Wave Packets

- Recall that for a wave packet $\Delta x\ \Delta k \sim 1$
- to localize a wave to some region Δx we need a spread of wavenumbers Δk
- de Broglie hypothesis $\lambda = h/p$ or $p = h/\lambda$
- but $k = 2\pi/\lambda$ and $p = (h/2\pi)(2\pi/\lambda) = \hbar k$
- h bar
- $\Delta p = \hbar \Delta k$ and $\Delta x\ \Delta p \sim \hbar$

- $\Delta x\ \Delta p \geq \hbar$ uncertainty principle

Heisenberg's Uncertainty Principle

- States that measured values cannot be assigned to the position r and momentum p of a particle simultaneously with unlimited precision
- introduce "h-bar" $\hbar = h/2\pi$
- $\Delta x\ \Delta p_x \geq \hbar$
- $\Delta y\ \Delta p_y \geq \hbar$
- $\Delta z\ \Delta p_z \geq \hbar$

Energy-Time Uncertainty

- Recall for photons E = hf
- uncertainty in energy => uncertainty in f
- $\Delta f = \Delta E/h$ $\Delta\omega = 2\pi\Delta f$ $\Delta\omega.\Delta t \approx 1$
- introduce "h-bar" $\hbar = h/2\pi$
- $\Delta E \,.\, \Delta t \geq \hbar$

- need infinite time to make an accurate measurement of energy
- eg. Ground state of atom has well defined energy
- excited state does not

Uncertainty Principle

- If $\Delta p_x = \Delta p_y = \Delta p_z = 0$ (free particle)
- then $\Delta x = \Delta y = \Delta z = \infty$
- an electron has kinetic energy K = 12.0 eV
- $K=(1/2)mv^2 = 12.0$ eV => $v = 2.05 \times 10^6$ m/s
- suppose the electron is moving // x and we can measure its speed with .50% precision (one part in 200)
- what is the uncertainty in its position?

- p_x=mv =(.911x 10^{-31} kg)(2.05 x 10^6 m/s)
  = 1.87 x 10^{-24} kg.m/s
- Δp_x = .005 (1.87 x 10^{-24} kg.m/s)
  = 9.35 x10^{-27} kg.m/s
- $\Delta x \sim \hbar / \Delta p_x$
  =(6.63 x 10-34 J.s)/(2π)(9.35 x10^{-27} kg.m/s)
  = 11.0 nm ~ 100 atomic diameters

Problem

- Imagine a universe in which Planck's constant h = .60 J.s
- quantum baseball
- consider a .50 kg baseball that is moving at 20 m/s along an axis
- if the uncertainty in its speed is 0.5 m/s, what is the uncertainty in its position?
- Δp = (.5 kg)(0.5 m/s) = .25 kg.m/s
- $\Delta x \geq \hbar / (\Delta p)$ = (.60/2π)/.25 = .38 m

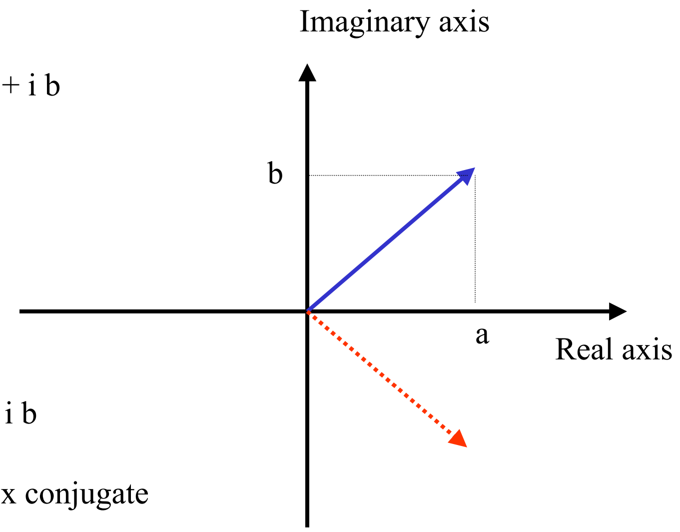
Wave function

- What kind of function is $\psi(x,y,z,t)$?
- Travelling wave? Standing wave?

- Let's learn something about complex numbers
- sqrt(-1) is a complex number $i^2 = -1$
- a complex number has the general form $a + i\,b$
- 'a' is the real part and 'b' is the imaginary part
- a complex function has the form $f_1(x) + i\,f_2(x)$

Real part

Imaginary part



$z = a + i\,b$

$z^* = a - i\,b$

Complex conjugate

Modulus $|a + i\,b| = \text{sqrt}\,(a^2 + b^2)$

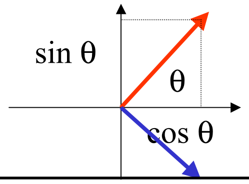
$zz^* = (a + ib)(a - ib) = a^2 - i^2\,b^2 = a^2 + b^2 = |z|^2$

Complex function

- $e^x = 1 + x + x^2/2 + x^3/6 + \ldots + x^n/n! + \ldots$

- $e^{\,i\theta} = 1 + i\theta + (i\theta)^2/2 + (i\,\theta)^3/6 + (i\,\theta)^4/24 + \ldots$
- $= 1 + i\,\theta - \theta^2/2 \;\; -i\,\theta^3/6 + (\theta)^4/24 + \ldots$
- $= (1 - \theta^2/2 + \theta^4/24 - \ldots) + i(\theta - \theta^3/6 + \ldots)$
- $= \cos\theta + i\sin\theta$
- $e^{-i\theta} = \cos\theta - i\sin\theta$

$e^{\,i\theta} = \cos\theta \;\; + i\sin\theta$



Complex Waves

- $\psi(x,t) = f(x)\,g(t)$ a standing wave
- string: $y(x,t)=2y_m\sin(kx)\cos(\omega t)$

- matter wave: $\psi(x,t) = f(x)\,e^{-i\omega t}$
- $= f(x)[\cos\omega t \;\; - i\sin\omega t\,]$

Schrödinger's Quantum Mechanics

- Probability waves are described by a wave function $\psi(x,y,z,t)$
- Schrödinger adopted de Broglie's equations
- $\lambda = h/p$ wavelength of a particle
- $f = E/h$ frequency of a particle
- non-relativistic energy is
- $E = \underset{\text{Rest energy}}{mc^2} + \underset{\text{K.E.}}{p^2/2m} + \underset{\text{P.E.}}{V}$

Quantum Mechanics

- Choose mc^2 as the reference energy
- $E = p^2/2m + V$ particle
- $hf = h^2/2m\lambda^2 + V$ wave
- relates f to λ
- what is the equation for the wave function $\psi(x,t)$?

Schrödinger Equation

$$\frac{d^2\Psi}{dx^2}+\frac{8\pi^2 m}{h^2}(E-V)\Psi=0$$

- Schrödinger Equation 1926
- $\psi(x,t)$ is a solution of this equation
- the wave equation for matter waves

Free Particle

- Consider the case of V=0
- hence $E=(1/2)mv^2 = p^2/2m$

$$\frac{d^2\Psi}{dx^2}+\frac{8\pi^2 m}{h^2}(E-V)\Psi=0$$

$$\frac{d^2\Psi}{dx^2}+\left(\frac{2\pi\, p}{h}\right)^2\Psi=0$$

Free Particle

$$\boxed{\frac{d^2\Psi}{dx^2} + \left(\frac{2\pi\ p}{h}\right)^2 \Psi = 0}$$

- But p=h/λ and k=2π/λ

$$\boxed{\frac{d^2\Psi}{dx^2} + k^2\Psi = 0}$$

Can we solve this equation?