PHYS 2380 Quantum Physics I

Term Test March 6th, 2012, 19:00 - 21:00 hrs

519 Allen Bldg.

1. The probability distribution of energies for atoms of a gas at temperature T is given by:

   $$P(E) = Ce^{-E/kT}$$

   Where *k* is Boltzmann's constant, *E* is the energy of the particle and *T* is the temperature in kelvin. The probability of finding a particle with an energy between *E* and *E+dE* is given by *P(E)dE.*

   a. Determine the constant *C* by normalizing *P(E).* (5 marks)

   b. Use *P(E)* to find the average energy for the atoms in the gas. (5 marks)

2. A spherical object at a temperature of 1800 K and a radius of 10.0 cm radiates like a black body.

   a. What is the total power radiated by this object? (5 marks)

   b. If the radius of the object is reduced to 5.00 cm while keeping the power radiated by the object the same, what is the new temperature of the object? (5 marks)

   c. At what wavelength does the new spectral distribution have its maximum value? (5 marks)

3. A neutron is confined to a region of space that is 2.0×10^{-12} m wide.

   a. What is the minimum uncertainty in its momentum? (5 marks)

   b. What is the minimum kinetic energy that this electron could have? Express your answer in eV. (5 marks)

4. A particle is described by a wave function $\psi(x) = A\sin\left(\frac{4\pi x}{L}\right)e^{-i\omega t}$ in the region $0 \le x \le L$. It is zero everywhere else.

   a. Show that $A = \sqrt{\frac{2}{L}}$ is required to normalize the wave function. (5 marks)

   b. Given that the potential energy of the particle is zero in this region, determine ω in terms of *L* and other constants. (2 marks)

   c. What is the probability density as a function of position? (5 marks)

   d. Sketch the probability density for the range $0 \le x \le L$. Clearly show the maxima and minima and their locations. (5 marks)

   e. What is the probability of finding the particle in the range $\frac{3}{8}L \le x \le \frac{5}{8}L$ (5 marks)

   f. What is the average position of the particle? (5 marks)

The End

Appendix: Some information from the text and lectures:

Special Relativity:

Relativistic momentum and energy:

$$\vec{p} = \gamma m\vec{v}$$
$$E = \gamma mc^2 = mc^2 + K$$
$$E^2 = c^2p^2 + m^2c^4$$

Electromagnetic radiation:

Power received by a detector from a wave:

$$P = \left(\frac{1}{\mu_0 c}\right)E_0^2 A\sin^2(kz - \omega t + \phi)$$
$$P_{ave} = \frac{1}{T_0}\int_0^{T_0} P\,dt = \frac{E_0^2 A}{2\mu_0 c}, \quad I = \frac{P_{ave}}{A} = \frac{E_0^2}{2\mu_0 c}$$

Where P is the instantaneous power, P_{ave} is the average power delivered to a detector of area A and I is the intensity of the light.

Interference and diffraction:

Pattern Type	Bright Fringes	Dark Fringes
Single slit (width *w*)	$\frac{w}{2}\sin\theta = m\lambda$	$\frac{w}{2}\sin\theta = \left(m + \frac{1}{2}\right)\lambda$
Double slit (spacing *d*)	$d\sin\theta = m\lambda$	
Grating (lines spaced *d* apart)	$d\sin\theta = m\lambda$	
Bragg (layers of atoms *d* apart)	$2d\sin\theta = m\lambda$	
Circular object		First fringe at $1.22\lambda/d$

Photons and light:

$$\lambda\nu = c$$
$$E_{ph} = h\nu = cp_{ph}$$
$$p_{ph} = h/\lambda$$

Where K is the kinetic energy of the emitted electrons, ϕ is the work function of the material and V_s is the stopping potential.

Photoelectric effect: $K = h\nu - \phi = eV_s$

Black body radiation:

$$I = \sigma T^4$$
$$\lambda_{\max}T = 2.898\times10^{-3}\,m\bullet K$$
$$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$
$$R(\lambda) = \frac{c}{4}u(\lambda)$$
$$dI = R(\lambda)d\lambda$$

Compton Scattering:

$$\lambda' - \lambda = \frac{h}{m_e c}\left(1 - \cos\theta\right)$$

Bremsstrahlung: $\lambda_{\min} = \frac{hc}{eV}$

Wavelike properties of particles:

De Broglie wavelength: $\lambda = h/p$

Heisenberg uncert. relationships:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

Wave packets:

$$p = h/\lambda = \hbar k$$

$$\hbar\omega = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$v_{group} = \frac{d\omega}{dk}$$

$$v_{phase} = \frac{\omega}{k}$$

Schrödinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$

Time independent: $-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$

Probability Current: $S(x,t) = \frac{i\hbar}{2m}\left\{\Psi\frac{\partial \Psi^*}{\partial x} - \Psi^*\frac{\partial \Psi}{\partial x}\right\}$

Normalization (1-D): $\int_{-\infty}^{+\infty} \Psi^* \Psi dx = 1$

Nuclear Physics:

$$R = R_0 A^{1/3} = 1.2 A^{1/3}\, fm$$

$$B = \left[Zm\left({}^1_1H\right) + Nm_n - m\left({}^A_zX\right)\right]c^2$$

$$Q = [M_{parent} - M_{Daughter} - M_{emitted}]c^2$$

$$N = N_0 e^{-\lambda t}$$

$$A = \lambda N$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Rutherford Scattering:

$$b = \frac{zZ}{2K}\frac{e^2}{4\pi\varepsilon_0}\cot\tfrac{1}{2}\theta$$

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(\frac{b^2v^2}{r_{\min}^2}\right) + \frac{e^2}{4\pi\varepsilon_0}\frac{zZ}{r_{\min}}$$

$$d = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{K}$$

Bohr model:

$$E_n = -\frac{Z_{eff}^2 me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2} = -\frac{13.6Z_{eff}^2}{n^2}eV$$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$R = R_\infty\left(\frac{1}{1 + m/M}\right)$$

$$R_\infty = \frac{mk^2e^4}{4\pi c\hbar^3}$$

- *Where R_∞ = 1.0973732 X 10^7 m^{-1} is the Rydberg constant*

X-rays:

K-series: $E_{photon} = (13.6eV)\left(\frac{1}{1^2} - \frac{1}{n^2}\right)(Z-1)^2$

L-series: $E_{photon} = (13.6eV)\left(\frac{1}{2^2} - \frac{1}{n^2}\right)(Z-3)^2$

M-series: $E_{photon} = (13.6eV)\left(\frac{1}{3^2} - \frac{1}{n^2}\right)(Z-5)^2$

Some useful mathematical relations:

$$\sqrt{\left(1 - u^2/c^2\right)} \approx 1 - \frac{1}{2}\frac{u^2}{c^2}$$

$$\frac{1}{\sqrt{\left(1 - u^2/c^2\right)}} \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$$

For u^2/c^2 <<1

$$\sin(A+B)=\sin(A)\cos(B)+\cos(A)\sin(B)$$

$$\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$$

$$\sin(2A)=2\sin(A)\cos(A)$$

$$e^{x}=1+x+\frac{x^{2}}{2!}+\frac{x^{3}}{3!}+\cdots$$

Some useful Integrals:

$$\int \sin^{2}(u)\,du=\frac{1}{2}\left(u-\sin u\cos u\right)$$

$$\int \sin u\cos u\,du=\frac{1}{2}\sin^{2}u$$

$$\int \cos^{2}u\,du=\frac{1}{2}\left(u+\sin u\cos u\right)$$

$$\int u\sin^{2}u\,du=\frac{u^{2}}{4}-\frac{u\sin 2u}{4}-\frac{\cos 2u}{8}$$

$$\int u\cos^{2}u\,du=\frac{u^{2}}{4}+\frac{u\sin 2u}{4}+\frac{\cos 2u}{8}$$

$$\int u^{2}\sin^{2}u\,du=\frac{u^{3}}{6}-\left(\frac{u^{2}}{4}-\frac{1}{8}\right)\sin 2u-\frac{u\cos 2u}{4}$$

$$\int u^{2}\cos^{2}u\,du=\frac{u^{3}}{6}+\left(\frac{u^{2}}{4}-\frac{1}{8}\right)\sin 2u+\frac{u\cos 2u}{4}$$

$$\int_{0}^{\infty} u^{n}e^{-u}\,du=n!\quad for\ \ n>0$$

$$\int \cos^{n}u\sin u\,du=-\frac{\cos^{n+1}u}{n+1}\quad for\ \ n>0$$

$$\int \sin^{n}u\cos u\,du=\frac{\sin^{n+1}u}{n+1}\quad for\ \ n>0$$

Constants:

Constant	Standard value	Alternate units
Speed of light	$c = 2.998\times10^{8} m/s$	
Electronic charge	$e = 1.602\times10^{-19} C$	
Boltzmann constant	$k = 1.381\times10^{-23} J/K$	$8.617\times10^{-5} eV/K$
Planck's constant	$h = 6.626\times10^{-34} J\cdot s$	$4.136\times10^{-15} eV\cdot s$
	$\hbar = 1.055\times10^{-34} J\cdot s$	$0.652\times10^{-15} eV\cdot s$
Avogadro's constant	$N_A = 6.022\times10^{23} mole^{-1}$	
Stefan-Boltzmann constant	$\sigma = 5.67\times10^{-8}\ W/m^2\cdot K^4$	
Electron mass	$m_e = 5.49\times10^{-4} u\ or\ 9.11\times10^{-31} kg$	$0.511 MeV/c^2$
Proton mass	$1.007276\ u\ or\ 1.67262171\times10^{-27}\ kg$	$938.3 MeV/c^2$
Neutron mass	$1.008665\ u\ or\ 1.67492728\times10-27\, kg$	$939.6 MeV/c^2$
Mass of ^{4}He	*4.002603 u*	
Bohr radius	$a_0 = 4\pi\varepsilon_0\hbar^2 / m_e e^2 = 0.0529 nm$	
Hydrogen ionization energy	$13.6 eV$	
	$hc = 1240 ev\cdot nm$	
	$1eV = 1.602\times10^{-19} J$	
Atomic mass unit (dalton)	$1u = 931.5 MeV/c^2$	$1.661\times10^{-27} kg$
	$kT = 0.02525 eV \approx \frac{1}{40} eV$ at T=293 K	
	$\frac{1}{4\pi\varepsilon_0} = 8.988\times10^{9} N\cdot m^2\cdot C^{-2}$	
	$\frac{e^2}{4\pi\varepsilon_0} = 1.44 eV\cdot nm$	