UNIVERSITY OF MANITOBA

April 16th, 2012 **Final Exam**

NO. OF PAGES: 7

LOCATION: 405 Allen Bldg.

DEPT. AND COURSE NO.: PHYS 2380 **TIME: 09:00 to 12:00 (3 hours)**

EXAMINATION: Quantum Physics 1 **Examiners: K.S. Sharma**

Instructions:

- Answer all questions.
- Start each question on a new page.
- Show all of your steps in arriving at the solution. Do not just state the answer. State your arguments clearly and indicate any assumptions you make.
- Include a sentence or two at the end of each problem summarizing your solution.
- Each part of a question carries equal weight.

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1. An x-ray spectrometer on board a satellite measures the wavelength at the maximum intensity emitted by a particular star to be $\lambda_m = 82.8\,nm$. Assume that the star radiates like a blackbody:
   a) What is the star's surface temperature?
   b) What is the ratio of the intensity radiated at $\lambda = 70.0\,nm$ and at $\lambda = 100.0\,nm$ to that radiated at λ_m.

   (8 marks)

2. A 511 keV photon is Compton scattered from an electron at an angle of $110°$ to its original direction. Express your answers in eV and nanometers.
   a) Show that in general for such a Compton scattered photon $E' = \dfrac{E}{\left(E/mc^2\right)\left(1-\cos\theta\right)+1}$ where E', E are the energies of the scattered and incident photons, θ the angle the scattered photon makes with the incident photon, *m* is the mass of the electron and *c* is the speed of light.
   b) What is the wavelength of the incident photon?
   c) What is the energy and wavelength of the photon after it was scattered?
   d) What is the kinetic energy of the scattered electron?
   e) What de Broglie wavelength of the scattered electron?
   (Hint: is this a relativistic situation?)

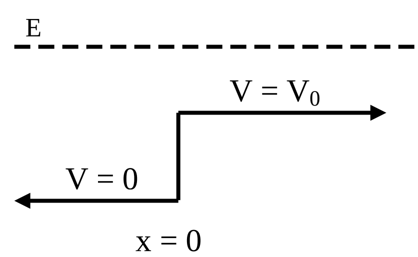
   (15 marks)

3. An electron is confined in an infinite square well potential of width *a*. You may find it useful to consider such a potential as extending over the region $0 \leq x \leq a$.

   a) Solve the S. equation for the wave functions of such a particle and derive an expression for the allowed values of its energy.
   b) Calculate the average value for x (where x is the coordinate of the trapped particle) from the wave functions in part (a) and show that is a/2 for all states.
   c) Calculate the average value for x^2 (where x is the coordinate of the trapped particle) using the same technique.
   d) From the results to parts (b) and (c) calculate the uncertainty in the position of the particle, Δx.

   (20 marks)

4. A wave is incident from the left on the potential function, V(x), shown here:



   a) Write down the "space-part" wave function for a particle with energy $E > V_0$ in both regions.
      Identify the incident and reflected components in these wave functions. Do any of the components of the wave functions have a zero amplitude (explain)?
   b) What are the corresponding "time-parts" of the wave function?
   c) Given that $V_0 = \frac{5}{9}E$ provide expressions for the wave number (k) in each region.
      Express of them in terms of the other.
   d) Apply the boundary conditions and determine the amplitudes for all the component waves in terms of one of them.
   e) From the coefficients determined in the previous parts derive an expression for the probability density as a function of x.
   f) Sketch the probability density and locate the maxima, minima and constant levels if any. Assign relative values to maxima, minima and constant levels.

   (20 marks)

5. The normalized wave functions for the n=2 state of the hydrogen atom are:

$$\psi_{nlm} = R_{nl}(r)\Theta_{lm}(\theta)\Theta(\phi) \ where:$$

$$R_{20} = \frac{1}{\sqrt{2a_0^3}}\left(1 - \frac{r}{2a_0}\right)e^{-r/2a_0}$$

$$R_{21} = \frac{C}{\sqrt{3a_0^3}}\left(\frac{r}{a_0}\right)e^{-r/2a_0}$$

$$and \ the \ angular \ dependence \ is \ given \ by:$$

$$\Theta_{11}\Phi_1 = \sqrt{\frac{3}{8\pi}}\sin\theta e^{i\phi}$$

$$\Theta_{10}\Phi_0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$\Theta_{11}\Phi_{-1} = \sqrt{\frac{3}{8\pi}}\sin\theta e^{-i\phi}$$

where a_0 is the Bohr radius.

a) Calculate the expectation value for *r* for the state described by R_{20} and compare it with the radius predicted by Bohr's theory for this energy level and comment.
b) Determine the constant C so that R_{21} is normalized by itself.
c) Calculate the angle made by the angular momentum vector relative to the z-axis for each of the three angular states. Provide a sketch to illustrate your results.

(15 marks)

The End

Appendix: Some information from the text and lectures:

Special Relativity:

Relativistic momentum and energy:

$$\vec{p} = \gamma m\vec{v}$$
$$E = \gamma mc^2 = mc^2 + K$$
$$E^2 = c^2 p^2 + m^2 c^4$$

Electromagnetic radiation:

Power received by a detector from a wave:

$$P = \left(\frac{1}{\mu_0 c}\right) E_0^2 A \sin^2(kz - \omega t + \phi)$$
$$P_{ave} = \frac{1}{T_0}\int_0^{T_0} P\,dt = \frac{E_0^2 A}{2\mu_0 c}, \quad I = \frac{P_{ave}}{A} = \frac{E_0^2}{2\mu_0 c}$$

Where P is the instantaneous power, P_{ave} is the average power delivered to a detector of area A and I is the intensity of the light.

Interference and diffraction:

Pattern Type	**Bright Fringes**	**Dark Fringes**
Single slit (width *w*)	$\frac{w}{2}\sin\theta = m\lambda$	$\frac{w}{2}\sin\theta = \left(m + \frac{1}{2}\right)\lambda$
Double slit (spacing *d*)	$d\sin\theta = m\lambda$	
Grating (lines spaced *d* apart)	$d\sin\theta = m\lambda$	
Bragg (layers of atoms *d* apart)	$2d\sin\theta = m\lambda$	
Circular object		First fringe at $1.22\lambda/d$

Photons and light:

$$\lambda\nu = c$$
$$E_{ph} = h\nu = cp_{ph}$$
$$p_{ph} = h/\lambda$$

Photoelectric effect: $K = h\nu - \phi = eV_s$

Where K is the kinetic energy of the emitted electrons, ϕ is the work function of the material and V_s is the stopping potential.

Black body radiation:

$$I = \sigma T^4$$
$$\lambda_{\max} T = 2.898\times10^{-3}\, m \bullet K$$
$$u(\lambda) = \left(\frac{8\pi hc}{\lambda^5}\right)\left[\frac{1}{e^{hc/\lambda kT} - 1}\right]$$
$$dI = \frac{c}{4}u(\lambda)\,d\lambda$$

Where *u(λ)* is the energy density, *h* is Planck's constant, *k* is the Botzmann constant and *c* is the speed of light.

Compton Scattering: $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$

Bremsstrahlung: $\lambda_{\min} = \frac{hc}{eV}$

Wavelike properties of particles:

De Broglie wavelength: $\lambda = h/p$

Hiesenberg uncertainty relationships:

$$\Delta E \Delta t \geq \frac{\hbar}{2} \qquad \Delta x = \left(\langle x^2\rangle - \langle x\rangle^2\right)^{1/2}$$
$$\Delta p_x \Delta x \geq \frac{\hbar}{2} \qquad \Delta p = \left(\langle p^2\rangle - \langle p\rangle^2\right)^{1/2}$$

Wave packets:

$p = h/\lambda = \hbar k$

$\hbar\omega = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$

$v_{group} = \frac{d\omega}{dk}$

$v_{phase} = \frac{\omega}{k}$

Quantum Mechanics:

Schrödinger equation:

Complete: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$

Time independent: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$

Probability Current: $S(x,t) = \frac{i\hbar}{2m}\left\{\Psi\frac{\partial\Psi^*}{\partial x} - \Psi^*\frac{\partial\Psi}{\partial x}\right\}$

Normalization (1-D): $\int_{-\infty}^{+\infty}\Psi^*\Psi dx = 1$

Normalization (3-D): $\int_0^{+\infty}\int_0^{\pi}\int_0^{2\pi}\Psi^*\Psi\, r^2\sin\theta\, dr\, d\theta\, d\phi = 1$ or $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Psi^*\Psi\, dx\, dy\, dz = 1$

H-atom: $\Psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$ each of these components may be normalized individually.

Radial probability distribution: $P(r) = r^2 R^*(r) R(r)$

Nuclear Physics:

$R = R_0 A^{1/3} = 1.2 A^{1/3}\, fm$

$B = \left[Zm\left({}_1^1H\right) + Nm_n - m\left({}_z^AX\right)\right]c^2$

$Q = [M_{parent} - M_{Daughter} - M_{emitted}]c^2$

$N = N_0 e^{-\lambda t}$

$A = \lambda N$

$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$

Nuclear Atom:

$F = \frac{(ze)(Ze)}{4\pi\varepsilon_0 r^2}$

$U = \frac{(ze)(Ze)}{4\pi\varepsilon_0 r}$

Rutherford Scattering:

$b = \frac{zZ}{2K}\frac{e^2}{4\pi\varepsilon_0}\cot\tfrac{1}{2}\theta$

$\tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(\frac{b^2v^2}{r_{\min}^2}\right) + \frac{e^2}{4\pi\varepsilon_0}\frac{zZ}{r_{\min}}$

$d = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{K}$

Bohr model: $E_n = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2} = -\frac{13.6 Z_{eff}^2}{n^2}eV$

$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$R = R_\infty\left(\frac{1}{1+m/M}\right)$

$R_\infty = \frac{mk^2e^4}{4\pi c\hbar^3}$ where $k = \frac{1}{4\pi\varepsilon_0}$

Bohr radius: $a_0 = \frac{\hbar^2}{4\pi\varepsilon_0 me^2}$

$r_n = n^2 a_0 / Z$

X-rays:

K-series: $E_{photon} = (13.6eV)\left(\frac{1}{1^2} - \frac{1}{n^2}\right)(Z-1)^2$

L-series: $E_{photon} = (13.6eV)\left(\frac{1}{2^2} - \frac{1}{n^2}\right)(Z-3)^2$

M-series: $E_{photon} = (13.6eV)\left(\frac{1}{3^2} - \frac{1}{n^2}\right)(Z-5)^2$

Some useful mathematical relations:

$$\sqrt{\left(1-u^2/c^2\right)} \approx 1-\frac{1}{2}\frac{u^2}{c^2}$$

$$\frac{1}{\sqrt{\left(1-u^2/c^2\right)}} \approx 1+\frac{1}{2}\frac{u^2}{c^2}$$

For $u^2/c^2 << 1$

Some useful trigonometric relations:

$$\sin(A+B) = \sin(A)\cos(B)+\cos(A)\sin(B)$$

$$\cos(A+B) = \cos(A)\cos(B)-\sin(A)\sin(B)$$

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$\sin^2(A) = \frac{1-\cos(2A)}{2}$$

$$\cos^2(A) = \frac{1+\cos(2A)}{2}$$

$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

Some useful Integrals:

$$\int \sin^2(u)\,du = \frac{1}{2}\left(u-\sin u\cos u\right)$$

$$\int \sin u\cos u\,du = \frac{1}{2}\sin^2 u$$

$$\int \cos^2 u\,du = \frac{1}{2}\left(u+\sin u\cos u\right)$$

$$\int u\sin^2 u\,du = \frac{u^2}{4}-\frac{u\sin 2u}{4}-\frac{\cos 2u}{8}$$

$$\int u\cos^2 u\,du = \frac{u^2}{4}+\frac{u\sin 2u}{4}+\frac{\cos 2u}{8}$$

$$\int u^2\sin^2 u\,du = \frac{u^3}{6}-\left(\frac{u^2}{4}-\frac{1}{8}\right)\sin 2u-\frac{u\cos 2u}{4}$$

$$\int u^2\cos^2 u\,du = \frac{u^3}{6}+\left(\frac{u^2}{4}-\frac{1}{8}\right)\sin 2u+\frac{u\cos 2u}{4}$$

$$\int \sin^3 u\,du = -\frac{1}{3}\cos u\left[\sin^2 u+2\right]$$

$$\int \cos^3 u\,du = \frac{1}{3}\sin u\left[\cos^2 u+2\right]$$

$$\int_0^\infty u^n e^{-u}\,du = n! \quad for\ \ n>0$$

$$\int \cos^n u\sin u\,du = -\frac{\cos^{n+1}u}{n+1} \quad for\ \ n>0$$

$$\int \sin^n u\cos u\,du = \frac{\sin^{n+1}u}{n+1} \quad for\ \ n>0$$

n	$I_n = \int_0^\infty x^n e^{-\lambda x^2}dx$
0	$\frac{1}{2}\pi^{1/2}\lambda^{-1/2}$
1	$\frac{1}{2}\lambda^{-1}$
2	$\frac{1}{4}\pi^{1/2}\lambda^{-3/2}$
3	$\frac{1}{2}\lambda^{-2}$

n	$I_n = \int_0^\infty x^n e^{-\lambda x^2}dx$
4	$\frac{3}{8}\pi^{1/2}\lambda^{-5/2}$
5	λ^{-3}
If n is even	$\int_{-\infty}^{\infty} x^n e^{-\lambda x^2}dx = 2I_n$
If n is odd	$\int_{-\infty}^{\infty} x^n e^{-\lambda x^2}dx = 0$

Constants:

Constant	Standard value	Alternate units
Speed of light	$c = 2.998\times10^{8} m/s$	
Electronic charge	$e = 1.602\times10^{-19} C$	
Boltzmann constant	$k = 1.381\times10^{-23} J/K$	$8.617\times10^{-5} eV/K$
Planck's constant	$h = 6.626\times10^{-34} J\cdot s$	$4.136\times10^{-15} eV\cdot s$
	$\hbar = 1.055\times10^{-34} J\cdot s$	$0.652\times10^{-15} eV\cdot s$
Avogadro's constant	$N_A = 6.022\times10^{23} mole^{-1}$	
Stefan-Boltzmann constant	$\sigma = 5.67\times10^{-8}\ W/m^2\cdot K^4$	
Electron mass	$m_e = 5.49\times10^{-4} u\ or\ 9.109\times10^{-31} kg$	$0.511 MeV/c^2$
Proton mass	$1.007276\ u\ or\ 1.673\times10^{-27}\ kg$	$938.3 MeV/c^2$
Neutron mass	$1.008665\ u\ or\ 1.675\times10^{-27}\ kg$	$939.6 MeV/c^2$
Mass of ^{4}He	*4.002603 u*	
Bohr radius	$a_0 = 4\pi\varepsilon_0\hbar^2 / m_e e^2 = 0.0529 nm$	
Hydrogen ionization energy	$13.6 eV$	
	$hc = 1240 ev\cdot nm$	
	$1 eV = 1.602\times10^{-19} J$	
Atomic mass unit (dalton)	$1u = 931.5 MeV/c^2$	$1.661\times10^{-27} kg$
	$kT = 0.02525 eV \approx \frac{1}{40} eV$ at T=293 K	
	$\frac{1}{4\pi\varepsilon_0} = 8.988\times10^{9} N\cdot m^2\cdot C^{-2}$	
	$\frac{e^2}{4\pi\varepsilon_0} = 1.44 eV\cdot nm$	