PHYS 2380 Quantum Physics I
Term Test March 5th, 2009, 19:00 hrs to 21:00 hrs
519 Allen Bldg.

1. Planck's law states that the energy density inside a cavity at a temperature T is given by :

   $$u(\lambda)=\left(\frac{8\pi hc}{\lambda^5}\right)\left(\frac{1}{e^{hc/\lambda kT}-1}\right)$$

   Show that this law leads to Wein's displacement law that states that the wavelength, λ_m, where $u(\lambda)$ is a maximum and the temperature, T are related by: $\lambda_m T \cong hc/5k$.

   Hint: you may assume that at λ_m $\dfrac{e^{hc/\lambda_m kT}}{e^{hc/\lambda_m kT}-1}\cong 1$ (5 points)

2. The work function for cesium (Cs) is 2.7 eV, the lowest of any metal.

   a. Find the threshold wavelength and frequency for the photoelectric effect. (2 points)

   b. What is the maximum energy of electrons (in eV) emitted from a Cs surface when it is struck by photons with a wavelength of 300 nm? (2 points)

3. Bohr model for the hydrogen atom gives the allowed energies for the electron as

   $$E_n=-\frac{13.6}{n^2}\ eV\ .$$

   a. What are the corresponding allowed energies for the carbon ion, C^{5+}?

      NB: a neutral carbon atom has 6 electrons in orbit. (3 points)

   b. What is the longest wavelength of the spectral series terminating on level n = 3 for this ion ? (2 points)

   c. What would be the kinetic energy of an electron (in eV) whose de Broglie wavelength would be the same as the photon from part (b) (there is no need for relativity) (2 points)

4. The position of an electron is known to be within a region of space that is 1 nm wide in the x direction. What is the minimum corresponding uncertainty in its momentum? Based on this, estimate its minimum kinetic energy. (6 points)

5. The time dependent Schroedinger equation is given by:

   $$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)+V(x,t)\Psi(x,t)=i\hbar\frac{\partial}{\partial t}\Psi(x,t)\ ,$$

   Find out by substitution if the following wave functions satisfy this equation:

   a. $\Psi(x,t)=Ae^{i(kx-\omega t)}$ (3 points)

   b. $\Psi(x,t)=A\sin(kx-\omega t)$ (3 points)

6. An electron is described by a wave function $\psi(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{3\pi x}{L}\right)$ in the region $0\le x\le L$. What is the probability of finding this electron in the region $0\le x\le L/3$? Illustrate your answer by sketching the probability distribution for the range $0\le x\le L$. (6 points)

Appendix: Some information from the text and lectures:

Special Relativity:

Relativistic momentum and energy:

$$\vec{p} = \gamma m \vec{v}$$
$$E = \gamma mc^2 = mc^2 + K$$
$$E^2 = c^2 p^2 + m^2 c^4$$

Electromagnetic radiation:

Power received by a detector from a wave:

$$P = \left(\frac{1}{\mu_0 c}\right) E_0^2 A \sin^2(kz - \omega t + \phi)$$
$$P_{ave} = \frac{1}{T_0}\int_0^{T_0} P dt = \frac{E_0^2 A}{2\mu_0 c}, \quad I = \frac{P_{ave}}{A} = \frac{E_0^2}{2\mu_0 c}$$

Where P is the instantaneous power, P_{ave} is the average power delivered to a detector of area A and I is the intensity of the light.

Interference and diffraction:

Pattern Type	**Bright Fringes**	**Dark Fringes**
Single slit (width *w*)	$\frac{w}{2}\sin\theta = m\lambda$	$\frac{w}{2}\sin\theta = \left(m + \frac{1}{2}\right)\lambda$
Double slit (spacing *d*)	$d\sin\theta = m\lambda$	
Grating (lines spaced *d* apart)	$d\sin\theta = m\lambda$	
Bragg (layers of atoms *d* apart)	$2d\sin\theta = m\lambda$	
Circular object		First fringe at $1.22\lambda/d$

Photons and light:

$$\lambda \nu = c$$
$$E_{ph} = h\nu = cp_{ph}$$
$$p_{ph} = h/\lambda$$

Where K is the kinetic energy of the emitted electrons, ϕ is the work function of the material and V_s is the stopping potential.

Photoelectric effect: $K = h\nu - \phi = eV_s$

Black body radiation:

$$I = \sigma T^4$$
$$\lambda_{\max} T = 2.898 \times 10^{-3} m \bullet K$$
$$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$
$$R(\lambda) = \frac{c}{4} u(\lambda)$$
$$dI = R(\lambda) d\lambda$$

Compton Scattering: $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$

Bremsstrahlung: $\lambda_{\min} = \frac{hc}{eV}$

Wavelike properties of particles:

De Broglie wavelength: $\lambda = h/p$

Heisenberg uncertainty relationships:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

Wave packets:

$$p = h/\lambda = \hbar k$$

$$\hbar\omega = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$v_{group} = \frac{d\omega}{dk}$$

$$v_{phase} = \frac{\omega}{k}$$

Quantum Mechanics:

Schrödinger equation:

Complete: $-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$

Time independent: $-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$

Probability Current: $S(x,t) = \frac{i\hbar}{2m}\left\{\Psi\frac{\partial \Psi^*}{\partial x} - \Psi^*\frac{\partial \Psi}{\partial x}\right\}$

Normalization (1-D): $\int_{-\infty}^{+\infty} \Psi^*\Psi dx = 1$

Normalization (3-D): $\int_0^{+\infty}\int_0^{\pi}\int_0^{2\pi} \Psi^*\Psi\, r^2 \sin\theta\, dr\, d\theta\, d\phi = 1$ or $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \Psi^*\Psi\, dx\, dy\, dz = 1$

Nuclear Physics:

$$R = R_0 A^{1/3} = 1.2 A^{1/3}\, fm$$

$$B = \left[Zm\left({}_1^1H\right) + Nm_n - m\left({}_z^AX\right)\right]c^2$$

$$Q = [M_{parent} - M_{Daughter} - M_{emitted}]c^2$$

$$N = N_0 e^{-\lambda t}$$

$$A = \lambda N$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Nuclear Atom:

$$F = \frac{(ze)(Ze)}{4\pi\varepsilon_0 r^2}$$

$$U = \frac{(ze)(Ze)}{4\pi\varepsilon_0 r}$$

Rutherford Scattering:

$$b = \frac{zZ}{2K}\frac{e^2}{4\pi\varepsilon_0}\cot\tfrac{1}{2}\theta$$

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(\frac{b^2v^2}{r_{\min}^2}\right) + \frac{e^2}{4\pi\varepsilon_0}\frac{zZ}{r_{\min}}$$

$$d = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{K}$$

Bohr model: $E_n = -\frac{Z_{eff}^2 me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2} = -\frac{13.6Z_{eff}^2}{n^2}eV$

X-rays:

K-series: $E_{photon} = (13.6eV)\left(\frac{1}{1^2} - \frac{1}{n^2}\right)(Z-1)^2$

L-series: $E_{photon} = (13.6eV)\left(\frac{1}{2^2} - \frac{1}{n^2}\right)(Z-3)^2$

M-series: $E_{photon} = (13.6eV)\left(\frac{1}{3^2} - \frac{1}{n^2}\right)(Z-5)^2$

Some useful mathematical relations:

$$\sqrt{\left(1-u^2/c^2\right)} \approx 1-\frac{1}{2}\frac{u^2}{c^2}$$

$$\frac{1}{\sqrt{\left(1-u^2/c^2\right)}} \approx 1+\frac{1}{2}\frac{u^2}{c^2}$$

For $u^2/c^2 << 1$

$$\sin(A+B)=\sin(A)\cos(B)+\cos(A)\sin(B)$$

$$\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$$

$$\sin(2A)=2\sin(A)\cos(A)$$

$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

Some useful Integrals:

$$\int \sin^2(u)\,du=\frac{1}{2}\left(u-\sin u\cos u\right)$$

$$\int \sin u\cos u\,du=\frac{1}{2}\sin^2 u$$

$$\int \cos^2 u\,du=\frac{1}{2}\left(u+\sin u\cos u\right)$$

$$\int u\sin^2 u\,du=\frac{u^2}{4}-\frac{u\sin 2u}{4}-\frac{\cos 2u}{8}$$

$$\int u\cos^2 u\,du=\frac{u^2}{4}+\frac{u\sin 2u}{4}+\frac{\cos 2u}{8}$$

$$\int u^2\sin^2 u\,du=\frac{u^3}{6}-\left(\frac{u^2}{4}-\frac{1}{8}\right)\sin 2u-\frac{u\cos 2u}{4}$$

$$\int u^2\cos^2 u\,du=\frac{u^3}{6}+\left(\frac{u^2}{4}-\frac{1}{8}\right)\sin 2u+\frac{u\cos 2u}{4}$$

$$\int_0^\infty u^n e^{-u}\,du=n! \quad for\ \ n>0$$

$$\int \cos^n u\sin u\,du=-\frac{\cos^{n+1}u}{n+1} \quad for\ \ n>0$$

$$\int \sin^n u\cos u\,du=\frac{\sin^{n+1}u}{n+1} \quad for\ \ n>0$$

Constants:

Constant	Standard value	Alternate units
Speed of light	$c = 2.998\times10^{8} m/s$	
Electronic charge	$e = 1.602\times10^{-19} C$	
Boltzmann constant	$k = 1.381\times10^{-23} J/K$	$8.617\times10^{-5} eV/K$
Planck's constant	$h = 6.626\times10^{-34} J\cdot s$	$4.136\times10^{-15} eV\cdot s$
	$\hbar = 1.055\times10^{-34} J\cdot s$	$0.652\times10^{-15} eV\cdot s$
Avogadro's constant	$N_A = 6.022\times10^{23} mole^{-1}$	
Stefan-Boltzmann constant	$\sigma = 5.67\times10^{-8}\ W/m^2\cdot K^4$	
Electron mass	$m_e = 5.49\times10^{-4} u\ or\ 9.11\times10^{-31} kg$	$0.511 MeV/c^2$
Proton mass	$1.007276\ u$	$938.3 MeV/c^2$
Neutron mass	$1.008665\ u$	$939.6 MeV/c^2$
Mass of ^{4}He	*4.002603 u*	
Bohr radius	$a_0 = 4\pi\varepsilon_0\hbar^2 / m_e e^2 = 0.0529 nm$	
Hydrogen ionization energy	$13.6 eV$	
	$hc = 1240 ev\cdot nm$	
	$1 eV = 1.602\times10^{-19} J$	
Atomic mass unit (dalton)	$1u = 931.5 MeV/c^2$	$1.661\times10^{-27} kg$
	$kT = 0.02525 eV \approx \frac{1}{40} eV$ at T=293 K	
	$\frac{1}{4\pi\varepsilon_0} = 8.988\times10^{9} N\cdot m^2\cdot C^{-2}$	
	$\frac{e^2}{4\pi\varepsilon_0} = 1.44 eV\cdot nm$	