UNIVERSITY OF MANITOBA

April 12th, 2013

Final Exam

NO. OF PAGES: 7

LOCATION: 405 Allen Bldg.

DEPT. AND COURSE NO.: PHYS 2380

TIME: 09:00 to 12:00 (3 hours)

EXAMINATION: Quantum Physics 1

Examiners: K.S. Sharma

Instructions:

- Answer all questions.
- Start each question on a new page.
- Show all of your steps in arriving at the solution. Do not just state the answer. State your arguments clearly and indicate any assumptions you make.
- Include a sentence or two at the end of each problem summarizing your solution.
- Each part of a question carries equal weight.

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1. A particle is confined to a region on the x-axis that is $2\times10^{-15}m$ wide.

   a) What is the corresponding uncertainty in its momentum, Δp? Express your answer in SI units (*kg m/s*).

   Based on the result from (a), What is the minimum kinetic energy that the particle can have if it is:

   b) A neutron *(Express your answer in eV)*.

   c) An electron *(Express your answer in eV)*.

   (9 marks)

2. X-ray photons of wavelength 2.480×10^{-11} *m* are incident on a target and Compton scattered photons are observed at 90^0 to the direction of the incident photon.

   a) What is the wavelength of the scattered photon?

   b) What is the momentum of the incident and scattered photons?

   c) What is the kinetic energy of the scattered electron?

   d) What is the momentum (magnitude and direction) of the scattered electron?

   Express your answers in eV and nanometers.

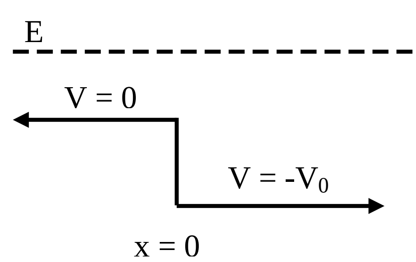
   (12 marks)

3. An electron is confined in an infinite square well potential of width *a*. You may find it useful to consider such a potential as extending over the region $0 \leq x \leq a$.
   a) Solve the S. equation for the wave functions of such a particle and derive an expression for the allowed values of its energy.
   b) Calculate the average value for x (where x is the coordinate of the trapped particle) from the wave functions in part (a) and show that is a/2 for all states.
   c) Calculate the average value for x^2 (where x is the coordinate of the trapped particle) using the same technique.
   d) From the results to parts (b) and (c) calculate the uncertainty in the position of the particle, Δx.
   e) Calculate the expectation value for $p^2, \langle p^2 \rangle$.
   f) Given that the expectation value for *p* is *0*, calculate the uncertainty in the momentum of the particle (Δp) using your result from part (c)
   g) Compare your results with the Heisenberg uncertainty relation?

   (20 marks)

4. A particle with energy, E, is incident from the left on the potential function, V(x), where:

$$V = 0 \quad for \ x \leq 0$$
$$V = -V_0 = -3E \quad for \ x > 0$$



   a) Write down the most general "space" wave functions in both regions. Identify the incident and reflected components in these wave functions. Should any of the components of the wave functions have a zero amplitude (explain)?
   b) What are the corresponding "time-parts" of the wave function?
   c) Provide expressions for the wave number (*k*) in each region. Express one of them in terms of the other.
   d) Apply the boundary conditions and determine the amplitudes for all the component waves in terms of one of them.
   e) From the coefficients determined in the previous parts derive an expression for the probability density as a function of *x*.
   f) Sketch the probability density and locate the maxima, minima and constant levels if any. Assign relative values to maxima, minima and constant levels.

   Using the coefficients for the incident, reflected and transmitted waves from part (d), directly calculate

   g) The transmission coefficient *T*.
   h) The reflection coefficient *R*.
   i) From your results, demonstrate that $T + R = 1$.

   (20 marks)

5. For states of the hydrogen atom with $n = 3$:
   a) What are the allowed values for the orbital angular momentum quantum number ℓ?
   b) What are the allowed quantum numbers for the component of angular momentum along the z-direction, m, for the possible values of ℓ?
   c) What is the largest magnitude that the orbital angular momentum can have (in units of $\hbar$?
   d) Draw a diagram showing all the possible orientations of the total angular momentum with respect to the z-axis for the state with the largest possible value of ℓ. Determine the angle that the angular momentum makes with the z-axis in each case.

   (16 marks)

The End

Appendix: Some information from the text and lectures:

Special Relativity:

Relativistic momentum and energy:

$$\vec{p} = \gamma m\vec{v}$$
$$E = \gamma mc^2 = mc^2 + K$$
$$E^2 = c^2p^2 + m^2c^4$$

Electromagnetic radiation:

Power received by a detector from a wave:

$$P = \left(\frac{1}{\mu_0 c}\right)E_0^2 A\sin^2(kz - \omega t + \phi)$$
$$P_{ave} = \frac{1}{T_0}\int_0^{T_0} P\,dt = \frac{E_0^2 A}{2\mu_0 c}, \quad I = \frac{P_{ave}}{A} = \frac{E_0^2}{2\mu_0 c}$$

Where P is the instantaneous power, P_{ave} is the average power delivered to a detector of area A and I is the intensity of the light.

Interference and diffraction:

Pattern Type	Bright Fringes	Dark Fringes
Single slit (width *w*)	$\frac{w}{2}\sin\theta = m\lambda$	$\frac{w}{2}\sin\theta = \left(m + \frac{1}{2}\right)\lambda$
Double slit (spacing *d*)	$d\sin\theta = m\lambda$	
Grating (lines spaced *d* apart)	$d\sin\theta = m\lambda$	
Bragg (layers of atoms *d* apart)	$2d\sin\theta = m\lambda$	
Circular object		First fringe at $1.22\lambda/d$

Photons and light:

$$\lambda\nu = c$$
$$E_{ph} = h\nu = cp_{ph}$$
$$p_{ph} = h/\lambda$$

Photoelectric effect: $K = h\nu - \phi = eV_s$

Where K is the kinetic energy of the emitted electrons, ϕ is the work function of the material and V_s is the stopping potential.

Black body radiation:

$$I = \sigma T^4$$
$$\lambda_{\max} T = 2.898 \times 10^{-3}\, m \bullet K$$
$$u(\lambda) = \left(\frac{8\pi hc}{\lambda^5}\right)\left[\frac{1}{e^{hc/\lambda kT} - 1}\right]$$
$$dI = \frac{c}{4}u(\lambda)d\lambda$$

Where *u(λ)* is the energy density, *h* is Planck's constant, *k* is the Botzmann constant and *c* is the speed of light.

Compton Scattering: $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$

Bremsstrahlung: $\lambda_{\min} = \frac{hc}{eV}$

Wavelike properties of particles:

De Broglie wavelength: $\lambda = h/p$

Hiesenberg uncertainty relationships:

$$\Delta E\Delta t \geq \frac{\hbar}{2} \qquad \Delta x = \left(\langle x^2\rangle - \langle x\rangle^2\right)^{1/2}$$
$$\Delta p_x \Delta x \geq \frac{\hbar}{2} \qquad \Delta p = \left(\langle p^2\rangle - \langle p\rangle^2\right)^{1/2}$$

Wave packets:

$$v_{group} = \frac{d\omega}{dk}$$

$$v_{phase} = \frac{\omega}{k}$$

$$p = h/\lambda = \hbar k$$

$$\hbar\omega = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

Quantum Mechanics:

Schrödinger equation:

Complete: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$

Time independent: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$

Probability Current: $S(x,t) = \frac{i\hbar}{2m}\left\{\Psi\frac{\partial\Psi^*}{\partial x} - \Psi^*\frac{\partial\Psi}{\partial x}\right\}$

Normalization (1-D): $\int_{-\infty}^{+\infty}\Psi^*\Psi dx = 1$

Normalization (3-D): $\int_0^{+\infty}\int_0^{\pi}\int_0^{2\pi}\Psi^*\Psi\, r^2\sin\theta\, dr\, d\theta\, d\phi = 1$ or $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Psi^*\Psi\, dx\, dy\, dz = 1$

H-atom: $\Psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$ each of these components may be normalized individually.

Radial probability distribution: $P(r) = r^2 R^*(r)R(r)$

Nuclear Physics:

$$R = R_0 A^{1/3} = 1.2A^{1/3}\, fm$$

$$B = \left[Zm\left({}_1^1H\right) + Nm_n - m\left({}_Z^AX\right)\right]c^2$$

$$Q = [M_{parent} - M_{Daughter} - M_{emitted}]c^2$$

$$N = N_0 e^{-\lambda t}$$

$$A = \lambda N$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Nuclear Atom:

$$F = \frac{(ze)(Ze)}{4\pi\varepsilon_0 r^2}$$

$$U = \frac{(ze)(Ze)}{4\pi\varepsilon_0 r}$$

Rutherford Scattering:

$$b = \frac{zZ}{2K}\frac{e^2}{4\pi\varepsilon_0}\cot\tfrac{1}{2}\theta$$

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(\frac{b^2v^2}{r_{\min}^2}\right) + \frac{e^2}{4\pi\varepsilon_0}\frac{zZ}{r_{\min}}$$

$$d = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{K}$$

Bohr model: $E_n = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2} = -\frac{13.6Z_{eff}^2}{n^2}eV$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$R = R_\infty\left(\frac{1}{1+m/M}\right)$$

$$R_\infty = \frac{mk^2e^4}{4\pi c\hbar^3} \text{ where } k = \frac{1}{4\pi\varepsilon_0}$$

Bohr radius: $a_0 = \frac{\hbar^2}{4\pi\varepsilon_0 me^2}$

$$r_n = n^2 a_0/Z$$

X-rays:

K-series: $E_{photon} = (13.6eV)\left(\frac{1}{1^2} - \frac{1}{n^2}\right)(Z-1)^2$

L-series: $E_{photon} = (13.6eV)\left(\frac{1}{2^2} - \frac{1}{n^2}\right)(Z-3)^2$

M-series: $E_{photon} = (13.6eV)\left(\frac{1}{3^2} - \frac{1}{n^2}\right)(Z-5)^2$

Some useful mathematical relations:

$$\sqrt{\left(1-u^2/c^2\right)} \approx 1-\frac{1}{2}\frac{u^2}{c^2}$$

$$\frac{1}{\sqrt{\left(1-u^2/c^2\right)}} \approx 1+\frac{1}{2}\frac{u^2}{c^2}$$

For $u^2/c^2 << 1$

Some useful trigonometric relations:

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$\sin^2(A) = \frac{1-\cos(2A)}{2}$$

$$\cos^2(A) = \frac{1+\cos(2A)}{2}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Some useful Integrals:

$$\int \sin^2(u)\,du = \frac{1}{2}\left(u - \sin u \cos u\right)$$

$$\int \sin u \cos u\, du = \frac{1}{2}\sin^2 u$$

$$\int \cos^2 u\, du = \frac{1}{2}\left(u + \sin u \cos u\right)$$

$$\int u \sin^2 u\, du = \frac{u^2}{4} - \frac{u \sin 2u}{4} - \frac{\cos 2u}{8}$$

$$\int u \cos^2 u\, du = \frac{u^2}{4} + \frac{u \sin 2u}{4} + \frac{\cos 2u}{8}$$

$$\int u^2 \sin^2 u\, du = \frac{u^3}{6} - \left(\frac{u^2}{4} - \frac{1}{8}\right)\sin 2u - \frac{u \cos 2u}{4}$$

$$\int u^2 \cos^2 u\, du = \frac{u^3}{6} + \left(\frac{u^2}{4} - \frac{1}{8}\right)\sin 2u + \frac{u \cos 2u}{4}$$

$$\int \sin^3 u\, du = -\frac{1}{3}\cos u\left[\sin^2 u + 2\right]$$

$$\int \cos^3 u\, du = \frac{1}{3}\sin u\left[\cos^2 u + 2\right]$$

$$\int_0^\infty u^n e^{-u}\, du = n! \quad for\ \ n > 0$$

$$\int \cos^n u \sin u\, du = -\frac{\cos^{n+1} u}{n+1} \quad for\ \ n > 0$$

$$\int \sin^n u \cos u\, du = \frac{\sin^{n+1} u}{n+1} \quad for\ \ n > 0$$

n	$I_n = \int_0^\infty x^n e^{-\lambda x^2} dx$
0	$\frac{1}{2}\pi^{1/2}\lambda^{-1/2}$
1	$\frac{1}{2}\lambda^{-1}$
2	$\frac{1}{4}\pi^{1/2}\lambda^{-3/2}$
3	$\frac{1}{2}\lambda^{-2}$

n	$I_n = \int_0^\infty x^n e^{-\lambda x^2} dx$
4	$\frac{3}{8}\pi^{1/2}\lambda^{-5/2}$
5	λ^{-3}
If n is even	$\int_{-\infty}^{\infty} x^n e^{-\lambda x^2} dx = 2I_n$
If n is odd	$\int_{-\infty}^{\infty} x^n e^{-\lambda x^2} dx = 0$

Constants:

Constant	Standard value	Alternate units
Speed of light	$c = 2.998\times10^{8} m/s$	
Electronic charge	$e = 1.602\times10^{-19} C$	
Boltzmann constant	$k = 1.381\times10^{-23} J/K$	$8.617\times10^{-5} eV/K$
Planck's constant	$h = 6.626\times10^{-34} J\cdot s$	$4.136\times10^{-15} eV\cdot s$
	$\hbar = 1.055\times10^{-34} J\cdot s$	$0.652\times10^{-15} eV\cdot s$
Avogadro's constant	$N_A = 6.022\times10^{23} mole^{-1}$	
Stefan-Boltzmann constant	$\sigma = 5.67\times10^{-8}\ W/m^2\cdot K^4$	
Electron mass	$m_e = 5.49\times10^{-4} u\ or\ 9.109\times10^{-31} kg$	$0.511 MeV/c^2$
Proton mass	$1.007276\ u\ or\ 1.673\times10^{-27} kg$	$938.3 MeV/c^2$
Neutron mass	$1.008665\ u\ or\ 1.675\times10^{-27} kg$	$939.6 MeV/c^2$
Mass of ^{4}He	*4.002603 u*	
Bohr radius	$a_0 = 4\pi\varepsilon_0\hbar^2 / m_e e^2 = 0.0529 nm$	
Hydrogen ionization energy	$13.6 eV$	
	$hc = 1240 ev\cdot nm$	
	$1 eV = 1.602\times10^{-19} J$	
Atomic mass unit (dalton)	$1u = 931.5 MeV/c^2$	$1.661\times10^{-27} kg$
	$kT = 0.02525 eV \approx \frac{1}{40} eV$ at T=293 K	
	$\frac{1}{4\pi\varepsilon_0} = 8.988\times10^{9} N\cdot m^2\cdot C^{-2}$	
	$\frac{e^2}{4\pi\varepsilon_0} = 1.44 eV\cdot nm$	