PHYS 2380 Quantum Physics I
Term Test March 2nd, 2010, 19:00 hrs to 21:00 hrs
519 Allen Bldg.

1. Planck's law states that the energy density inside a black body cavity at a temperature T is given by :

$$u(\lambda) = \left(\frac{8\pi hc}{\lambda^5}\right)\left(\frac{1}{e^{hc/\lambda kT} - 1}\right)$$

A manufacturer wishes to use this relation to make a thermometer to measure the temperature of an object that is radiating as a black body. The device would measure the ratio of the intensities of the emitted light at two frequencies: $R = I(\lambda_1)/I(\lambda_2)$.

   a. Given that $\lambda_2 = 2\,\lambda_1$, derive an expression for the observed ratio, R. (3 points)

   b. Calculate the temperature of the body from the observed ratio in terms of λ_1 and the known fundamental constants as required. (3 points)

2. The work function for copper (Cu) is 4.70 eV.

   a. Find the threshold wavelength and frequency for the photoelectric effect. (2 points)

   b. What is the maximum energy of electrons (in eV) emitted from a copper surface when it is struck by photons with a wavelength of 180 nm? (2 points)

   c. If the copper surface was charged to a potential of +1.5 V, what would be the new threshold wavelength? (2 points)

3. The longest wavelength emitted by hydrogen (Z = 1) for the Lyman series (terminating on the ground state) is 121.568 nm. An astronomer measures the longest wavelength of a spectral series, terminating on the ground state, in the optical spectra from an ion which consists of a nucleus and a single electron in orbit as 30.381 nm.

   Hint: *Preserve at least 6 significant figures in your intermediate calculations*

   a. What is the atomic number (Z) of the nucleus of this ion (identify the element)? (3 points)

   b. Show that the measured wavelength is consistent with the spectra of an ion that has an approximate mass that is 3 times the mass of a proton. (3 points)

4. Consider a neutron (of mass m = 1.674927 x 10^{-27} kg).

   a. At what kinetic energy (in eV) would the wavelength associated with the neutron be 0.100 nm? (2 points)

   If the neutron were to be confined into a region of space that is 10^{-15} m wide in the x-direction:

   b. What is the minimum corresponding uncertainty in its momentum? (2 points)

   c. Based on part (b), estimate its minimum kinetic energy (in eV). (2 points)

5. An electron is described by a wave function $\psi(x) = A\sin\left(\frac{3\pi x}{L}\right)e^{-i\omega t}$ in the region $0 \le x \le L$. It is zero everywhere else.

   a. Show that $A = \sqrt{\frac{2}{L}}$ is required to normalize the wave function. (3 points)

   b. Sketch the probability density for the range $0 \le x \le L$. (3 points)

   c. What is the probability of finding this electron in the region $0 \le x \le L/3$? (2 points)

The End

Appendix: Some information from the text and lectures:

Special Relativity:

Relativistic momentum and energy:

$$\vec{p} = \gamma m\vec{\mathrm{v}}$$
$$E = \gamma mc^2 = mc^2 + K$$
$$E^2 = c^2p^2 + m^2c^4$$

Electromagnetic radiation:

Power received by a detector from a wave:

$$P = \left(\frac{1}{\mu_0 c}\right)E_0^2 A\sin^2(kz - \omega t + \phi)$$
$$P_{ave} = \frac{1}{T_0}\int_0^{T_0} P dt = \frac{E_0^2 A}{2\mu_0 c}, \quad I = \frac{P_{ave}}{A} = \frac{E_0^2}{2\mu_0 c}$$

Where P is the instantaneous power, P_{ave} is the average power delivered to a detector of area A and I is the intensity of the light.

Interference and diffraction:

Pattern Type	Bright Fringes	Dark Fringes
Single slit (width *w*)	$\frac{w}{2}\sin\theta = m\lambda$	$\frac{w}{2}\sin\theta = \left(m + \frac{1}{2}\right)\lambda$
Double slit (spacing *d*)	$d\sin\theta = m\lambda$	
Grating (lines spaced *d* apart)	$d\sin\theta = m\lambda$	
Bragg (layers of atoms *d* apart)	$2d\sin\theta = m\lambda$	
Circular object		First fringe at $1.22\lambda/d$

Photons and light:

$$\lambda\nu = c$$
$$E_{ph} = h\nu = cp_{ph}$$
$$p_{ph} = h/\lambda$$

Where K is the kinetic energy of the emitted electrons, ϕ is the work function of the material and V_s is the stopping potential.

Photoelectric effect: $K = h\nu - \phi = eV_s$

Black body radiation:

$$I = \sigma T^4$$
$$\lambda_{\max} T = 2.898\times10^{-3}\, m \bullet K$$
$$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$
$$R(\lambda) = \frac{c}{4}u(\lambda)$$
$$dI = R(\lambda)d\lambda$$

Compton Scattering:

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

Bremsstrahlung: $\lambda_{\min} = \frac{hc}{eV}$

Wavelike properties of particles:

De Broglie wavelength: $\lambda = h/p$

Heisenberg uncert. relationships:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

Wave packets:

$$p = h/\lambda = \hbar k$$

$$\hbar\omega = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$v_{group} = \frac{d\omega}{dk}$$

$$v_{phase} = \frac{\omega}{k}$$

Schrödinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$

Time independent: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$

Probability Current: $S(x,t) = \frac{i\hbar}{2m}\left\{\Psi\frac{\partial\Psi^*}{\partial x} - \Psi^*\frac{\partial\Psi}{\partial x}\right\}$

Normalization (1-D): $\int_{-\infty}^{+\infty}\Psi^*\Psi dx = 1$

Nuclear Physics:

$$R = R_0 A^{1/3} = 1.2 A^{1/3}\, fm$$

$$B = \left[Zm\left({}^1_1H\right) + Nm_n - m\left({}^A_zX\right)\right]c^2$$

$$Q = [M_{parent} - M_{Daughter} - M_{emitted}]c^2$$

$$N = N_0 e^{-\lambda t}$$

$$A = \lambda N$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Rutherford Scattering:

$$b = \frac{zZ}{2K}\frac{e^2}{4\pi\varepsilon_0}\cot\tfrac{1}{2}\theta$$

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(\frac{b^2v^2}{r_{\min}^2}\right) + \frac{e^2}{4\pi\varepsilon_0}\frac{zZ}{r_{\min}}$$

$$d = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{K}$$

Bohr model:

$$E_n = -\frac{Z_{eff}^2 me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2} = -\frac{13.6Z_{eff}^2}{n^2}eV$$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$R = R_\infty\left(\frac{1}{1+m/M}\right)$$

$$R_\infty = \frac{mk^2e^4}{4\pi c\hbar^3}$$

- *Where R_∞ = 1.0973732 X 10^7 m^{-1} is the Rydberg constant*

X-rays:

K-series: $E_{photon} = (13.6eV)\left(\frac{1}{1^2} - \frac{1}{n^2}\right)(Z-1)^2$

L-series: $E_{photon} = (13.6eV)\left(\frac{1}{2^2} - \frac{1}{n^2}\right)(Z-3)^2$

M-series: $E_{photon} = (13.6eV)\left(\frac{1}{3^2} - \frac{1}{n^2}\right)(Z-5)^2$

Some useful mathematical relations:

$$\sqrt{\left(1-u^2/c^2\right)} \approx 1 - \frac{1}{2}\frac{u^2}{c^2}$$

$$\frac{1}{\sqrt{\left(1-u^2/c^2\right)}} \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$$

For $u^2/c^2 << 1$

$$\sin(A+B)=\sin(A)\cos(B)+\cos(A)\sin(B)$$

$$\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$$

$$\sin(2A)=2\sin(A)\cos(A)$$

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

Some useful Integrals:

$$\int \sin^2(u)\,du=\frac{1}{2}\left(u-\sin u\cos u\right)$$

$$\int \sin u\cos u\,du=\frac{1}{2}\sin^2 u$$

$$\int \cos^2 u\,du=\frac{1}{2}\left(u+\sin u\cos u\right)$$

$$\int u\sin^2 u\,du=\frac{u^2}{4}-\frac{u\sin 2u}{4}-\frac{\cos 2u}{8}$$

$$\int u\cos^2 u\,du=\frac{u^2}{4}+\frac{u\sin 2u}{4}+\frac{\cos 2u}{8}$$

$$\int u^2\sin^2 u\,du=\frac{u^3}{6}-\left(\frac{u^2}{4}-\frac{1}{8}\right)\sin 2u-\frac{u\cos 2u}{4}$$

$$\int u^2\cos^2 u\,du=\frac{u^3}{6}+\left(\frac{u^2}{4}-\frac{1}{8}\right)\sin 2u+\frac{u\cos 2u}{4}$$

$$\int_0^\infty u^n e^{-u}\,du=n! \quad for\ \ n>0$$

$$\int \cos^n u\sin u\,du=-\frac{\cos^{n+1}u}{n+1} \quad for\ \ n>0$$

$$\int \sin^n u\cos u\,du=\frac{\sin^{n+1}u}{n+1} \quad for\ \ n>0$$

Constants:

Constant	**Standard value**	**Alternate units**
Speed of light	$c = 2.998\times10^{8}\,m/s$	
Electronic charge	$e = 1.602\times10^{-19}\,C$	
Boltzmann constant	$k = 1.381\times10^{-23}\,J/K$	$8.617\times10^{-5}\,eV/K$
Planck's constant	$h = 6.626\times10^{-34}\,J\cdot s$	$4.136\times10^{-15}\,eV\cdot s$
	$\hbar = 1.055\times10^{-34}\,J\cdot s$	$0.652\times10^{-15}\,eV\cdot s$
Avogadro's constant	$N_A = 6.022\times10^{23}\,mole^{-1}$	
Stefan-Boltzmann constant	$\sigma = 5.67\times10^{-8}\ W/m^2\cdot K^4$	
Electron mass	$m_e = 5.49\times10^{-4}\,u\ or\ 9.11\times10^{-31}\,kg$	$0.511\,MeV/c^2$
Proton mass	$1.007276\ u\ or\ 1.67262171\times10^{-27}\ kg$	$938.3\,MeV/c^2$
Neutron mass	$1.008665\ u\ or\ 1.67492728\times10-27\,kg$	$939.6\,MeV/c^2$
Mass of ^{4}He	*4.002603 u*	
Bohr radius	$a_0 = 4\pi\varepsilon_0\hbar^2 \big/ m_e\,e^2 = 0.0529\,nm$	
Hydrogen ionization energy	$13.6\,eV$	
	$hc = 1240\,ev\cdot nm$	
	$1\,eV = 1.602\times10^{-19}\,J$	
Atomic mass unit (dalton)	$1u = 931.5\,MeV/c^2$	$1.661\times10^{-27}\,kg$
	$kT = 0.02525\,eV \approx \frac{1}{40}\,eV$ at T=293 K	
	$\frac{1}{4\pi\varepsilon_0} = 8.988\times10^{9}\,N\cdot m^2\cdot C^{-2}$	
	$\frac{e^2}{4\pi\varepsilon_0} = 1.44\,eV\cdot nm$	