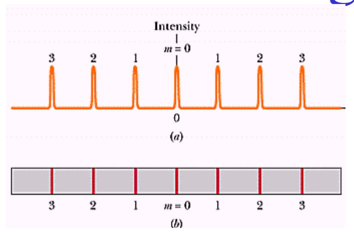
Diffraction Gratings

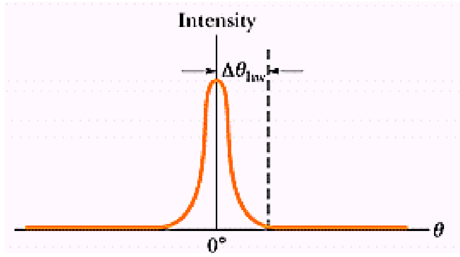


- As N increases, the pattern changes to narrow maxima separated by wide dark regions
- slits of a grating are called "rulings"
- if W= total width of N rulings, then $d = W/N$
- can be used to determine the wavelength of light
- the narrow maxima are called "lines"
- $d \sin\theta = m\lambda$ gives location of m^{th} order line

Problem

- A 3.00 cm wide diffraction grating produces second order lines at an angle of 33.0^0 with light of wavelength 600 nm. How many lines (rulings) on the grating?
- $d \sin\theta = m\lambda$ gives location of m^{th} order line
- $d=2(600\text{ nm})/\sin(33.0^0) = 2.203\ \mu\text{m}$
- $d = W/N$
- $N=W/d = 3.00\text{ cm}/2.203\ \mu\text{m} =13{,}600$ rulings

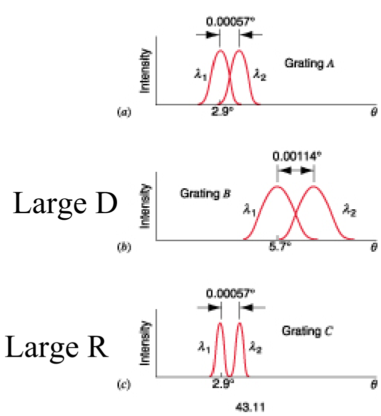
Width of Lines



- Half width of the central line is measured from the center to the adjacent minimum
- $\sin(N\delta/2)/\sin(\delta/2) = 0$ first when $\delta/2 = \pi/N$
- $\delta/2 = kd\sin(\theta)/2 = \pi d \sin(\theta)/\lambda$
- $\Delta(\delta/2) = [\pi d \cos(\theta)/\lambda]\, \Delta\theta = \pi/N$
- $\Delta\theta_{hw} = \lambda/[Nd \cos(\theta)]$ =>better resolution for larger N

Dispersion and Resolving Power

- Precise measurements of wavelengths using a grating depend on (1) the angular separation Δθ between lines that differ in wavelength by a small amount Δλ and (2) the width or sharpness of the lines
- dispersion $D = \frac{\Delta\theta}{\Delta\lambda}$
- maxima for $d\sin(\theta) = m\lambda$
- hence $d\cos(\theta)\Delta\theta = m\Delta\lambda$ (1)
- $D = \frac{m}{d\cos\theta}$
- larger D if smaller d
- resolving power $R = \frac{\lambda}{\Delta\lambda}$



Resolving Power

- Dispersion $$d\cos(\theta)\Delta\theta = m\Delta\lambda$$
- Width of maxima $$\Delta\theta = \frac{\lambda}{Nd\cos\theta}$$
- hence $$R = \frac{\lambda}{\Delta\lambda} = Nm$$

Applications of Gratings

Grating spectroscopes are used to measure wavelengths of light sources

$d\sin\theta = m\lambda$ all λ have a line at $\theta = 0$

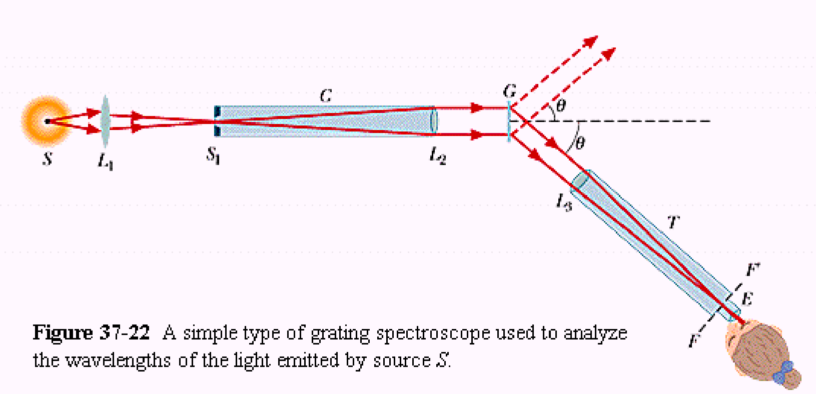
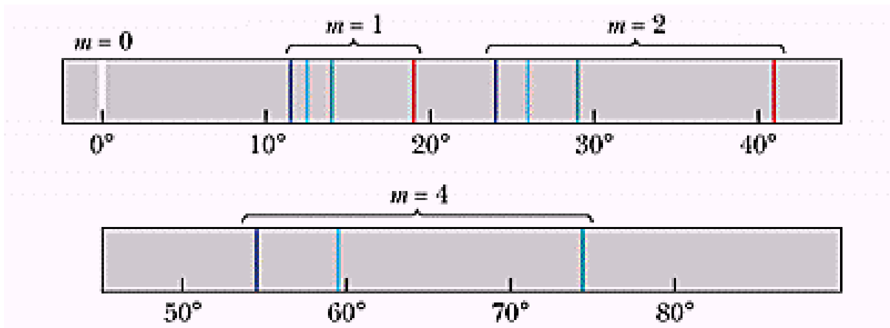
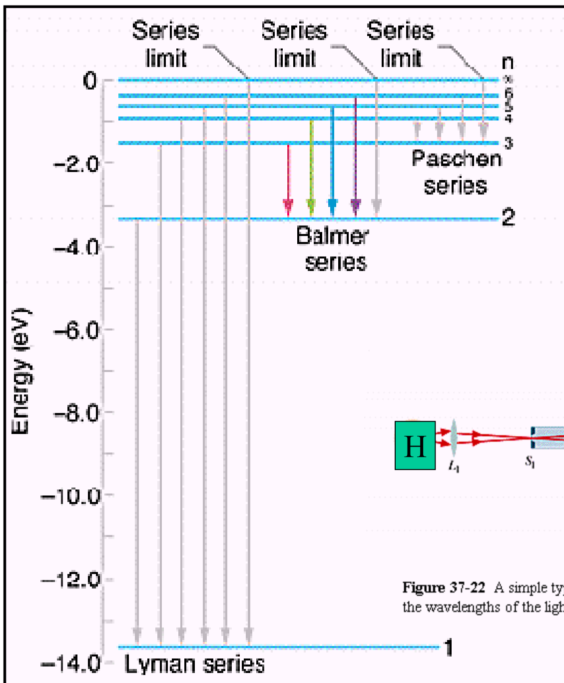


Figure 37-22 A simple type of grating spectroscope used to analyze the wavelengths of the light emitted by source *S*.

Hydrogen emission lines



Larger λ lines (red) are shifted more



Hydrogen atom (Lab 5)

$\Delta E \propto 1/\lambda$

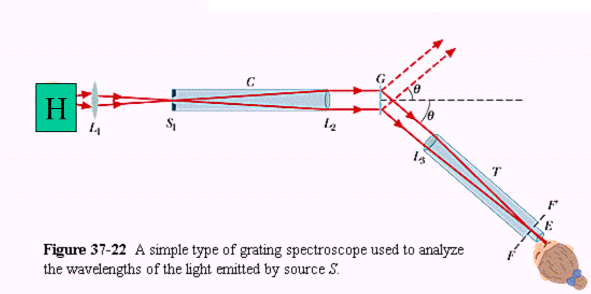
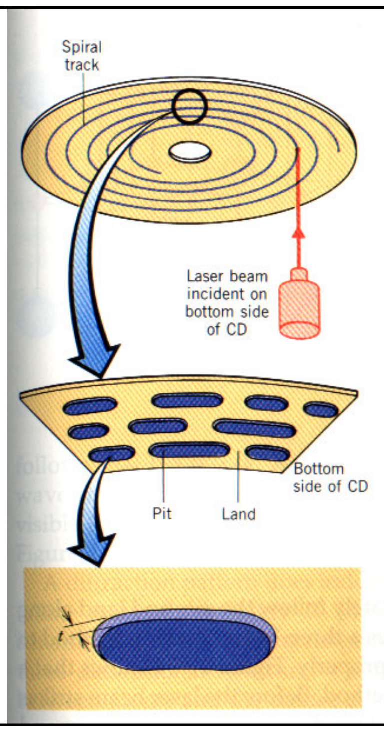
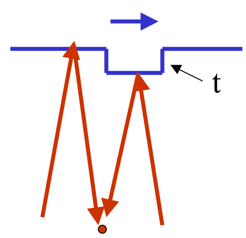


Figure 37-22 A simple type of grating spectroscope used to analyze the wavelengths of the light emitted by source *S*.

Compact disc uses interference effects



λ~790 nm in vacuum
plastic coating has n=1.5

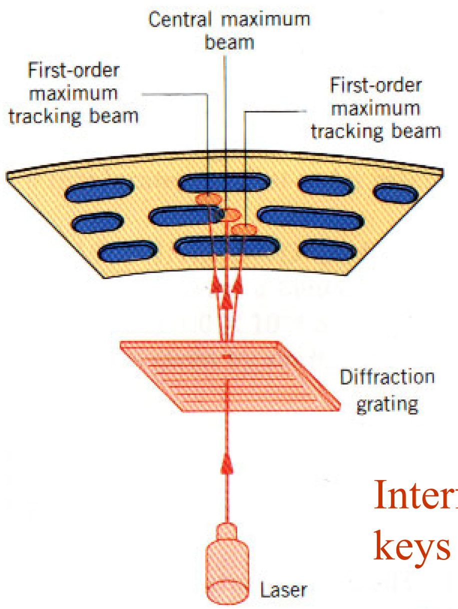


Path difference= $2t = \lambda/2$
produces a minimum => ‘0’

Path difference =0 produces
a maximum => ‘1’

Rotation produces a binary series of 011001

$t = \lambda_{coating}/4 = (790nm)/4(1.5) = .13\mu m$

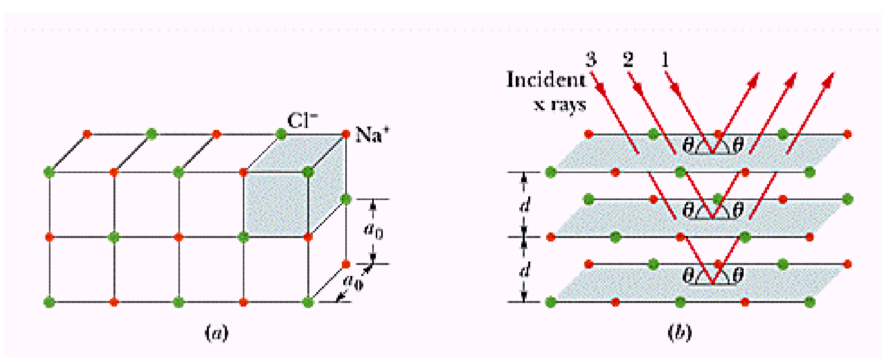


Three beam tracking method

First order diffraction lines keep the central beam aligned

Interference and diffraction are keys elements of a CD player

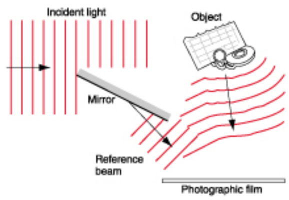
X-ray diffraction

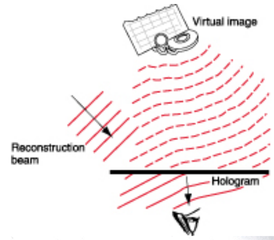


$2d \sin\theta = m\lambda$

Can determine the crystal structure from the diffraction pattern

Holograms





Normal photography records intensity but no information about phase

Must use a reference beam which is coherent to produce an interference pattern

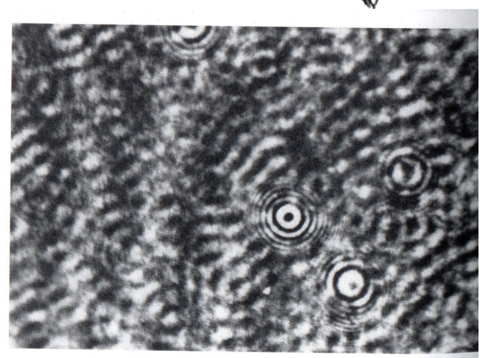


FIGURE 43-21. A close-up view of a hologram, showing the interference pattern.