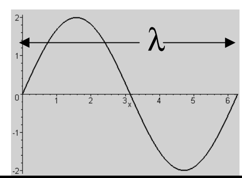
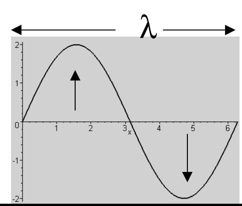
Standing Waves

- $y(x,t) = y_m [\sin(kx-\omega t) + \sin(kx+\omega t)]$
- $\sin A + \sin B = 2 \sin[(A+B)/2] \cos[(A-B)/2]$
- $y(x,t) = [2 y_m \sin(kx)] \cos[-\omega t]$
- amplitude depends on position x
- not of the form f(x-vt) but rather g(x)h(t)
- not a travelling wave!
- Amplitude is zero when $kx=n\pi$, n=0,1,2,…
- but $k= 2\pi/\lambda$ => 'nodes' at $x=n \lambda/2$
- separated by $\lambda/2$

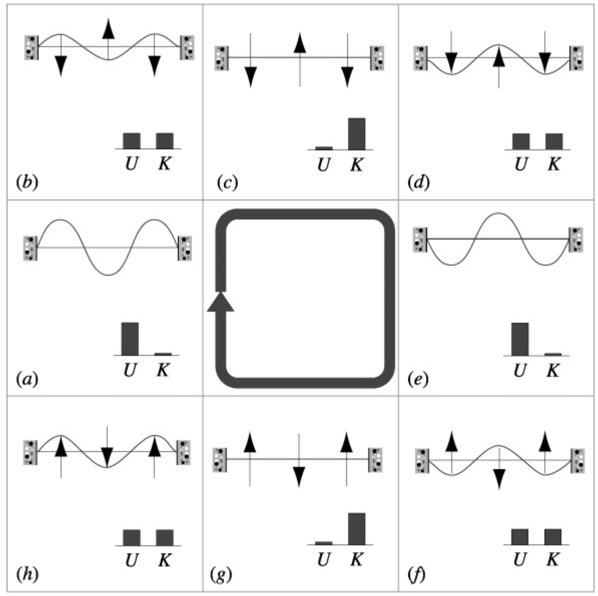


Standing Waves

- $y(x,t) = [2 y_m \sin(kx)] \cos[-\omega t]$
- amplitude is maximum at $kx =(2n+1) \pi/2$
- that is, when $x= (2n+1) \lambda/4$ 'anti-nodes'

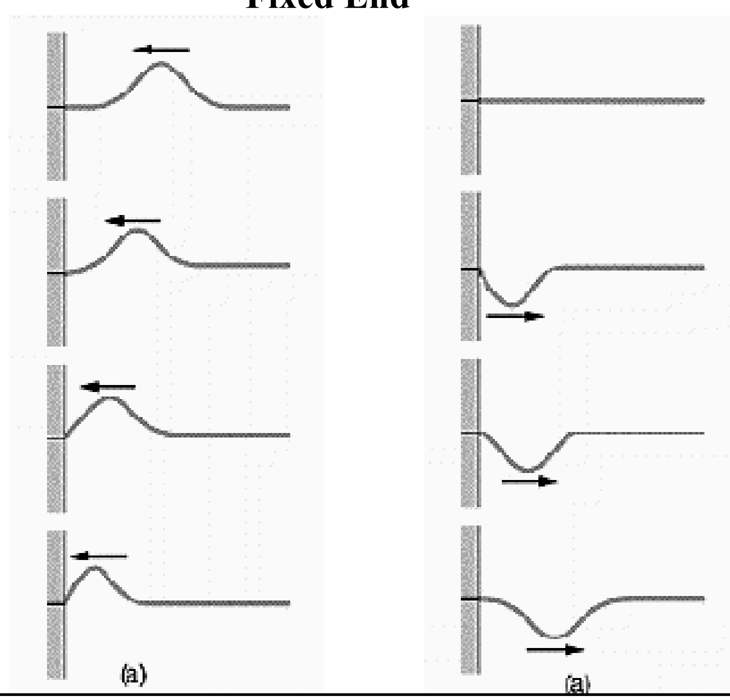


standing

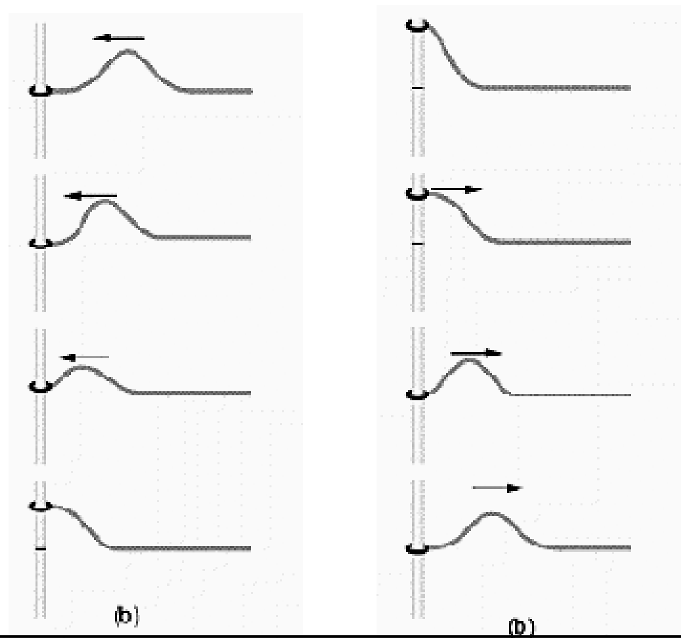


18.18

Fixed End

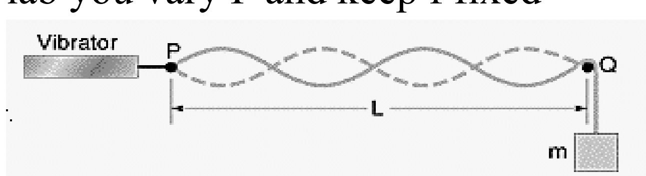


Free End



Resonance

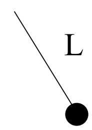
- send a sinusoidal wave down a string with the far end fixed
- frequency f is determined by the source
- wave reflects and interferes with incident wave
- for given tension F and mass density μ the speed is determined by the medium $v=(F/\mu)^{1/2}$
- since $v=\lambda f$, then λ is determined
- in the lab you vary F and keep f fixed



Resonance

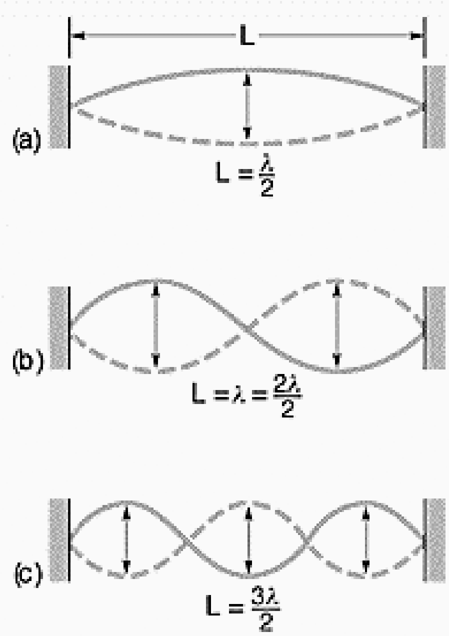
- Consider varying $\omega=2\pi f$ and keep tension fixed
- a standing wave pattern is only produced for certain frequencies called 'resonant' frequencies
- the system has a large response only at these frequencies => a 'resonance'
- e.g. a swing $T= 2\pi(L/g)^{1/2}= 2\pi/\omega$
- 'one' *natural frequency* $\omega= (g/L)^{1/2}$

  "Push" at this frequency



- consider a string instrument with a wire under tension and fixed at both ends

- what are the 'natural' frequencies?



Speed determined by medium
Need to adjust f so that $\lambda=2L$

$f=v/\lambda = v/2L$
'fundamental mode'

$v^2= F/\mu$ => $F = \mu\, 4L^2f^2$

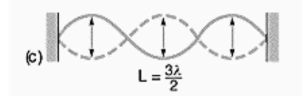
$\lambda=L$ for next stable pattern
second harmonic

$\boxed{\lambda=2L/n}$ in general
n is the number of loops

$f_n = n\, v/2L = n\, f_1$ 'harmonics'

natural frequencies of the wire

Problem



- a guitar string with μ=7.2 g/m has a tension F=150 N. The fixed supports are 90 cm apart and it oscillates in a 3-loop pattern
- what is a) speed b)wavelength c)frequency of the component waves?
- a) $v=(F/\mu)^{1/2} = (150N/7.2x10^{-3}\ kg/m)^{1/2} =$ 144m/s
- b) λ= 2L/3 = 2(90.0cm)/3 = 60 cm
- c) $f=\omega/2\pi = vk/2\pi = v/\lambda$ =(144m/s)/.6m =240 Hz

Suggested Homework

- Chapter 18
- MC: 1, 3, 4, 7, 8, 13, 15
- EX: 1, 3, 6, 10, 11, 15, 17, 20, 22, 26, 28
- P: 2, 4, 8, 14, 20

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