**PHYS 2380 Quantum Physics I**
**Term Test March 5$^{th}$, 2014, 19:00 - 21:00 hrs**
**519 Allen Bldg.**

1. Light of wavelength 350 *nm* strikes a metal plate and photoelectrons are produced moving with a maximum possible speed of 6.0 x 10$^{3}$ *m/s*.

    a. What is the work function of the metal (express your answer in *eV*)? *(5 marks)*

    b. What is the threshold wavelength for photoelectric emission from this metal? *(2 marks)*

2. An electron bound to a hydrogen atom is in the n=5 energy level. Using the Bohr model:

    a. What is the shortest wavelength that can be emitted by the atom as the electron goes to lower energy levels? *(3 marks)*

    b. What is the longest wavelength that can be emitted as the electron goes to lower energy levels? *(3 marks)*

    *c.* How many different wavelengths are potentially observed in the spectrum as this atom de-excites to the ground state? Draw a sketch of the energy levels to illustrate your reasoning. *(3 marks)*

3. A spherical object at a temperature of 1800 *K* and a radius of 10.0 *cm* radiates like a black body.

    a. What is the total power radiated by this object? *(5 marks)*

    b. At what wavelength does the spectral distribution have its maximum value? *(3 marks)*

    c. If the radius of the object is reduced to 5.00 *cm* while keeping the power radiated by the object the same, what is the new temperature of the object? *(6 marks)*

4. An electron is confined to a region of space that is $2.0\times10^{-12}$ *m* wide.

    a. What is the minimum uncertainty in its momentum? *(3 marks)*

    b. What is the minimum kinetic energy that this particle could have? Express your answer in *eV*. *(3 marks)*

    c. If the particle were a neutron instead of an electron what would its minimum kinetic energy be? *(3 marks)*

5. A particle is described by a wave function $\Psi(x,t)=\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)e^{-i\omega t}$ in the region $0\le x\le L$ . It is zero everywhere else.

    a. The potential energy of the particle is zero in this region. Use this wave function and the S. equation to determine ω in terms of *L* and other constants. *(5 marks)*

    b. For the case with n=7, plot the probability density as a function of position? Sketch the probability density for the range $0\le x\le L$ . Clearly show the maxima and minima and their magnitudes and locations on your plot. What is the probability of finding the particle in the range $\frac{4L}{7}\le x\le\frac{5L}{7}$ ? *(5 marks)*

    c. Use the given wave function and the momentum operator, $p_{op}=-i\hbar\frac{\partial}{\partial x}$ , to determine the average momentum of the particle? *(5 marks)*

**The End**

## Appendix: Some information from the text and lectures:

**Special Relativity:**

Relativistic momentum and energy:

$$\vec{p} = \gamma m \vec{v}$$
$$E = \gamma mc^2 = mc^2 + K$$
$$E^2 = c^2 p^2 + m^2 c^4$$

**Electromagnetic radiation:**

Power received by a detector from a wave:

$$P = \left(\frac{1}{\mu_0 c}\right) E_0^2 A \sin^2(kz - \omega t + \phi)$$
$$P_{ave} = \frac{1}{T_0}\int_0^{T_0} P dt = \frac{E_0^2 A}{2\mu_0 c}, \quad I = \frac{P_{ave}}{A} = \frac{E_0^2}{2\mu_0 c}$$

*Where P is the instantaneous power, $P_{ave}$ is the average power delivered to a detector of area A and I is the intensity of the light.*

**Interference and diffraction:**

| Pattern Type | Bright Fringes | Dark Fringes |
|---|---|---|
| Single slit (width *w*) | $\frac{w}{2}\sin\theta = m\lambda$ | $\frac{w}{2}\sin\theta = \left(m + \frac{1}{2}\right)\lambda$ |
| Double slit (spacing *d*) | $d \sin\theta = m\lambda$ | |
| Grating (lines spaced *d* apart) | $d \sin\theta = m\lambda$ | |
| Bragg (layers of atoms *d* apart) | $2d \sin\theta = m\lambda$ | |
| Circular object | | First fringe at $1.22\lambda/d$ |

**Photons and light:**

$$\lambda \nu = c$$
$$E_{ph} = h\nu = cp_{ph}$$
$$p_{ph} = h/\lambda$$

**Photoelectric effect:** $K = h\nu - \phi = eV_s$

*Where K is the kinetic energy of the emitted electrons, $\phi$ is the work function of the material and $V_s$ is the stopping potential.*

**Black body radiation:**

$$I = \sigma T^4$$
$$\lambda_{\max} T = 2.898 \times 10^{-3}\, m \bullet K$$
$$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$
$$R(\lambda) = \frac{c}{4} u(\lambda)$$
$$dI = R(\lambda) d\lambda$$

**Compton Scattering:** $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$

**Bremsstrahlung:** $\lambda_{\min} = \frac{hc}{eV}$

**Wavelike properties of particles:**

**De Broglie wavelength:** $\lambda = h/p$

**Heisenberg uncert. relationships:**

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

**Wave packets:**

$$p = h/\lambda = \hbar k$$

$$\hbar\omega = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$v_{group} = \frac{d\omega}{dk}$$

$$v_{phase} = \frac{\omega}{k}$$

**Schrödinger equation:** $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$

**Time independent:** $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$

**Probability Current:** $S(x,t) = \frac{i\hbar}{2m}\left\{\Psi\frac{\partial\Psi^*}{\partial x} - \Psi^*\frac{\partial\Psi}{\partial x}\right\}$

**Normalization (1-D):** $\int_{-\infty}^{+\infty}\Psi^*\Psi dx = 1$

| **Rutherford Scattering:** | **Bohr model:** |
|---|---|
| $b = \frac{zZ}{2K}\frac{e^2}{4\pi\varepsilon_0}\cot\tfrac{1}{2}\theta$ <br> $\tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(\frac{b^2v^2}{r_{\min}^2}\right) + \frac{e^2}{4\pi\varepsilon_0}\frac{zZ}{r_{\min}}$ <br> $d = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{K}$ | $E_n = -\frac{Z_{eff}^2 me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2} = -\frac{13.6 Z_{eff}^2}{n^2}eV$ <br> $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ <br> $R = R_\infty\left(\frac{1}{1+m/M}\right)$ <br> $R_\infty = \frac{mk^2e^4}{4\pi c\hbar^3}$ <br> - *Where $R_\infty$ = 1.0973732 X $10^7$ $m^{-1}$ is the Rydberg constant* |

**X-rays:**

K-series: $E_{photon} = (13.6eV)\left(\frac{1}{1^2} - \frac{1}{n^2}\right)(Z-1)^2$

L-series: $E_{photon} = (13.6eV)\left(\frac{1}{2^2} - \frac{1}{n^2}\right)(Z-3)^2$

M-series: $E_{photon} = (13.6eV)\left(\frac{1}{3^2} - \frac{1}{n^2}\right)(Z-5)^2$

**Some useful mathematical relations:**

$$\sqrt{\left(1-u^2/c^2\right)} \approx 1-\frac{1}{2}\frac{u^2}{c^2}$$

$$\frac{1}{\sqrt{\left(1-u^2/c^2\right)}} \approx 1+\frac{1}{2}\frac{u^2}{c^2}$$

*For $u^2/c^2 \ll 1$*

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

**Some useful Integrals:**

$$\int \sin^2(u)\,du = \frac{1}{2}\left(u - \sin u \cos u\right)$$

$$\int \sin u \cos u\,du = \frac{1}{2}\sin^2 u$$

$$\int \cos^2 u\,du = \frac{1}{2}\left(u + \sin u \cos u\right)$$

$$\int u \sin^2 u\,du = \frac{u^2}{4} - \frac{u \sin 2u}{4} - \frac{\cos 2u}{8}$$

$$\int u \cos^2 u\,du = \frac{u^2}{4} + \frac{u \sin 2u}{4} + \frac{\cos 2u}{8}$$

$$\int u^2 \sin^2 u\,du = \frac{u^3}{6} - \left(\frac{u^2}{4} - \frac{1}{8}\right)\sin 2u - \frac{u \cos 2u}{4}$$

$$\int u^2 \cos^2 u\,du = \frac{u^3}{6} + \left(\frac{u^2}{4} - \frac{1}{8}\right)\sin 2u + \frac{u \cos 2u}{4}$$

$$\int_0^\infty u^n e^{-u}\,du = n! \quad for\ n > 0$$

$$\int \cos^n u \sin u\,du = -\frac{\cos^{n+1} u}{n+1} \quad for\ n > 0$$

$$\int \sin^n u \cos u\,du = \frac{\sin^{n+1} u}{n+1} \quad for\ n > 0$$

**Constants:**

| **Constant** | **Standard value** | **Alternate units** |
|---|---|---|
| Speed of light | $c = 2.998 \times 10^{8} m/s$ | |
| Electronic charge | $e = 1.602 \times 10^{-19} C$ | |
| Boltzmann constant | $k = 1.381 \times 10^{-23} J/K$ | $8.617 \times 10^{-5} eV/K$ |
| Planck's constant | $h = 6.626 \times 10^{-34} J \cdot s$ | $4.136 \times 10^{-15} eV \cdot s$ |
| | $\hbar = 1.055 \times 10^{-34} J \cdot s$ | $0.652 \times 10^{-15} eV \cdot s$ |
| Avogadro's constant | $N_A = 6.022 \times 10^{23} mole^{-1}$ | |
| Stefan-Boltzmann constant | $\sigma = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4$ | |
| Electron mass | $m_e = 5.49 \times 10^{-4} u\ or\ 9.11 \times 10^{-31}\ kg$ | $0.511 MeV/c^2$ |
| Proton mass | $1.007276\ u\ or\ 1.67262171 \times 10^{-27}\ kg$ | $938.3 MeV/c^2$ |
| Neutron mass | $1.008665\ u\ or\ 1.67492728 \times 10^{-27}\ kg$ | $939.6 MeV/c^2$ |
| Mass of $^4$He | *4.002603 u* | |
| Bohr radius | $a_0 = 4\pi\varepsilon_0\hbar^2 / m_e e^2 = 0.0529 nm$ | |
| Hydrogen ionization energy | $13.6 eV$ | |
| | $hc = 1240 ev \cdot nm$ | |
| | $1 eV = 1.602 \times 10^{-19} J$ | |
| Atomic mass unit (dalton) | $1u = 931.5 MeV/c^2$ | $1.661 \times 10^{-27} kg$ |
| | $kT = 0.02525 eV \approx \frac{1}{40} eV$ at T=293 K | |
| Coulomb constant | $\frac{1}{4\pi\varepsilon_0} = 8.988 \times 10^{9} N \cdot m^2 \cdot C^{-2}$ | |
| | $\frac{e^2}{4\pi\varepsilon_0} = 1.44 eV \cdot nm$ | |