UNIVERSITY OF MANITOBA

April 14th, 2014 | **Final Exam**

NO. OF PAGES: 7

LOCATION: 402 Allen Bldg.

DEPT. AND COURSE NO.: PHYS 2380 | **TIME: 09:00 to 12:00 (3 hours)**

EXAMINATION: Quantum Physics 1 | **Examiners: K.S. Sharma**

Instructions:

- Answer all questions.
- Start each question on a new page.
- Show all of your steps in arriving at the solution. Do not just state the answer. State your arguments clearly and indicate any assumptions you make.
- Include a sentence or two at the end of each problem summarizing your solution.
- Each part of a question carries equal weight.

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1. The surface temperature of the sun is approximately 5800 K.
   a) At what wavelength is the peak of the radiated energy?
   b) If the radius of the sun is 6.96 x 10^8 m, what is the total power radiated by the sun?
   c) The human eye is sensitive to a narrow range of wavelengths from 400 nm to 700 nm. Assuming that the intensity of emitted light is constant at a value near the wavelength in part (a) what fraction of the total energy emitted falls within this window.

   (9 marks)

2. A photon with energy 2.00 MeV is scattered by an electron at 90^0 to its initial direction.
   a) What is the wavelength of the scattered photon?
   b) What is the momentum of the incident and scattered photons?
   c) What is the kinetic energy of the scattered electron?
   d) What is the momentum (magnitude and direction) of the scattered electron?

   Express your answers in eV and nanometers.

   (12 marks)

3. The wave function for the harmonic oscillator potential $\left(V(x) = \frac{C}{2}x^2\right)$ for n=0 is given by:

   Where C is a constant and x is the position of the particle.

$$\Psi_0(x,t) = A_0 e^{-m\omega x^2/2\hbar} e^{-iEt/\hbar}$$
$$where\ \omega = \sqrt{C/m}$$
$$E = (n+1/2)\hbar\omega$$

   a) Verify that this wave function is a solution to the Schrodinger Equation by directly substituting it into the equation.
   b) Determine A_0 by normalizing the wave function.

c) Without computing the integrals, argue from the form of the integrand that $\langle x \rangle$ and $\langle p \rangle$ are zero.

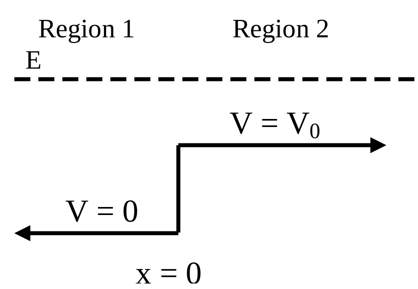
d) Evaluate the average values $\langle x^2 \rangle$ and $\langle p^2 \rangle$ for this state.

e) Using the results from part (d) obtain values for the average values for potential and kinetic energies and their sum. Express your answers in terms of $\hbar$ and ω.

f) Use the results from part (c,d) to calculate the uncertainty in *x* and *p* and verify that the Heisenberg uncertainty principle is satisfied.

(20 marks)

4. A wave is incident from the left on the potential function, V(x), shown here:



a) Write down the “space-part” wave function for a particle with energy $E > V_0$ in both regions.

Identify the incident and reflected components in these wave functions. Do any of the components of the wave functions have a zero amplitude (explain)?

b) What are the corresponding “time-parts” of the wave function?

c) Given that $V_0 = \frac{5}{9}E$ provide expressions for the wave number (k) in each region.

Express of them in terms of the other. What is the ratio of k_1 to k_2?

d) Apply the boundary conditions and determine the amplitudes for all the component waves in terms of one of them.

e) From the coefficients determined in part (d) derive an expression for the probability density as a function of x.

f) Sketch the probability density and locate the maxima, minima and constant levels if any. Assign relative values to maxima, minima and constant levels.

g) Using the coefficients for the incident, reflected and transmitted waves from part (d), directly calculate the transmission coefficient *T*.

h) Using the coefficients for the incident, reflected and transmitted waves from part (d), directly calculate the reflection coefficient *R*.

i) From your results for parts (g) and (h), demonstrate that *T* + *R* = *1*.

(20 marks)

5. The normalized wave function for the $n = 3, l = 2, m = 0$ state of the hydrogen atom has the form:

$$\psi_{320} = \frac{1}{81\sqrt{6\pi}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}}\left(\frac{1}{a_0^2}\right) r^2 e^{-r/3a_0}\left(3\cos^2\theta - 1\right)e^{i\phi}$$

a) Calculate the expectation value for r, <*r*>.
b) Determine the radius at which the radial probability density function is a maximum.
c) Compare your answers to parts (a) and (b) with the predictions of the Bohr theory and comment on any differences.
d) For an $l = 2$ state, sketch all the allowed orientations of the angular momentum relative to the z-axis. Give values for the angles and magnitudes.

(20 marks)

The End

Appendix: Some information from the text and lectures:

Special Relativity:

Relativistic momentum and energy:

$$\vec{p} = \gamma m\vec{v}$$
$$E = \gamma mc^2 = mc^2 + K$$
$$E^2 = c^2 p^2 + m^2 c^4$$

Electromagnetic radiation:

Power received by a detector from a wave:

$$P = \left(\frac{1}{\mu_0 c}\right) E_0^2 A \sin^2(kz - \omega t + \phi)$$
$$P_{ave} = \frac{1}{T_0}\int_0^{T_0} P\,dt = \frac{E_0^2 A}{2\mu_0 c}, \quad I = \frac{P_{ave}}{A} = \frac{E_0^2}{2\mu_0 c}$$

Where P is the instantaneous power, P_{ave} is the average power delivered to a detector of area A and I is the intensity of the light.

Interference and diffraction:

Pattern Type	**Bright Fringes**	**Dark Fringes**
Single slit (width *w*)	$\frac{w}{2}\sin\theta = m\lambda$	$\frac{w}{2}\sin\theta = \left(m + \frac{1}{2}\right)\lambda$
Double slit (spacing *d*)	$d\sin\theta = m\lambda$	
Grating (lines spaced *d* apart)	$d\sin\theta = m\lambda$	
Bragg (layers of atoms *d* apart)	$2d\sin\theta = m\lambda$	
Circular object		First fringe at $1.22\lambda/d$

Photons and light:

$$\lambda\nu = c$$
$$E_{ph} = h\nu = cp_{ph}$$
$$p_{ph} = h/\lambda$$

Photoelectric effect: $K = h\nu - \phi = eV_s$

Where K is the kinetic energy of the emitted electrons, ϕ is the work function of the material and V_s is the stopping potential.

Black body radiation:

$$I = \sigma T^4$$
$$\lambda_{\max} T = 2.898\times 10^{-3}\, m \bullet K$$
$$u(\lambda) = \left(\frac{8\pi hc}{\lambda^5}\right)\left[\frac{1}{e^{hc/\lambda kT} - 1}\right]$$
$$dI = \frac{c}{4} u(\lambda)\, d\lambda$$

Where *u(λ)* is the energy density, *h* is Planck's constant, *k* is the Botzmann constant and *c* is the speed of light.

Compton Scattering: $\lambda' - \lambda = \frac{h}{m_e c}\left(1 - \cos\theta\right)$

Bremsstrahlung: $\lambda_{\min} = \frac{hc}{eV}$

Wavelike properties of particles:

De Broglie wavelength: $\lambda = h/p$

Hiesenberg uncertainty relationships:

$$\Delta E \Delta t \ge \frac{\hbar}{2} \qquad \Delta x = \left(\langle x^2\rangle - \langle x\rangle^2\right)^{1/2}$$
$$\Delta p_x \Delta x \ge \frac{\hbar}{2} \qquad \Delta p = \left(\langle p^2\rangle - \langle p\rangle^2\right)^{1/2}$$

Wave packets:

$$p = h/\lambda = \hbar k$$
$$\hbar\omega = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$v_{group} = \frac{d\omega}{dk}$$
$$v_{phase} = \frac{\omega}{k}$$

Quantum Mechanics:

Schrödinger equation:

Complete: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$

Time independent: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$

Probability Current: $S(x,t) = \frac{i\hbar}{2m}\left\{\Psi\frac{\partial\Psi^*}{\partial x} - \Psi^*\frac{\partial\Psi}{\partial x}\right\}$

Normalization (1-D): $\int_{-\infty}^{+\infty}\Psi^*\Psi dx = 1$

Normalization (3-D): $\int_0^{+\infty}\int_0^{\pi}\int_0^{2\pi}\Psi^*\Psi\, r^2\sin\theta\, dr\, d\theta\, d\phi = 1$ or $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Psi^*\Psi\, dx\, dy\, dz = 1$

H-atom: $\Psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$ each of these components may be normalized individually.

Radial probability distribution: $P(r) = r^2 R^*(r)R(r)$

Nuclear Physics:

$$R = R_0 A^{1/3} = 1.2A^{1/3}\,fm$$
$$B = \left[Zm\left({}_1^1H\right) + Nm_n - m\left({}_Z^AX\right)\right]c^2$$
$$Q = [M_{parent} - M_{Daughter} - M_{emitted}]c^2$$

$$N = N_0 e^{-\lambda t}$$
$$A = \lambda N$$
$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Nuclear Atom:

$$F = \frac{(ze)(Ze)}{4\pi\varepsilon_0 r^2}$$
$$U = \frac{(ze)(Ze)}{4\pi\varepsilon_0 r}$$

Rutherford Scattering:

$$b = \frac{zZ}{2K}\frac{e^2}{4\pi\varepsilon_0}\cot\tfrac{1}{2}\theta$$
$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(\frac{b^2v^2}{r_{\min}^2}\right) + \frac{e^2}{4\pi\varepsilon_0}\frac{zZ}{r_{\min}}$$
$$d = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{K}$$

Bohr model: $E_n = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2} = -\frac{13.6Z_{eff}^2}{n^2}eV$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$
$$R = R_\infty\left(\frac{1}{1+m/M}\right)$$
$$R_\infty = \frac{mk^2e^4}{4\pi c\hbar^3}\ where\ k = \frac{1}{4\pi\varepsilon_0}$$

Bohr radius: $a_0 = \frac{\hbar^2}{4\pi\varepsilon_0 me^2}$

$$r_n = n^2 a_0/Z$$

X-rays:

K-series: $E_{photon} = (13.6eV)\left(\frac{1}{1^2} - \frac{1}{n^2}\right)(Z-1)^2$

L-series: $E_{photon} = (13.6eV)\left(\frac{1}{2^2} - \frac{1}{n^2}\right)(Z-3)^2$

M-series: $E_{photon} = (13.6eV)\left(\frac{1}{3^2} - \frac{1}{n^2}\right)(Z-5)^2$

Some useful mathematical relations:

$$\sqrt{\left(1-u^2/c^2\right)} \approx 1 - \frac{1}{2}\frac{u^2}{c^2}$$

$$\frac{1}{\sqrt{\left(1-u^2/c^2\right)}} \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$$

For $u^2/c^2 << 1$

Some useful trigonometric relations:

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$\sin^2(A) = \frac{1-\cos(2A)}{2}$$

$$\cos^2(A) = \frac{1+\cos(2A)}{2}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Some useful Integrals:

$$\int \sin^2(u)\,du = \frac{1}{2}\left(u - \sin u \cos u\right)$$

$$\int \sin u \cos u\,du = \frac{1}{2}\sin^2 u$$

$$\int \cos^2 u\,du = \frac{1}{2}\left(u + \sin u \cos u\right)$$

$$\int u \sin^2 u\,du = \frac{u^2}{4} - \frac{u \sin 2u}{4} - \frac{\cos 2u}{8}$$

$$\int u \cos^2 u\,du = \frac{u^2}{4} + \frac{u \sin 2u}{4} + \frac{\cos 2u}{8}$$

$$\int u^2 \sin^2 u\,du = \frac{u^3}{6} - \left(\frac{u^2}{4} - \frac{1}{8}\right)\sin 2u - \frac{u \cos 2u}{4}$$

$$\int u^2 \cos^2 u\,du = \frac{u^3}{6} + \left(\frac{u^2}{4} - \frac{1}{8}\right)\sin 2u + \frac{u \cos 2u}{4}$$

$$\int \sin^3 u\,du = -\frac{1}{3}\cos u\left[\sin^2 u + 2\right]$$

$$\int \cos^3 u\,du = \frac{1}{3}\sin u\left[\cos^2 u + 2\right]$$

$$\int_0^\infty u^n e^{-u}\,du = n! \quad for\ \ n > 0$$

$$\int \cos^n u \sin u\,du = -\frac{\cos^{n+1} u}{n+1} \quad for\ \ n > 0$$

$$\int \sin^n u \cos u\,du = \frac{\sin^{n+1} u}{n+1} \quad for\ \ n > 0$$

n	$I_n = \int_0^\infty x^n e^{-\lambda x^2}\,dx$
0	$\frac{1}{2}\pi^{1/2}\lambda^{-1/2}$
1	$\frac{1}{2}\lambda^{-1}$
2	$\frac{1}{4}\pi^{1/2}\lambda^{-3/2}$
3	$\frac{1}{2}\lambda^{-2}$

n	$I_n = \int_0^\infty x^n e^{-\lambda x^2}\,dx$
4	$\frac{3}{8}\pi^{1/2}\lambda^{-5/2}$
5	λ^{-3}
If n is even	$\int_{-\infty}^{\infty} x^n e^{-\lambda x^2}\,dx = 2I_n$
If n is odd	$\int_{-\infty}^{\infty} x^n e^{-\lambda x^2}\,dx = 0$

Constants:

Constant	Standard value	Alternate units
Speed of light	$c = 2.998 \times 10^{8} m/s$	
Electronic charge	$e = 1.602 \times 10^{-19} C$	
Boltzmann constant	$k = 1.381 \times 10^{-23} J/K$	$8.617 \times 10^{-5} eV/K$
Planck's constant	$h = 6.626 \times 10^{-34} J \cdot s$	$4.136 \times 10^{-15} eV \cdot s$
	$\hbar = 1.055 \times 10^{-34} J \cdot s$	$0.652 \times 10^{-15} eV \cdot s$
Avogadro's constant	$N_A = 6.022 \times 10^{23} mole^{-1}$	
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4$	
Electron mass	$m_e = 5.49 \times 10^{-4} u\ or\ 9.109 \times 10^{-31} kg$	$0.511 MeV/c^2$
Proton mass	$1.007276\ u\ or\ 1.673 \times 10^{-27}\ kg$	$938.3 MeV/c^2$
Neutron mass	$1.008665\ u\ or\ 1.675 \times 10^{-27}\ kg$	$939.6 MeV/c^2$
Mass of ^{4}He	*4.002603 u*	
Bohr radius	$a_0 = 4\pi\varepsilon_0\hbar^2 / m_e e^2 = 0.0529 nm$	
Hydrogen ionization energy	$13.6 eV$	
	$hc = 1240 ev \cdot nm$	
	$1eV = 1.602 \times 10^{-19} J$	
Atomic mass unit (dalton)	$1u = 931.5 MeV/c^2$	$1.661 \times 10^{-27} kg$
	$kT = 0.02525 eV \approx \frac{1}{40} eV$ at T=293 K	
	$\frac{1}{4\pi\varepsilon_0} = 8.988 \times 10^{9} N \cdot m^2 \cdot C^{-2}$	
	$\frac{e^2}{4\pi\varepsilon_0} = 1.44 eV \cdot nm$	