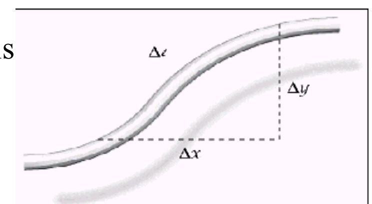
Potential Energy

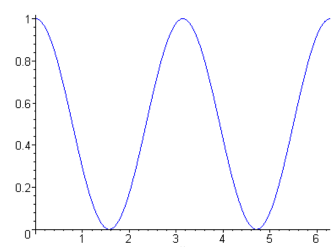
- Length $dl = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + (dy/dx)^2} \approx dx + (1/2)(dy/dx)^2 dx$
- hence $dl - dx = (1/2)\,(dy/dx)^2\,dx$
- $dU = (1/2)\,F\,(dy/dx)^2\,dx$ *potential energy of element dx*

- $y(x,t) = y_m \sin(kx - \omega t)$
- $dy/dx = y_m k \cos(kx - \omega t)$ *keeping t fixed!*
- Since $F = \mu v^2 = \mu\omega^2/k^2$ we find
- $dU = (1/2)\,\mu dx\,\omega^2 y_m^2 \cos^2(kx - \omega t)$
- $dK = (1/2)\,\mu dx\,\omega^2 y_m^2 \cos^2(kx - \omega t)$
- $dE = \mu\omega^2 y_m^2 \cos^2(kx - \omega t)\,dx$
- average of $\cos^2$ over one period is
- $dE_{av} = (1/2)\,\mu\,\omega^2 y_m^2\,dx$



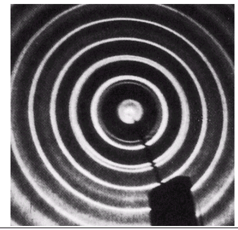
Power and Energy

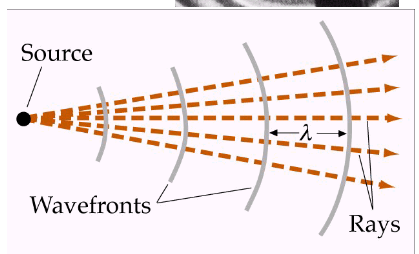
cos²(x)



- $dE_{av} = (1/2)\,\mu\,\omega^2 y_m^2\,dx$
- rate of change of total energy is power P
- average power = $P_{av} = (1/2)\,\mu v\,\omega^2\,y_m^2$
  -depends on medium and source of wave
- general result for *all* waves
- power varies as ω^2 and y_m^2

Waves in Three Dimensions





- Wavelength is distance between successive wave crests
- wavefronts separated by λ
- in three dimensions these are concentric spherical surfaces
- at distance *r* from source, energy is distributed uniformly over area $A=4\pi r^2$
- power/unit area I=P/A is the intensity
- intensity in any direction decreases as $1/r^2$

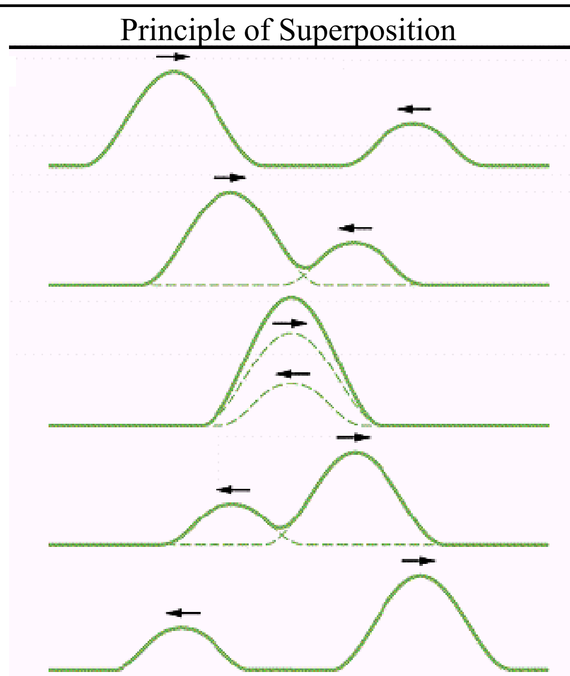
Principle of Superposition of Waves

- What happens when two or more waves pass simultaneously?
- E.g. - Concert has many instruments
  - TV receivers detect many broadcasts
  - a lake with many motor boats
- net displacement is the sum of the that due to individual waves

Superposition

- Let $y_1(x,t)$ and $y_2(x,t)$ be the displacements due to two waves
- at each point x and time t, the net displacement is the algebraic sum

  $y(x,t)= y_1(x,t) + y_2(x,t)$
- <u>Principle of superposition</u>: net effect is the sum of individual effects

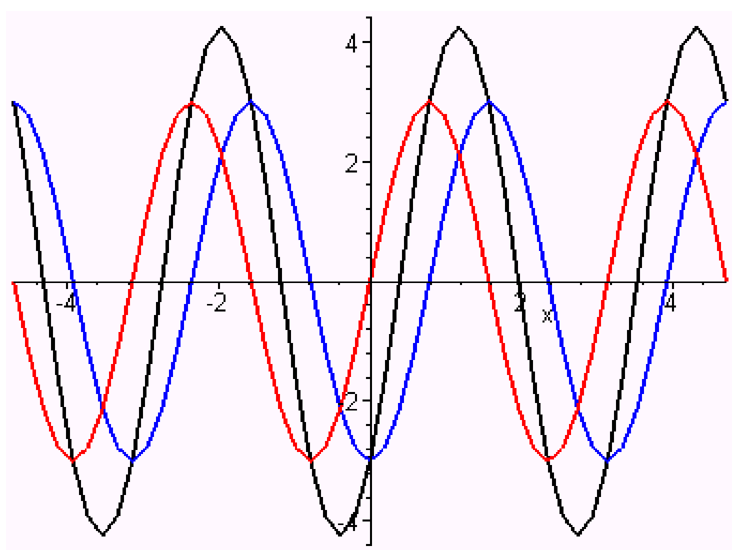


Interference of Waves

- Consider a sinusoidal wave travelling to the right on a stretched string
- $y_1(x,t)=y_m \sin(kx-\omega t)$

  $k=2\pi/\lambda$, $\omega=2\pi/T$, $\omega = v\,k$
- consider a second wave travelling in the same direction with the same wavelength, speed and amplitude but *different phase*
- $y_2(x,t)=y_m \sin(kx- \omega t-\phi)$ $y_2(0,0)=y_m \sin(-\phi)$
- phase shift $-\phi$ corresponds to sliding one wave with respect to the other

interfere



Interference

- $y(x,t) = y_1(x,t) + y_2(x,t)$
- $y(x,t) = y_m [\sin(kx - \omega t - \phi_1) + \sin(kx - \omega t - \phi_2)]$
- $\sin A + \sin B = 2 \sin[(A+B)/2] \cos[(A-B)/2]$
- $y(x,t) = 2 y_m [\sin(kx - \omega t - \phi`)] \cos[-(\phi_1 - \phi_2)/2]$
- $y(x,t) = [2 y_m \cos(\Delta\phi/2)] [\sin(kx - \omega t - \phi`)]$
- result is a sinusoidal wave travelling in same direction with
  ‘amplitude’ $2 y_m |\cos(\Delta\phi/2)|$ $\Delta\phi = \phi_2 - \phi_1$
  ‘phase’ $(kx - \omega t - \phi`)$ $\phi` = (\phi_1 + \phi_2)/2$

Problem

- Two sinusoidal waves, identical except for phase, travel in the same direction and interfere to produce
  $y(x,t) = (3.0\text{mm}) \sin(20x - 4.0t + .820)$
  where x is in metres and t in seconds
- what are a) wavelength b)phase difference and c) amplitude of the two component waves?
- recall $y = y_1 + y_2 = 2y_m \cos(\Delta\phi/2) \sin(kx - \omega t - \phi`)$
- $k = 20 = 2\pi/\lambda \Rightarrow \lambda = 2\pi/20 = .31$ m
- $\omega = 4.0$ rads/s
- $\phi` = (\phi_1 + \phi_2)/2 = -.820 \Rightarrow \Delta\phi = -1.64$ rad $(\phi_1 = 0)$
- $2y_m \cos(\Delta\phi/2) = 3.0\text{mm} \Rightarrow$
  $y_m = |3.0\text{mm}/2\cos(\Delta\phi/2)| = 2.2\text{mm}$

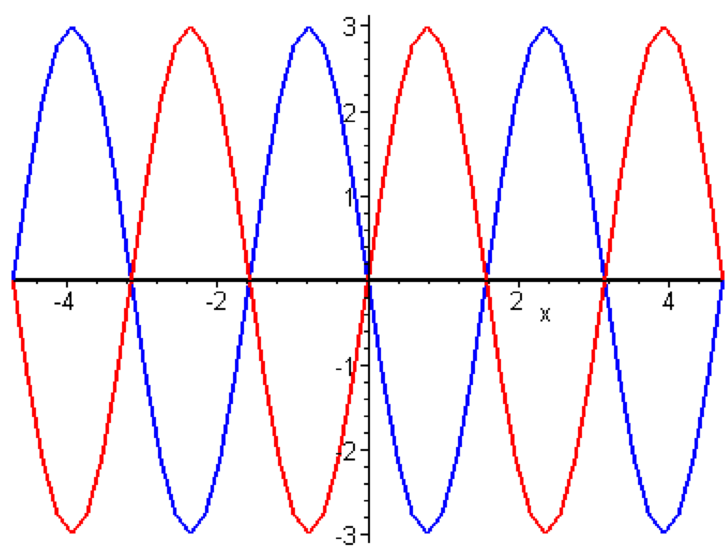
Interference

$$y(x,t) = [2\, y_m \cos(\Delta\phi/2)]\,[\sin(kx - \omega t - \phi`)]$$

- if $\Delta\phi = 0$, waves are in phase and amplitude is doubled
- largest possible => constructive interference

- if $\Delta\phi = \pi$, then $\cos(\Delta\phi/2) = 0$ and waves are exactly out of phase => exact cancellation
- => destructive interference $y(x,t)=0$
- 'nothing' = sum of two waves

nothing

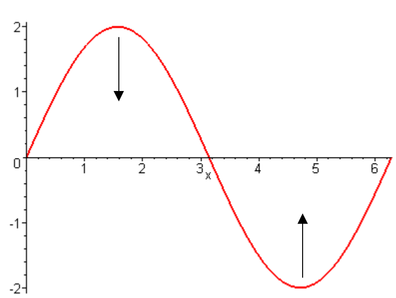


Standing Waves

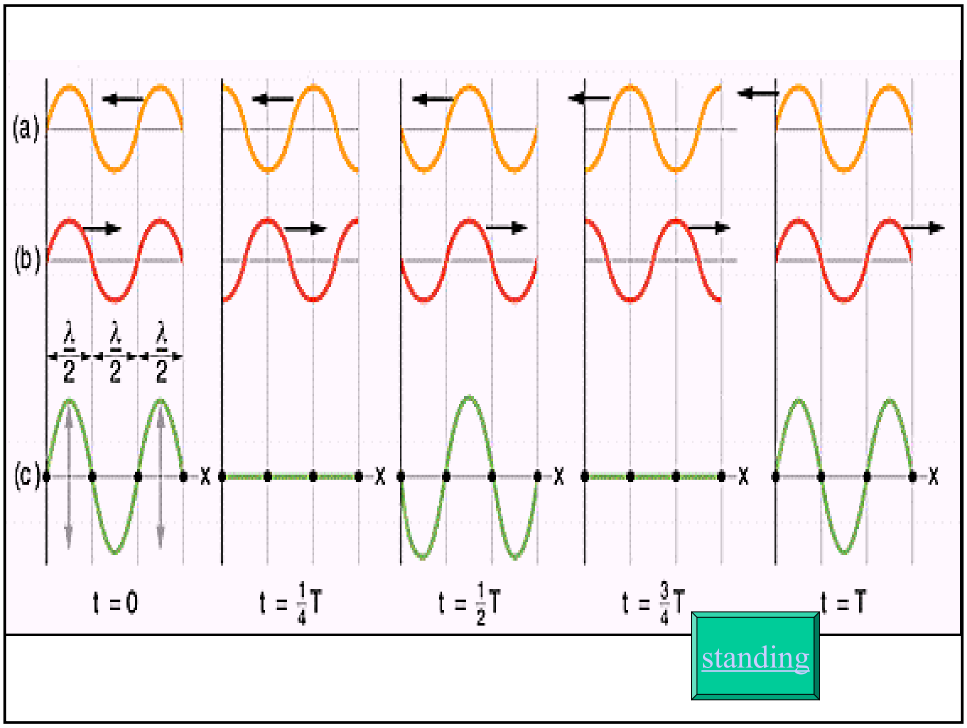
- Consider two sinusoidal waves moving in opposite directions
- $y(x,t)= y_1(x,t) + y_2(x,t)$
- $y(x,t) = y_m [\sin(kx-\omega t) + \sin(kx+\omega t)]$
- at $t=0$, the waves are in phase $y=2y_m \sin(kx)$
- at $t\neq 0$, the waves are out of phase
- phase difference $= (kx+\omega t) - (kx-\omega t) = 2\omega t$
- interfere constructively when $2\omega t= m2\pi$
- hence $t= m2\pi/2\omega = mT/2$ (same as $t=0$)

Standing Waves

- interfere constructively when $2\omega t= m2\pi$
- Destructive interference when
- phase difference$=2\omega t= \pi, 3\pi, 5\pi$, etc.



string is ‘flat’


(a)
(b)
(c)
λ/2
λ/2
λ/2
x
t = 0
t = ¼T
t = ½T
t = ¾T
t = T
standing