12th International Conference on p-Adic Functional Analysis
Abstracts of Talks

José Aguayo (Universidad de Concepción, Chile) and Miguel Nova (Universidad Catolica de la Santisima, Chile) and Khodr Shamseddine (University of Manitoba, Canada)

Title: Characterization of Compact and self-adjoint operators on Free Banach spaces of countable type over the complex Levi-Civita field

Abstract: Let $\mathcal{C}$ be the complex Levi-Civita field and let E be a free Banach space over $\mathcal{C}$ of countable type. Since any such free Banach space is isometrically isomorphic to $c_0(\mathbb{N}, \mathcal{C}, s)$, where $s : \mathbb{N} \to (0, \infty)$, it is enough to study $c_0(\mathbb{N}, \mathcal{C})$ [taking s to be the constant function 1], which will be denoted by $c_0(\mathcal{C})$ or, simply, c_0. In this talk, we define a natural inner product on c_0 which induces the sup-norm of c_0. Of course, c_0 is not orthomodular, so we characterize those closed subspaces of c_0 with an orthonormal complement with respect to this inner product; that is, those closed subspaces M of c_0 such that $c_0 = M \oplus M^{\perp}$. Such a subspace, together with its orthonormal complement, defines a special kind of projection, the so-called normal projection. We present a characterization of such normal projections as well as a characterization of another kind of operators, the compact operators on c_0.

Kamal Boussaf and Alain Escassut (Université Blaise Pascal, France)

Title: Zeros of the derivative of a p-Adic meromorphic function

Abstract: Let K be a complete algebraically closed field of characteristic 0 and let f be a transcendental meromorphic function in K. A conjecture suggests that f' takes every value infinitely many times, what was proved when f has finitely many multiple poles. Here we can generalize the conclusion just by assuming that there exists positive constants c, d such that number of multiple poles inside the disk $|x| \leq r$ is less than cr^d for all $r \geq 1$. Applications are given to entire functions g in K such that g' divides g, to links between residues and zeros of functions admitting primitives and finally to the p-Adic Hayman conjecture in the cases that are not yet solved.

Abdelbaki Boutabaa (Université Blaise Pascal, France)

Title: Factorization of ultrametric meromorphic functions

Abstract: Given an ultrametric meromorphic function, we are interested in how many ways it can be expressed as a composite of other meromorphic functions. Is this always possible? And when this happens, what can we say about the uniqueness of this decomposition? Etc...

Alain Escassut (Université Blaise Pascal, France) and Jacqueline Ojeda (Universidad de Concepción, Chile)

Title: Branched values for p-Adic meromorphic functions

Abstract: Let K be an algebraically closed field of characteristic 0, complete with respect to an ultrametric absolute value. We show that a transcendental meromorphic function in K or an unbounded meromorphic function inside an open disk, cannot admit more than 4 perfectly branched-values. An unbounded analytic function inside an open disk cannot admit more than 2 perfectly branched-values. And an entire function cannot admit more than 1 perfectly branched-value.

Completing a previous result by K. Boussaf and J. Ojeda, we prove that if all zeroes and poles are of order ≥ 2 but finitely many, then f assumes all non-zero value infinitely often. If f is meromorphic and "unbounded" in an open disk, we can prove that if all zeroes and poles are of order 2 but finitely many, then f assumes all non-zero value infinitely often.

Helge Glöckner (Universität Paderborn, Germany)
Title: Invariant manifolds for analytic dynamical systems over ultrametric fields
Abstract: We construct invariant manifolds around a fixed point, in the case of time-discrete, analytic dynamical systems over a complete ultrametric field $\mathbb{K}$.
Typically, we consider an analytic manifold M modelled on an ultrametric Banach space over $\mathbb{K}$, an analytic diffeomorphism $f\colon M \to M$, and a fixed point p of f. Under suitable assumptions on the tangent map $T_p(f)$, we construct a centre-stable manifold, a centre manifold, respectively, an a-stable manifold around p, for a given real number $a \in]0,1]$.

The invariant manifolds can be used to transfer some classical results by J.S.P. Wang [5] concerning p-Adic Lie groups to the case of Lie groups over local fields of positive characteristic, under suitable hypotheses (cf. [4]). For instance, they can be used to see that every Lie group admitting a contractive automorphism is nilpotent [3]. Under mild hypotheses, they also enable the "scale function" and "tidy subgroups" of a Lie group over a local field to be calculated (cf. [4]), which are fundamental concepts in the structure theory of totally disconnected, locally compact topological groups initiated by G. Willis [6]. So far, this was only possible for p-Adic Lie groups [2] and for semi-simple algebraic groups over local fields [1].

References

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William Grafton and Khodr Shamseddine (University of Manitoba, Canada)
Title: On Fourier series for functions on the Levi-Civita field
Abstract: The foundations for Fourier analysis on the Levi-Civita field will be presented. After reviewing basic knowledge about the field, we show how to find the Fourier series for periodic functions, where the period may be finite, infinitely small or infinitely large and the function satisfies Dirichlet-like conditions on a period. Then we apply that to get Fourier series for non-periodic real-valued functions on the field of real numbers $\mathbb{R}$ as well as for functions on the Levi-Civita field that reduce to the Dirac delta function, when restricted to $\mathbb{R}$.

Hans A. Keller (Hochschule Luzern, Switzerland)
Title: Subfields of valued, complete fields
Abstract: Let K be a field of any characteristic and φ a valuation on K. φ may be Archimedian or non-Archimedian. Suppose that K is complete in the valuation topology. A basic theorem of valuation theory states that if L is an extension field of finite degree over K then φ extends uniquely to a valuation ψ on L and the field (L, ψ) is complete too. This is the look from below to above. In our talk we propose to look in the opposite direction from above to below. Specifically, assume (K, φ) to be complete, moreover, suppose that the valuation topology admits a countable base of neighborhoods of 0. We ask: under which conditions does it happen that every subfield $H \subset K$ with degree $[K : H] < \infty$ is itself complete (in the induced topology)? We will show that for a large variety of fields this is indeed so, but there are exceptions.

It turns out that we have to treat two cases separately, depending on whether $\mathrm{char}(K) = 0$ or $\mathrm{char}(K) = p \neq 0$. In both cases we provide necessary and sufficient conditions which are easily checked. The proofs rely on a previous result by the author on continuity of automorphism and involve reasonings of different kinds.

Nicolas Mainetti (Unviersité d'Auvergne, France) and Alain Escassut (Université Blaise Pascal, France)
Title: Maximal ideals on algebras of continuous functions
Abstract: Let $\mathbb{K}$ be a field complete for an ultrametric absolute value $|\,.\,|$, and let E and E' be two complete ultrametric spaces. We denote by A (resp. A') the algebra of bounded continuous functions from E (resp. E') to $\mathbb{K}$, and by B (resp. B') the algebra of bounded uniformly continuous functions from E (resp. E') to $\mathbb{K}$. In some previous work, we described maximal ideals of A (reps. B) by classes of ultrafilters on E for some equivalence relations. Here, we first study properties of morphisms from A to A' (resp. from B to B') and their actions on maximal ideals. We next study properties of maximal ideals of these algebras. We show in particular that any maximal ideal of finite codimension of A or B has codimension 1. We also show that A (resp. B) has maximal ideals of infinite codimension if and only if $\mathbb{K}$ is not locally compact. More precisely, we also show that if E is separable and $\mathbb{K}$ is not locally compact, then a non convergent ultrafilter on E defines a maximal ideal of infinite codimension.

Enno Nagel (Université Pierre et Marie Curie, France)
Title: Fractional differentiability and (locally) polynomial functions
Abstract: Recently the notion of an r-times differentiable function for a real number $r \geq 0$ has proven to be a valuable tool in the representation theory of p-Adic Lie groups. We will first off briefly introduce those through a description by iterated difference quotients and recall their basic properties.
We will then show the space of locally polynomial functions of degree $\leq r$ to be dense inside the associated space $\mathbb{K}$-Banach space $\mathcal{C}^r(X, \mathbb{K})$. Consequently we recover the original definition of r-fold differentiability on $\mathbb{Z}_p$ through a growth condition of the function's Mahler coefficients and infer the theorem of Amice-Velu-Vishik describing their dual space. Finally we obtain a complete characterization of the canonical (van der Put-)basis of locally polynomial functions in $\mathcal{C}^r(\mathbf{o}, \mathbb{K})$ on the ring of integers $\mathbf{o} \subseteq \mathbb{K}$ inside a locally compact non-Archimedeanly valued field $\mathbb{K}$.

Herminia Ochsenius and Elena Olivos (Universidad Catolica de Chile, Chile)
Title: A Generalized Keller space over a field with a valuation of rank $\rho > \omega$
Abstract: We describe a Banach space E whose norm is induced by an inner product but in which the Projection Theorem is not valid. The field of scalars has a non-Archimedean valuation of rank $\rho > \omega$. It is a generalized Keller space in the sense of Priess-Crampe.
We show explicitly how the construction of closed hyperplanes that are orthogonally dense is linked to continuous linear functionals that do not satisfy Riesz theorem, and to a non-denumerable set of Lipschitz operators that have no adjoint.

Jacqueline Ojeda (Universidad de Concepción, Chile) and Kamal Boussaf and Alain Escassut (Université Blaise Pascal, France)
Title: p-Adic meromorphic functions $f'P'(f), g'P'(g)$ sharing a small function
Abstract: Let K be a complete algebraically closed p-Adic field of characteristic zero. Let $f,\ g$ be two transcendental meromorphic functions in the whole field K or meromorphic

functions in an open disk, that are not quotients of bounded analytic functions in that disk. Let P be a polynomial of uniqueness for meromorphic functions in K or in an open disk and let α be a small meromorphic function with regards to f and g. If $f'P'(f)$ and $g'P'(g)$ share α counting multiplicity, then we show that $f = g$ provided the multiplicity order of zeroes of P' satisfy certain inequalities. If α is a Moebius function or a non-zero constant, we can obtain more general results on P.

If $f,\ g, \alpha$ are analytic functions, all results are obtained in much more general hypotheses.

Elena Olivos (Universidad Catolica de Chile, Chile) and W. H. Schikhof (University of Nijmegen, the Netherlands)

Title: All proper multiplications on the Dedekind completion of a totally ordered abelian group

Abstract: Let G be a multiplicative written totally ordered abelian group with unit 1_G and let $G^{\#}$ be its Dedekind completion. If G is not a subgroup of the positive real numbers, the formulas $x \cdot y := \sup_{G^{\#}}\{g_1 g_2 : g_1 \leq x \text{ and } g_2 \leq y\}$ and $x * y := \inf_{G^{\#}}\{g_1 g_2 : g_1 \geq x \text{ and } g_2 \geq y\} (x,\, y \in G^{\#})$ define two natural extensions of the multiplication $G \times G \to G$ to a multiplication $G^{\#} \times G^{\#} \to G^{\#}$. In this case $G^{\#}$ is not a group neither with respect to $\cdot$ nor to $*$ but only a commutative semigroup with unit 1_G. In this work we shall describe all proper extensions of the multiplication of G to the Dedekind completion $G^{\#}$ that make $G^{\#}$ a semigroup.

References

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Cristina Perez-Garcia (Universidad de Cantabria, Spain)

Title: The Grothendieck approximation theory in non-Archimedean functional analysis

Abstract: This paper consists of a survey of the most important results on non-Archimedean bounded approximation properties obtained in recent papers by the author (one of them jointly with W.H. Schikhof), together with some new results on the subject. As an application, an affirmative answer is given, for non-spherically complete base fields, to the following problem, posed by the author and W.H. Schikhof in 2010: Does there exist an absolutely convex edged set B in a non-Archimedean locally convex space such that the closure of B is not edged?

Yaroslav D. Sergeyev (Universita della Calabria, Italy)

Title: A new methodology for executing numerical computations with infinities and infinitesimals: Interrelations between mathematical objects and tools used for their observation

Abstract: A new methodology allowing one to execute numerical computations with finite, infinite, and infinitesimal numbers on a new type of a computer: the Infinity Computer, is introduced. A calculator using the Infinity Computer technology is presented during the talk. The new approach is based on the principle "The part is less than the whole" introduced by Ancient Greeks that is applied to all numbers (finite, infinite, and infinitesimal) and to all

sets and processes (finite and infinite). It is shown that it becomes possible to write down finite, infinite, and infinitesimal numbers by a finite number of symbols as particular cases of a unique framework (different from that of the non-standard Analysis). The new methodology evolves ideas of Cantor and Levi- Civita in a more applied way and (among other things) introduces infinite integers that possess both cardinal and ordinal properties as usual finite numbers. Note that foundations of the Set Theory dealing with infinity have been developed starting from the end of the XIX-th century until more or less the first decades of the XX-th century. Foundations of the classical Analysis dealing both with infinity and infinitesimal quantities have been developed even earlier, more than 200 years ago, with the goal to develop mathematical tools allowing one to solve problems arising in the real world in that remote time. As a result, these parts of Mathematics reflect ideas that people had about Physics (including definitions of the notions continuous and discrete) more than 200 years ago. Thus, these mathematical tools do not include numerous achievements of Physics of the XX-th century. Even the brilliant non-standard Analysis of Robinson made in the middle of the XX-th century has been also directed to a reformulation of the classical Analysis (i.e., Analysis created two hundred years before Robinson) in terms of infinitesimals and not to the creation of a new kind of Analysis that would incorporate new achievements of Physics. The point of view on infinite and infinitesimal quantities presented in this talk uses strongly two methodological ideas borrowed from the modern Physics: relativity and interrelations holding between the object of an observation and the tool used for this observation. The latter is directly related to connections between different numeral systems used to describe mathematical objects and the objects themselves.

Yaroslav D. Sergeyev (Universita della Calabria, Italy)
Title: A new methodology for executing numerical computations with infinities and infinitesimals: Applications and numerical examples
Abstract: The new computational methodology gives the possibility both to execute numerical (not symbolic) computations of a new type and simplifies fields of Mathematics where the usage of the infinity and/or infinitesimals is necessary. Numerous examples and applications are considered: analysis (divergent series, limits, derivatives, integrals, , etc.), measure theory, probability theory, fractals, approximation, differentiation, cellular automata, differential equations, linear and non-linear optimization, fractals, percolation, processes of growth in biological systems, etc. In particular, a number of results related to the First Hilbert Problem and Turing machines are established and a new concept of continuity better reflecting the modern view of physicists on the world around us is introduced.

Khodr Shamseddine (University of Manitoba, Canada) and José Aguayo (Universidad de Concepción, Chile) and Miguel Nova (Universidad Catolica de la Santisima, Chile)
Title: B*-algebras of operators and study of positive operators on a free Banach space of countable type over the complex Levi-Civita field
Abstract: Let $\mathcal{C}$ be the complex Levi-Civita field and let $c_0(\mathcal{C})$ or, simply, c_0 denote the space of all null sequences $x = (a_n)$, $a_n \in \mathcal{C}$. Then the natural inner product on c_0 induces the supnorm of c_0. In a previous talk, we presented characterizations of normal projections, adjoint operators and compact operators on c_0. In this talk, we work on some algebras of operators, including those mentioned above. Then we study the properties of positive operators on c_0, which we then use to introduce a partial order on the algebra of compact and self-adjoint operators on c_0 and study the properties of that partial order.

Todd Sierens and Trevor Rempel (University of Waterloo, Canada) and Khodr Shamseddine (University of Manitoba, Canada)
Title: On locally uniformly differentiable functions: the Inverse Function Theorem and the Implicit Function Theorem in a non-Archimedean setting
Abstract: Let $\mathcal{N}$ be a non-Archimedean field extension of the real numbers that is real closed and Cauchy complete in the topology induced by the order. In this talk, we study the properties of locally uniformly continuous (LUC) and locally uniformly differentiable (LUD) $\mathcal{N}$-valued functions on $\mathcal{N}$ and we use those properties to formulate and prove a version of the inverse function theorem as well as a local intermediate value theorem. Then we generalize the (LUD) concept to functions of several variables and we obtain an inverse function theorem for functions from $\mathcal{N}^n$ to $\mathcal{N}^n$ and an implicit function theorem for functions from $\mathcal{N}^n$ to $\mathcal{N}^m$ with $m < n$.

Sebastian Troncoso and Martin Berz (Michigan State University, USA), and Herminia Ochsenius (Universidad Catolica de Chile, Chile)
Title: Affine Invariant Lebesgue Measures in Levi-Civita Vector Spaces and the Erdös Obtuse Angle Theorem
Abstract: An interesting question posed by Paul Erdös around 1950 pertains to the maximal number of points in n-dimensional Euclidean Space so that no subset thereof of three points can be picked that form an obtuse angle. An unexpected and surprising solution was presented around a decade later, which in its core relies on properties of measures in n-dimensional space. In order to study the question whether the theorem holds in the same way in Levi-Civita vector spaces, we develop a Lebesgue measure in these spaces that is invariant under affine transformations and satisfies commonly expected properties of Lebesgue measures, in particular a substitution rule based on Jacobians of transformations. Using the tools from this measure theory, we will show that the Obtuse Angle Theorem also holds on the non-Archimedean Levi Civita vector spaces.

Alexander Wittig and Martin Berz (Michigan State University, USA)
Title: Inverse and Implicit Function Theorems on Levi-Civita Vector Spaces
Abstract: One of the fundamental questions of conventional multivariate calculus is existence and uniqueness of inverses in local neighborhoods of points with suitable Jacobians, as well as the local solvability of implicit function equations under related conditions. We study this question for the case of Levi Civita vector spaces and develop sufficient conditions for the existences of inverses and implicit functions mimicking as closely as possible the real case. Of particular importance in this case is the condition number, a tool from numerical analysis to assess the difficulty of performing certain numerical methods efficiently, which gets simpler the smaller the number is. In the Levi-Civita vector field case, this "soft" requirement becomes rigorous, in the sense that certain operations work if and only if the condition number is rigorous. As an application of these results, we utilize this approach to determine local implicit functions arising from problems in the theory of dynamical systems and derive rigorous statements about the behavior of certain manifolds around fixed points.