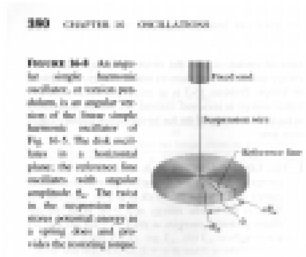
Lecture 3

Angular SHM
Torsion Pendulum

- "Springiness" replaced by "twisting"



Torsion Pendulum

- Restoring torque $\tau = -\kappa\ \theta$ *angular form of*
  'tau' = - 'kappa' 'theta' *Hooke's Law*
- replace F=ma by $\tau = I\alpha$
  I =moment of inertia,α=angular acceleration
- $I(d^2\theta/dt^2) = -\kappa\ \theta$ <==> $m(d^2x/dt^2) = -kx$
- $T=2\ \pi(I/\kappa)^{1/2}$ $T=2\ \pi(m/k)^{1/2}$

Pendula

- Now replace the spring by gravity
- energy is now kinetic and gravitational
- simple pendulum : particle of mass m at the end of a massless, non-elastic string of length L
- consider the forces involved

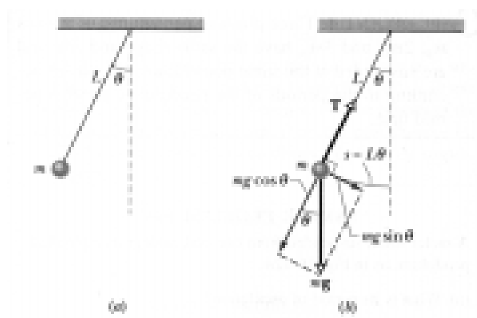
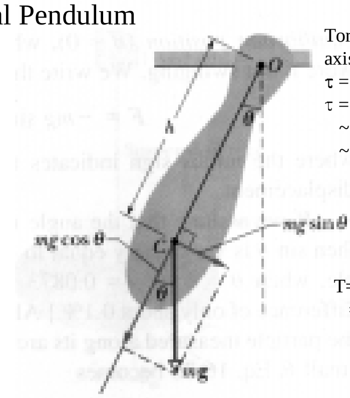


FIGURE 16-10 (*a*) A simple pendulum. (*b*) The forces acting on the bob are its weight *m***g** and the tension **T** in the string. The tangential component *mg* sin *θ* of the weight is a restoring force that brings the pendulum back to the central position.

Simple Pendulum

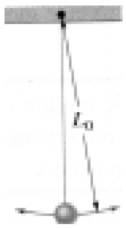
- The net force is $F = -mg\sin\theta$ and is tangential to the path and opposite to the displacement
- $\sin\theta \sim \theta - \theta^3/3 + \ldots$ (θ in radians!)
- displacement along path $s = L\ \theta$
- hence for small θ, $F \sim -mg\theta = -mg\ s/L$
- i.e. $F = -k\ s$ where $k = mg/L$
- ==> SHM for small θ
- Recall $T=2\ \pi(m/k)^{1/2}$ for mass-spring
- here $T=2\ \pi[m/(mg/L)]^{1/2} = 2\ \pi(L/g)^{1/2}$

Physical Pendulum



Torque about axis through O
$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$
$\tau = -(mg\sin\theta)(h)$
$\sim -(mgh)\theta$
$\sim -\kappa\ \theta$

$T=2\pi\ (I/\kappa)^{1/2}$
$=2\pi\ (I/mgh)^{1/2}$



$I = m(L_0)^2$
$h = L_0$

$T = 2\pi\,(I/mgh)^{1/2}$
$= 2\pi\,(L_0/g)^{1/2}$

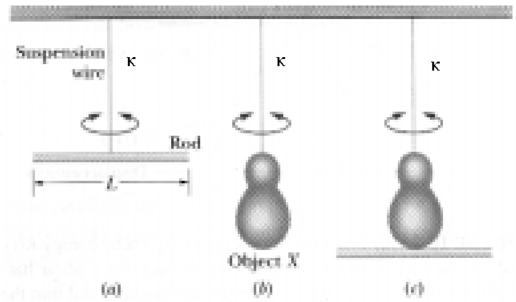
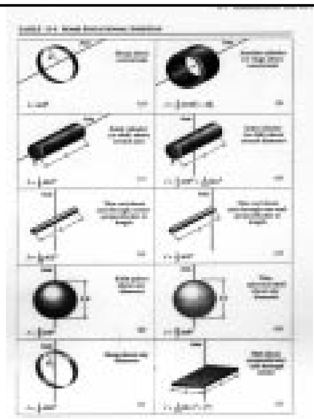


FIGURE 16-9 Sample Problem 16-5. Three torsion pendulums consisting of a wire and (a) a rod, (b) an irregularly shaped object, and (c) the rod and the object rigidly connected.

$I = \Sigma_i\, m_i\, r_i^2$

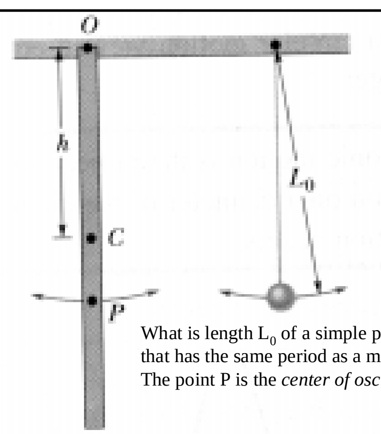


Example

- Length of rod L=12.4 cm and mass m=135g
- T_a is measured to be 2.53 s
- the object in (b) has a period of T_b = 4.76 s
- (1) what is I_b of the object?
- Solution: for (a) $I_a=(1/12)mL^2=1.73\times10^{-4}$ kg.m^2
- $T_a = 2\pi(I_a/\kappa)^{1/2}$ $T_b = 2\pi(I_b/\kappa)^{1/2}$
- since κ is the same(due to wire) we have
  $I_b = I_a\, T_b^2/T_a^2 = (1.73\times10^{-4}$ kg.m$^2)(4.76/2.53)^2$
  $=6.12\times10^{-4}$ kg.m^2

Cont'd

- (2)what is period is both are fastened together as in (c)?
- Since $I_c = I_a + I_b$ we have $T_c=T_a(I_c/I_a)^{1/2}$
- $I_c/I_a = (I_a + I_b)/\,I_a = 1 + I_b/I_a$
- T_c= 5.39 s



What is length L_0 of a simple pendulum that has the same period as a metre stick?
The point P is the *center of oscillation*

Solution

- $T = 2\pi(L_0/g)^{1/2} = 2\pi(I/mgh)^{1/2}$
- Hence $L_0 = I/mh$
- For metre stick $h=L/2$
  and $I = mL^2/3$
- hence $L_0 = 2L/3$ "sweet spot"
- and $T= 2\pi(2L/3g)^{1/2}$

Measuring g

- We can use <u>any</u> physical pendulum to measure 'g'
- For the metre stick, $I= mL^2/3$, $h=L/2$
- $T = 2\pi(I/mgh)^{1/2} = 2\pi(2L/3g)^{1/2}$
- Plot T^2 versus L ==> $T^2 = (8\pi^2/3g)L$