PHYS 2380 Quantum Physics I
Term Test March 5th, 2013, 19:00 - 21:00 hrs
519 Allen Bldg.

1. An electron bound to a hydrogen atom is in the n=5 energy level. Using the Bohr model:

    a. Find the ionization energy for this ion. (3 marks)

    b. What is the shortest wavelength that can be emitted by transitions as the electron goes to lower energy levels? (3 marks)

    c. What is the longest wavelength that can be emitted as the electron goes to lower energy levels? (3 marks)

2. Two stars, A and B, radiate energy like black bodies. The wavelength (λ_A) where the radiation is a maximum for star A is twice the wavelength (λ_B) where the radiation is a maximum for star B.

    a. What is the ratio of the temperatures of these stars, T_A/T_B? (3 marks)

    b. What is the ratio of their radii if they both radiate the same total amount of energy? (5 marks)

3. A neutron is confined to a region of space that is 2.0×10^{-12} m wide.

    a. What is the minimum uncertainty in its momentum? (3 marks)

    b. What is the minimum kinetic energy that this neutron could have? Express your answer in eV. (3 marks)

    c. If the particle were an electron instead of a neutron what would its minimum kinetic energy be? (3 marks)

4. A particle is described by a wave function $\Psi(x,t) = A\sin\left(\frac{3\pi x}{L}\right)e^{-i\omega t}$ in the region $0 \le x \le L$. It is zero everywhere else.

    a. Show that $A = \sqrt{\frac{2}{L}}$ is required to normalize the wave function. (5 marks)

    b. The potential energy of the particle is zero in this region, use the S. equation to determine ω in terms of *L* and other constants. (5 marks)

    c. What is the probability density as a function of position? (3 marks)

    d. Sketch the probability density for the range $0 \le x \le L$. Clearly show the maxima and minima and their locations. (3 marks)

    e. What is the probability of finding the particle in the range $\frac{L}{6} \le x \le \frac{L}{3}$ (3 marks)

    f. Use the given wavefunction to determine the average position of the particle? (5 marks)

The End

Appendix: Some information from the text and lectures:

Special Relativity:

Relativistic momentum and energy:

$$\vec{p} = \gamma m \vec{v}$$
$$E = \gamma mc^2 = mc^2 + K$$
$$E^2 = c^2 p^2 + m^2 c^4$$

Electromagnetic radiation:

Power received by a detector from a wave:

$$P = \left(\frac{1}{\mu_0 c}\right) E_0^2 A \sin^2(kz - \omega t + \phi)$$
$$P_{ave} = \frac{1}{T_0}\int_0^{T_0} P dt = \frac{E_0^2 A}{2\mu_0 c}, \quad I = \frac{P_{ave}}{A} = \frac{E_0^2}{2\mu_0 c}$$

Where P is the instantaneous power, P_{ave} is the average power delivered to a detector of area A and I is the intensity of the light.

Interference and diffraction:

Pattern Type	Bright Fringes	Dark Fringes
Single slit (width *w*)	$\frac{w}{2}\sin\theta = m\lambda$	$\frac{w}{2}\sin\theta = \left(m + \frac{1}{2}\right)\lambda$
Double slit (spacing *d*)	$d\sin\theta = m\lambda$	
Grating (lines spaced *d* apart)	$d\sin\theta = m\lambda$	
Bragg (layers of atoms *d* apart)	$2d\sin\theta = m\lambda$	
Circular object		First fringe at $1.22\lambda/d$

Photons and light:

$$\lambda\nu = c$$
$$E_{ph} = h\nu = cp_{ph}$$
$$p_{ph} = h/\lambda$$

Photoelectric effect: $K = h\nu - \phi = eV_s$

Where K is the kinetic energy of the emitted electrons, ϕ is the work function of the material and V_s is the stopping potential.

Black body radiation:

$$I = \sigma T^4$$
$$\lambda_{\max} T = 2.898\times10^{-3}\, m \bullet K$$
$$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$
$$R(\lambda) = \frac{c}{4} u(\lambda)$$
$$dI = R(\lambda) d\lambda$$

Compton Scattering: $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$

Bremsstrahlung: $\lambda_{\min} = \frac{hc}{eV}$

Wavelike properties of particles:

De Broglie wavelength: $\lambda = h/p$

Heisenberg uncert. relationships:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

Wave packets:

$$p = h/\lambda = \hbar k$$

$$\hbar\omega = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$v_{group} = \frac{d\omega}{dk}$$

$$v_{phase} = \frac{\omega}{k}$$

Schrödinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$

Time independent: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$

Probability Current: $S(x,t) = \frac{i\hbar}{2m}\left\{\Psi\frac{\partial\Psi^*}{\partial x} - \Psi^*\frac{\partial\Psi}{\partial x}\right\}$

Normalization (1-D): $\int_{-\infty}^{+\infty}\Psi^*\Psi dx = 1$

Rutherford Scattering:	**Bohr model:**
$b = \frac{zZ}{2K}\frac{e^2}{4\pi\varepsilon_0}\cot\frac{1}{2}\theta$ $\frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{b^2v^2}{r_{\min}^2}\right) + \frac{e^2}{4\pi\varepsilon_0}\frac{zZ}{r_{\min}}$ $d = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{K}$	$E_n = -\frac{Z_{eff}^2 me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2} = -\frac{13.6Z_{eff}^2}{n^2}eV$ $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ $R = R_\infty\left(\frac{1}{1+m/M}\right)$ $R_\infty = \frac{mk^2e^4}{4\pi c\hbar^3}$ - *Where* R_∞ *= 1.0973732 X 10⁷ m⁻¹ is the Rydberg constant*

X-rays:

K-series: $E_{photon} = (13.6eV)\left(\frac{1}{1^2} - \frac{1}{n^2}\right)(Z-1)^2$

L-series: $E_{photon} = (13.6eV)\left(\frac{1}{2^2} - \frac{1}{n^2}\right)(Z-3)^2$

M-series: $E_{photon} = (13.6eV)\left(\frac{1}{3^2} - \frac{1}{n^2}\right)(Z-5)^2$

Some useful mathematical relations:

$$\sqrt{\left(1-u^2/c^2\right)} \approx 1-\frac{1}{2}\frac{u^2}{c^2}$$

$$\frac{1}{\sqrt{\left(1-u^2/c^2\right)}} \approx 1+\frac{1}{2}\frac{u^2}{c^2}$$

For $u^2/c^2 << 1$

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Some useful Integrals:

$$\int \sin^2(u)\,du = \frac{1}{2}\left(u - \sin u \cos u\right)$$

$$\int \sin u \cos u\,du = \frac{1}{2}\sin^2 u$$

$$\int \cos^2 u\,du = \frac{1}{2}\left(u + \sin u \cos u\right)$$

$$\int u \sin^2 u\,du = \frac{u^2}{4} - \frac{u \sin 2u}{4} - \frac{\cos 2u}{8}$$

$$\int u \cos^2 u\,du = \frac{u^2}{4} + \frac{u \sin 2u}{4} + \frac{\cos 2u}{8}$$

$$\int u^2 \sin^2 u\,du = \frac{u^3}{6} - \left(\frac{u^2}{4} - \frac{1}{8}\right)\sin 2u - \frac{u \cos 2u}{4}$$

$$\int u^2 \cos^2 u\,du = \frac{u^3}{6} + \left(\frac{u^2}{4} - \frac{1}{8}\right)\sin 2u + \frac{u \cos 2u}{4}$$

$$\int_0^\infty u^n e^{-u}\,du = n! \quad for\ n > 0$$

$$\int \cos^n u \sin u\,du = -\frac{\cos^{n+1} u}{n+1} \quad for\ n > 0$$

$$\int \sin^n u \cos u\,du = \frac{\sin^{n+1} u}{n+1} \quad for\ n > 0$$

Constants:

Constant	**Standard value**	**Alternate units**
Speed of light	$c = 2.998\times10^{8} m/s$	
Electronic charge	$e = 1.602\times10^{-19} C$	
Boltzmann constant	$k = 1.381\times10^{-23} J/K$	$8.617\times10^{-5} eV/K$
Planck's constant	$h = 6.626\times10^{-34} J\cdot s$	$4.136\times10^{-15} eV\cdot s$
	$\hbar = 1.055\times10^{-34} J\cdot s$	$0.652\times10^{-15} eV\cdot s$
Avogadro's constant	$N_A = 6.022\times10^{23} mole^{-1}$	
Stefan-Boltzmann constant	$\sigma = 5.67\times10^{-8}\ W/m^2\cdot K^4$	
Electron mass	$m_e = 5.49\times10^{-4} u\ or\ 9.11\times10^{-31} kg$	$0.511 MeV/c^2$
Proton mass	$1.007276\ u\ or\ 1.67262171\times10^{-27}\ kg$	$938.3 MeV/c^2$
Neutron mass	$1.008665\ u\ or\ 1.67492728\times10^{-27}\ kg$	$939.6 MeV/c^2$
Mass of ^{4}He	*4.002603 u*	
Bohr radius	$a_0 = 4\pi\varepsilon_0\hbar^2 / m_e e^2 = 0.0529 nm$	
Hydrogen ionization energy	$13.6 eV$	
	$hc = 1240 ev\cdot nm$	
	$1 eV = 1.602\times10^{-19} J$	
Atomic mass unit (dalton)	$1u = 931.5 MeV/c^2$	$1.661\times10^{-27} kg$
	$kT = 0.02525 eV \approx \frac{1}{40} eV$ at T=293 K	
Coulomb constant	$\frac{1}{4\pi\varepsilon_0} = 8.988\times10^{9} N\cdot m^2\cdot C^{-2}$	
	$\frac{e^2}{4\pi\varepsilon_0} = 1.44 eV\cdot nm$	