PHYS 2380 Quantum Physics I
Term Test March 10th , 2011, 19:00 hrs to 21:00 hrs
403 Allen Bldg.

1. The Maxwell-Boltzmann distribution of velocities for atoms of a gas at temperature T is given by:

   $$h(v) = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$$

   Where N is the total number of particles in the system, m is the mass of an individual particle, k is Boltzmann's costant, v is the velocity of the particle and T is the temperature in Kelvins. The number of particles with a velocity between *v* and *v+dv* is given by *h(v)dv*. Use this definition of *h(v)* to find the most probable velocity for the atoms in the gas. (5 points)

2. A radiating cavity has the maximum of its spectral distribution of radiated power at a wavelength of 27.0 μm. The temperature of the cavity is then changed so that the total power radiated by the cavity doubles.

   a. What was the original temperature of the cavity? (2 marks)

   b. What is the new temperature of the cavity? (2 marks)

   c. At what wavelength does the new spectral distribution have its maximum value? (2 marks)

3. The work function for molybdenum (Mo) is 4.22 *eV*.

   a. Find the threshold wavelength and frequency for the photoelectric effect for light incident on a Mo surface. (3 points)

   b. What is the maximum energy of electrons (in *eV*) emitted from a Mo surface when it is struck by photons with a wavelength of 180 *nm*? (2 points)

4. The series of hydrogen spectral lines from transitions terminating on the n = 4 level from levels above is called the Brackett series.

   a. What are the two longest wavelengths in the series? (3 points)

   b. What is the shortest wavelength in this series? (2 points)

5. An electron is confined to a region of space that is 20 *nm* wide.

   a. What is the minimum uncertainty in its momentum? (2 points)

   b. What is the minimum kinetic energy that this electron could have? (3 points)

6. A particle is described by a wave function $\psi(x) = A\sin\left(\frac{5\pi x}{L}\right)e^{-i\omega t}$ in the region $0 \le x \le L$. It is zero everywhere else.

   a. Show that $A = \sqrt{\frac{2}{L}}$ is required to normalize the wave function. (3 points)

   b. What is the probability density as a function of position? Sketch the probability density for the range $0 \le x \le L$. Clearly show the maxima and minima and their locations. (3 points)

   c. Use the wavefunction to find the average position of the electron, $\bar{x}$. (3 points)

   d. Use this wave function and the momentum operator, $p_{op} = -i\hbar\frac{\partial}{\partial x}$, to find the average momentum of the particle, $\bar{p}$. (3 points)

The End

Appendix: Some information from the text and lectures:

Special Relativity:

Relativistic momentum and energy:

$$\vec{p} = \gamma m \vec{v}$$
$$E = \gamma mc^2 = mc^2 + K$$
$$E^2 = c^2 p^2 + m^2 c^4$$

Electromagnetic radiation:

Power received by a detector from a wave:

$$P = \left(\frac{1}{\mu_0 c}\right) E_0^2 A \sin^2(kz - \omega t + \phi)$$
$$P_{ave} = \frac{1}{T_0}\int_0^{T_0} P dt = \frac{E_0^2 A}{2\mu_0 c}, \quad I = \frac{P_{ave}}{A} = \frac{E_0^2}{2\mu_0 c}$$

Where P is the instantaneous power, P_{ave} is the average power delivered to a detector of area A and I is the intensity of the light.

Interference and diffraction:

Pattern Type	Bright Fringes	Dark Fringes
Single slit (width *w*)	$\frac{w}{2}\sin\theta = m\lambda$	$\frac{w}{2}\sin\theta = \left(m + \frac{1}{2}\right)\lambda$
Double slit (spacing *d*)	$d\sin\theta = m\lambda$	
Grating (lines spaced *d* apart)	$d\sin\theta = m\lambda$	
Bragg (layers of atoms *d* apart)	$2d\sin\theta = m\lambda$	
Circular object		First fringe at $1.22\lambda/d$

Photons and light:

$$\lambda \nu = c$$
$$E_{ph} = h\nu = cp_{ph}$$
$$p_{ph} = h/\lambda$$

Where K is the kinetic energy of the emitted electrons, ϕ is the work function of the material and V_s is the stopping potential.

Photoelectric effect: $K = h\nu - \phi = eV_s$

Black body radiation:

$$I = \sigma T^4$$
$$\lambda_{\max} T = 2.898 \times 10^{-3} m \bullet K$$
$$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$
$$R(\lambda) = \frac{c}{4} u(\lambda)$$
$$dI = R(\lambda) d\lambda$$

Compton Scattering:

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

Bremsstrahlung: $\lambda_{\min} = \frac{hc}{eV}$

Wavelike properties of particles:

De Broglie wavelength: $\lambda = h/p$

Heisenberg uncert. relationships:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

Wave packets:

$$p = h/\lambda = \hbar k$$

$$\hbar\omega = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$v_{group} = \frac{d\omega}{dk}$$

$$v_{phase} = \frac{\omega}{k}$$

Schrödinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$

Time independent: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$

Probability Current: $S(x,t) = \frac{i\hbar}{2m}\left\{\Psi\frac{\partial\Psi^*}{\partial x} - \Psi^*\frac{\partial\Psi}{\partial x}\right\}$

Normalization (1-D): $\int_{-\infty}^{+\infty}\Psi^*\Psi dx = 1$

Nuclear Physics:

$$R = R_0 A^{1/3} = 1.2 A^{1/3}\, fm$$

$$B = \left[Zm\left({}^1_1H\right) + Nm_n - m\left({}^A_zX\right)\right]c^2$$

$$Q = [M_{parent} - M_{Daughter} - M_{emitted}]c^2$$

$$N = N_0 e^{-\lambda t}$$

$$A = \lambda N$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Rutherford Scattering:

$$b = \frac{zZ}{2K}\frac{e^2}{4\pi\varepsilon_0}\cot\tfrac{1}{2}\theta$$

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(\frac{b^2v^2}{r_{\min}^2}\right) + \frac{e^2}{4\pi\varepsilon_0}\frac{zZ}{r_{\min}}$$

$$d = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{K}$$

Bohr model:

$$E_n = -\frac{Z_{eff}^2 me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2} = -\frac{13.6Z_{eff}^2}{n^2}eV$$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$R = R_\infty\left(\frac{1}{1 + m/M}\right)$$

$$R_\infty = \frac{mk^2e^4}{4\pi c\hbar^3}$$

- *Where R_∞ = 1.0973732 X 10^7 m^{-1} is the Rydberg constant*

X-rays:

K-series: $E_{photon} = (13.6eV)\left(\frac{1}{1^2} - \frac{1}{n^2}\right)(Z-1)^2$

L-series: $E_{photon} = (13.6eV)\left(\frac{1}{2^2} - \frac{1}{n^2}\right)(Z-3)^2$

M-series: $E_{photon} = (13.6eV)\left(\frac{1}{3^2} - \frac{1}{n^2}\right)(Z-5)^2$

Some useful mathematical relations:

$$\sqrt{\left(1 - u^2/c^2\right)} \approx 1 - \frac{1}{2}\frac{u^2}{c^2}$$

$$\frac{1}{\sqrt{\left(1 - u^2/c^2\right)}} \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$$

For $u^2/c^2 << 1$

$$\sin(A+B)=\sin(A)\cos(B)+\cos(A)\sin(B)$$

$$\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$$

$$\sin(2A)=2\sin(A)\cos(A)$$

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

Some useful Integrals:

$$\int \sin^2(u)\,du=\frac{1}{2}\left(u-\sin u\cos u\right)$$

$$\int \sin u\cos u\,du=\frac{1}{2}\sin^2 u$$

$$\int \cos^2 u\,du=\frac{1}{2}\left(u+\sin u\cos u\right)$$

$$\int u\sin^2 u\,du=\frac{u^2}{4}-\frac{u\sin 2u}{4}-\frac{\cos 2u}{8}$$

$$\int u\cos^2 u\,du=\frac{u^2}{4}+\frac{u\sin 2u}{4}+\frac{\cos 2u}{8}$$

$$\int u^2\sin^2 u\,du=\frac{u^3}{6}-\left(\frac{u^2}{4}-\frac{1}{8}\right)\sin 2u-\frac{u\cos 2u}{4}$$

$$\int u^2\cos^2 u\,du=\frac{u^3}{6}+\left(\frac{u^2}{4}-\frac{1}{8}\right)\sin 2u+\frac{u\cos 2u}{4}$$

$$\int_0^\infty u^n e^{-u}\,du=n!\quad for\ \ n>0$$

$$\int \cos^n u\sin u\,du=-\frac{\cos^{n+1}u}{n+1}\quad for\ \ n>0$$

$$\int \sin^n u\cos u\,du=\frac{\sin^{n+1}u}{n+1}\quad for\ \ n>0$$

Constants:

Constant	Standard value	Alternate units
Speed of light	$c = 2.998\times10^{8} m/s$	
Electronic charge	$e = 1.602\times10^{-19} C$	
Boltzmann constant	$k = 1.381\times10^{-23} J/K$	$8.617\times10^{-5} eV/K$
Planck's constant	$h = 6.626\times10^{-34} J\cdot s$	$4.136\times10^{-15} eV\cdot s$
	$\hbar = 1.055\times10^{-34} J\cdot s$	$0.652\times10^{-15} eV\cdot s$
Avogadro's constant	$N_A = 6.022\times10^{23} mole^{-1}$	
Stefan-Boltzmann constant	$\sigma = 5.67\times10^{-8}\ W/m^2\cdot K^4$	
Electron mass	$m_e = 5.49\times10^{-4} u\ or\ 9.11\times10^{-31} kg$	$0.511 MeV/c^2$
Proton mass	$1.007276\ u\ or\ 1.67262171\times10^{-27}\ kg$	$938.3 MeV/c^2$
Neutron mass	$1.008665\ u\ or\ 1.67492728\times10-27\, kg$	$939.6 MeV/c^2$
Mass of ^{4}He	*4.002603 u*	
Bohr radius	$a_0 = 4\pi\varepsilon_0\hbar^2 / m_e e^2 = 0.0529 nm$	
Hydrogen ionization energy	$13.6 eV$	
	$hc = 1240 ev\cdot nm$	
	$1 eV = 1.602\times10^{-19} J$	
Atomic mass unit (dalton)	$1u = 931.5 MeV/c^2$	$1.661\times10^{-27} kg$
	$kT = 0.02525 eV \approx \frac{1}{40} eV$ at T=293 K	
	$\frac{1}{4\pi\varepsilon_0} = 8.988\times10^{9} N\cdot m^2\cdot C^{-2}$	
	$\frac{e^2}{4\pi\varepsilon_0} = 1.44 eV\cdot nm$	