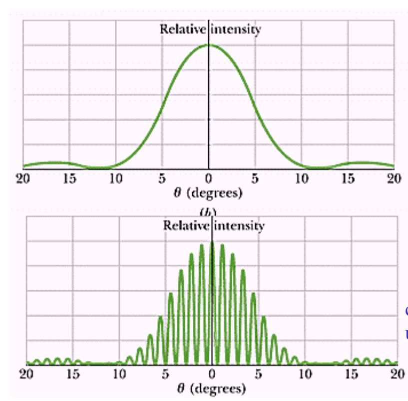
Single slit d=0
==> diffraction pattern





Intensity

- $I=I_0\,(\sin\alpha/\alpha)^2\,(\cos\beta)^2$
- diffraction and interference
- $\alpha = ka\sin(\theta)/2$ $\quad\beta = kd\sin(\theta)/2$
- limit $a \Rightarrow 0$ $\quad I = I_0\,(\cos\beta)^2$ $\quad$ Young's slits
- limit $d \Rightarrow 0$ $\quad I=I_0\,(\sin\alpha/\alpha)^2$ $\quad$ Single slit
- note: $\alpha/\beta = a/d$



$\alpha/\beta \sim 1/10$

Diffraction envelope

$\cos^2(\beta)$ term oscillates under the envelope

Diffraction Grating

$$E(r,t) = (E_m a)\frac{\sin\alpha}{\alpha}\frac{\sin(N\beta)}{\sin\beta}\sin[kr - \omega t + \alpha + (N-1)\beta]$$

$$I = \frac{I_0}{N^2}\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2 \qquad I_0 = (NE_m a)^2$$

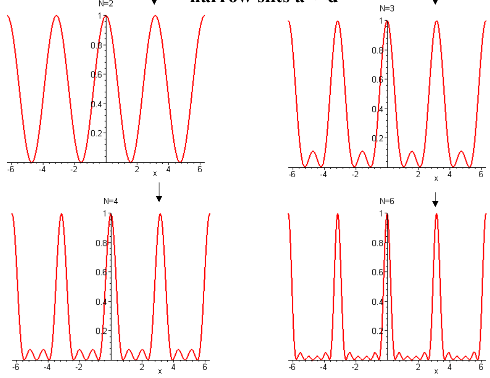
- $N=1 \Rightarrow I=I_0\,(\sin\alpha/\alpha)^2$ $\quad$ single slit diffraction
- $N=2 \Rightarrow I=I_0(\sin\alpha/\alpha)^2(\cos\beta)^2$ $\quad$ double slit diffraction
- For N narrow slits $\alpha \Rightarrow 0$, $\quad I = \frac{I_0}{N^2}\left(\frac{\sin N\beta}{\sin\beta}\right)^2$

Diffraction Grating

$$I = \frac{I_0}{N^2}\left(\frac{\sin N\beta}{\sin\beta}\right)^2 \qquad \beta = kd\sin(\theta)/2$$

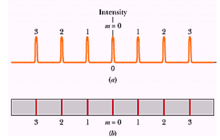
- at $\theta=0$, we have $\beta=0$ and $I = I_0=(NE_m a)^2$
- limit $\beta \Rightarrow 0$ $\;\sin N\beta/\sin\beta = N$
- calculus: l'hopital's rule for 0/0
- take derivative of numerator and divide by derivative of denominator
- = limit $\beta \Rightarrow 0$ $\quad N\cos(N\beta)/\cos\beta = N$
- let's plot I for N=3,4,5,...

narrow slits a<<d



Width of brightest fringes is decreasing

Diffraction Gratings



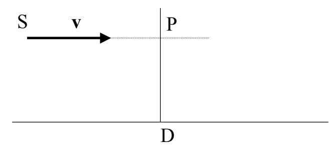
- As N increases, the pattern changes to narrow maxima separated by wide dark regions
- slits of a grating are called "rulings"
- if W= total width of N rulings, then d= W/N
- can be used to determine the wavelength of light
- the narrow maxima are called "lines"
- $d \sin\theta = m\lambda$ gives location of m^{th} order line

Doppler Effect for Light

- For source and detector separating
- $f = f_0 (1-\beta)^{1/2}/(1+\beta)^{1/2}$ red shift $\lambda > \lambda_0$
- For source and detector approaching
- $f = f_0 (1+\beta)^{1/2}/(1-\beta)^{1/2}$ blue shift $\lambda < \lambda_0$

Transverse Doppler Effect

- In the previous cases, the source and detector are moving along the same line ←s d→



- at point P, **v** is perpendicular to PD
- for sound emitted at P, there is no Doppler shift at D
- if light is emitted, there is a Doppler shift
- $f = f_0 (1 - \beta^2)^{1/2} = f_0/\gamma$ transverse Doppler effect

Modern Physics

- 1905 Einstein proposed:
- when an atom emits or absorbs light, energy
- is not transferred in a smooth continuous fashion but rather in discrete "packets" or "lumps" of energy
- "photons" have energy E=hf ← Frequency $c=\lambda f$

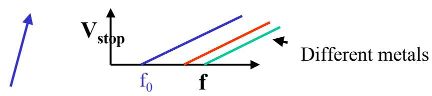
Planck's constant
$h=6.63\times10^{-34}$ J.s

Modern Physics

- h plays a similar role to c in relativity
- if $c \rightarrow \infty$ then no relativity! $v/c \ll 1$ always
  => signals transmitted instantaneously
- if $h \rightarrow 0$ then no quantum mechanics
  => no stable atoms!

Photoelectric Effect

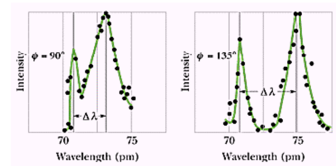
- Hence we need $hf > \Phi$ to just escape
- that is $f > \Phi/h = f_0$
- Einstein: $K_{max} = (hf - \Phi)$ if no other
  $= e V_{stop}$ losses of energy are involved
- $V_{stop} = (h/e) f - (\Phi/e) = (h/e)(f - f_0)$



Slope =h/e is independent of the metal!

Compton Scattering

- "loosely" bound electrons in Carbon are ejected and the x-rays are scattered
- $\lambda' - \lambda = (h/m_e c)(1 - \cos\phi)$
- "tightly" bound electrons are not ejected
  => photon interacts with entire carbon atom
- mass ~ 22,000 m_e => $\Delta\lambda$ reduced by this factor



Electrons and Matter Waves

- Light is a wave but can transfer energy and momentum to matter in "photon" sized lumps
- can a particle have the same properties?
- can it behave as a wave?
- "a matter wave"
- de Broglie (1924) suggested $\lambda = h/p$
- "de Broglie wavelength"
- a beam of electrons has a wavelength and should diffract if "slit width" comparable to $\lambda = h/p$
- 1927 Davisson and Germer observed diffraction of electrons from crystals

What is Waving?

- If particles behave as waves, what is waving?
- Wave on a string => particles in string execute SHM
- sound wave in air => air molecules oscillate in SHM
- light wave => electric and magnetic fields oscillate
- e.g. $\mathbf{E}(x,y,z,t)$ electric field varies from place to place and with time
- intensity $\propto |\mathbf{E}|^2$
- what varies from place to place for a matter wave?
- Wave function $\Psi(x,y,z,t)$ "psi"

Schrödinger's Quantum Mechanics

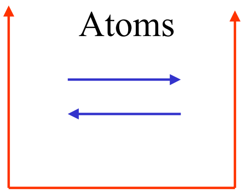
- Probability waves are described by a wave function $\Psi(x,y,z,t)$
- Schrödinger adopted de Broglie's equations
- $\lambda = h/p$ wavelength of a particle
- $f = E/h$ frequency of a particle
- non-relativistic energy is
- $E = mc^2 + p^2/2m + V$

Rest energy K.E. P.E.

Schrödinger Equation

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\Psi = 0$$

- Schrödinger Equation 1926
- $\Psi(x,t)$ is a solution of this equation
- the wave equation for matter waves

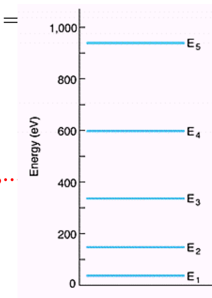


Atoms

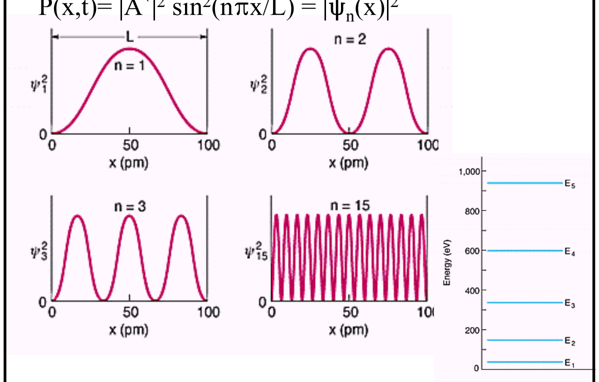
- Confined matter waves are standing waves
- 1926 Quantum mechanics was developed
- explained the structure of atoms and molecules
- electrons, protons treated as matter waves
- produce standing wave patterns

Atoms

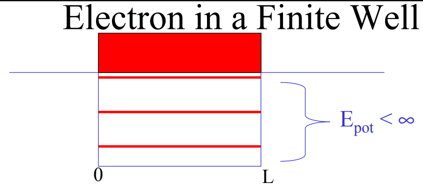
- $P(x,t)= |A`|^2 \sin^2(kx)$ => need $P(L,t)=0$
- $kL =(2\pi/\lambda)L = n\pi$ => $\lambda= (2L)/n$ "standing waves"
- only certain λ => only certain p =
- $p=h/\lambda = nh/2L$
- $E_n=p^2/2m = h^2n^2/8mL^2$ n=1,2,3,..



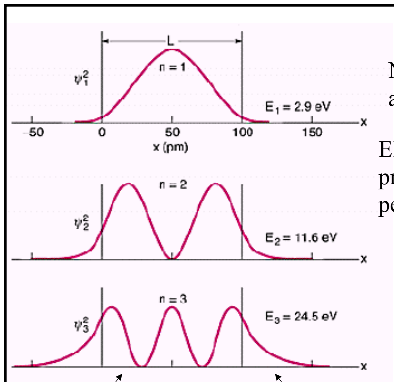
$P(x,t)= |A`|^2 \sin^2(n\pi x/L) = |\psi_n(x)|^2$



Electron in a Finite Well



- Only 3 levels with $E < E_{pot}$
- states with energies $E > E_{pot}$ are <u>not</u> confined
- all energies for $E > E_{pot}$ are allowed since the electron has enough kinetic energy to escape to infinity



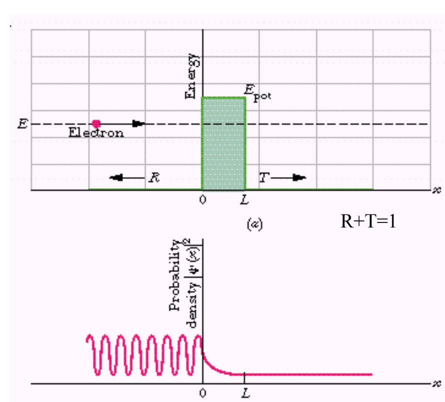
No longer a node at x=0 and x=L

Electron has small probability of penetrating the walls

Area under each curve is 1

oscillation

Exponential decay

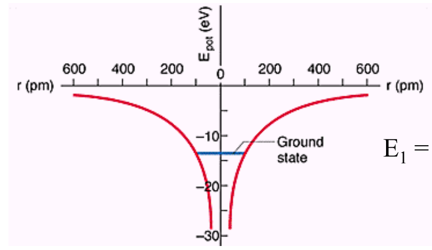


R+T=1

Tunneling

- Transmission coefficient $T = 16(E/U)(1-E/U)\, e^{-2kL}$
- $k = \{8\pi^2 m(U-E)/h^2\}^{1/2}$ Note: $E < U$
- if T=.02 then for every 1000 electrons hitting the barrier, about 20 will tunnel
- extremely sensitive to L and k
- width and height of barrier

$$E_{pot} = \frac{1}{4\pi\varepsilon_0}\frac{(e)(-e)}{r} < 0$$

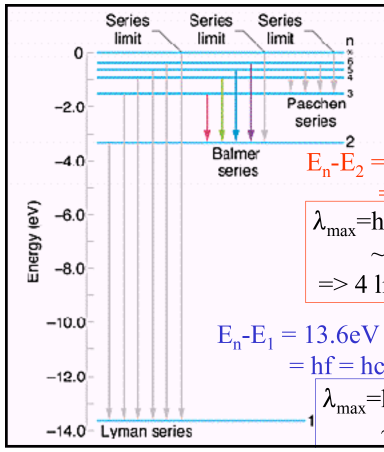


$E_1 = -13.6\text{eV}$

$E_n = -13.6\text{eV}/n^2 \quad n=1,2,3,...$

Hydrogen Atom

- $E_n = -(me^4/8\epsilon_0^2h^2)(1/n^2)$
- $E_n = -13.6\text{eV}/n^2 \quad n=1,2,3,\ldots$
- ground state has $E_1 = -13.6\text{eV}$
- ionization energy is $0 - E_1 = 13.6\text{eV}$
  => energy needed to remove electron
- excited state: n=2 $\quad E_2 = -(13.6/4)\text{eV}$
- electron must absorb a photon of energy
  $hf = E_2 - E_1 = hc/\lambda = (3/4)(13.6\text{eV})$



$E_n - E_2 = 13.6\text{eV}\ (1/4 - 1/n^2)$
$= hf = hc/\lambda$

$\lambda_{max} = hc/(13.6\text{eV})(5/36)$
~ 658 nm
=> 4 lines visible

$E_n - E_1 = 13.6\text{eV}\ (1 - 1/n^2)$
$= hf = hc/\lambda$

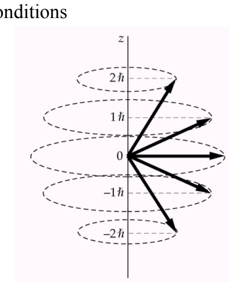
$\lambda_{max} = hc/(13.6\text{eV})(.75)$
~ 122 nm

Quantum Theory of Atoms

- For a particle in a cubic or rectangular box, Cartesian coordinates are more appropriate and there are three quantum numbers (n_1, n_2, n_3) needed to label a quantum state
- similarly in spherical coordinates, there are also three quantum numbers needed
- *n = 1, 2, 3, ...*
- *l = 0, 1, 2, ...,n-1* => *n values of l for a given value of n*
- *m = -l,(-l+1),...0,1,2,...,l* => *2l+1 values of m for a given l*
- *n* is the principal quantum number and is associated with the distance r of an electron from the nucleus
- *l* is the orbital quantum number and the angular momentum of the electron is given by $L = [l(l+1)]^{1/2}\,\hbar$
- *m* is the magnetic quantum number and the component of the angular momentum along the z-axis is $L_z = m\,\hbar$

Quantum Theory of Atoms

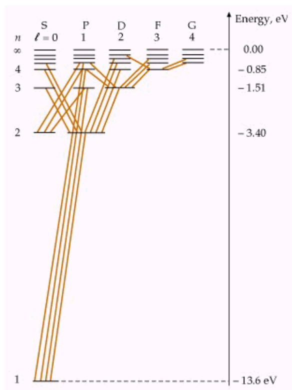
- The fact that both *l* and *m* are restricted to certain values is due to boundary conditions
- in the figure, *l=2* is shown
- hence $L = (2(2+1))^{1/2}\,\hbar = \hbar(6)^{1/2}$
- and m= -2, -1, 0, 1, 2

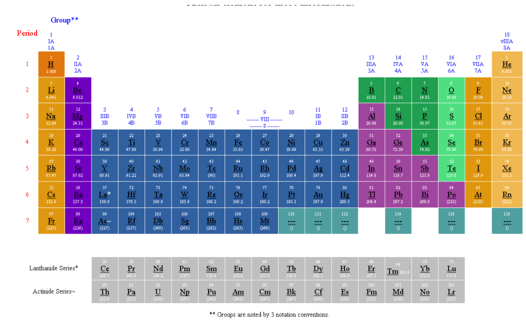


- the Schrödinger equation can be solved exactly for hydrogen
- the energies are the same as in the Bohr theory $E_n = -Z^2\,(13.6\text{ eV})/n^2$
- they do not depend on the value of *l* and *m*
- *this is a special property of an inverse-square law force*

- The lowest energy has n=1 => l=0 and m=0
- the second lowest energy has n=2 => $l=0$, $m=0$
  $l=1$, $m=-1,0,1$
- hence 4 states!

- Notation: $l=0$ *S-state*
- $l=1$ *P-state*
- $l=2$ *D-state*
- $l=3$ *F-state*
- $l=4$ *G-state*





configurations

Periodic table

Periodic Table

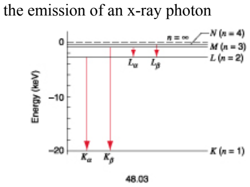
- Atoms with more than one electron cannot be solved exactly
- assume the Z electrons do not interact with one another but rather only see the nucleus with charge +Z
- the state of each electron is described by four quantum numbers
- n, l, m and m_s
- *the fourth number,* $m_s = \pm 1/2$ is the *spin quantum number*
- l = 0, 1, 2, 3, 4, 5, … correspond to
- s p d f g h
- Pauli principle: no two electrons can have the same set of values of n, l, m and m_s

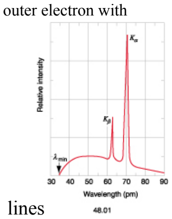
- eg. Hydrogen (Z=1) has only one electron
- lowest energy state has n=1 => l=0 => m=0 and $m_s = \pm 1/2$
- 1s state

X-ray spectrum

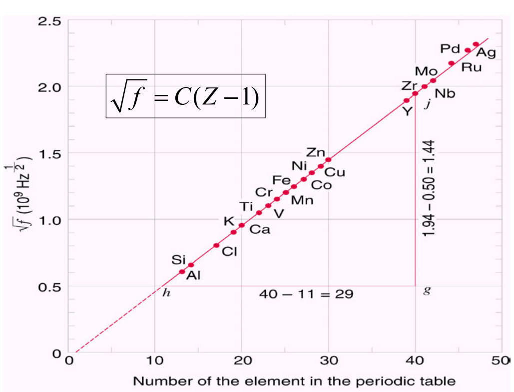
- If electron loses all its energy, $eV_{accel} = hf_{max} = hc/\lambda_{min}$
- λ_{min} is independent of the material and depends only on KE of electrons
- note that if h=0, then $\lambda_{min} = hc/eV_{accel}$ would be zero!

- the peaks at larger λ depend on the material
- arise when the incident electron knocks out an *inner electron*
- this leaves a hole in an inner shell which is filled by an outer electron with the emission of an x-ray photon





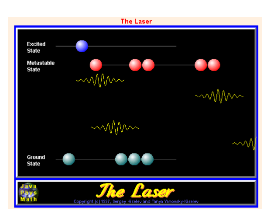
Note K_α K_β lines

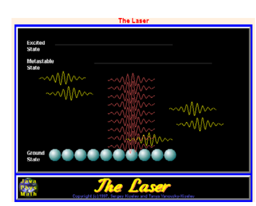


Lasers

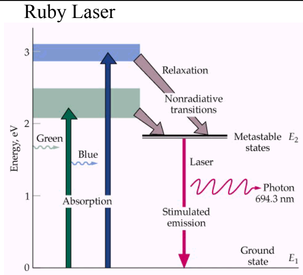
- Light amplification by stimulated emission of radiation
- produces a beam of coherent photons by stimulated emission

laser





Ruby Laser



Normally all atoms are in the ground state E_1

For the laser to work, we need more atoms in an excited state
--> called population inversion

Optical pumping is used to excite electrons to higher levels which then relax to the state E_2