Supplementary Material for

Multimode propagation in phononic crystals with overlapping Bragg and hybridization effects

C. Croënne,[1,2] E. J. S. Lee,[2,a)] and J. H. Page[2,b)]

[1)] Université Lille, CNRS, Centrale Lille, Junia, Université Polytechnique Hauts-de-France, UMR 8520 - IEMN, F-59000 Lille, France

[2)] Department of Physics and Astronomy, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada

ABSTRACT

In this supplementary material, we present additional information on the phononic crystal that was constructed from nylon rods, including a schematic of the crystal structure, the conventional band structure along the propagation direction investigated in this work, and the experimental setup.

The phononic crystal was fabricated from cylindrical nylon rods arranged in a two-dimensional (2D) triangular lattice and immersed in water. The rod diameter was 0.46 mm, and the lattice constant *a* was 0.98 mm, corresponding to a nylon filling fraction of 20%. The 2D crystal structure (direct lattice) is illustrated schematically in Fig. S1(a).

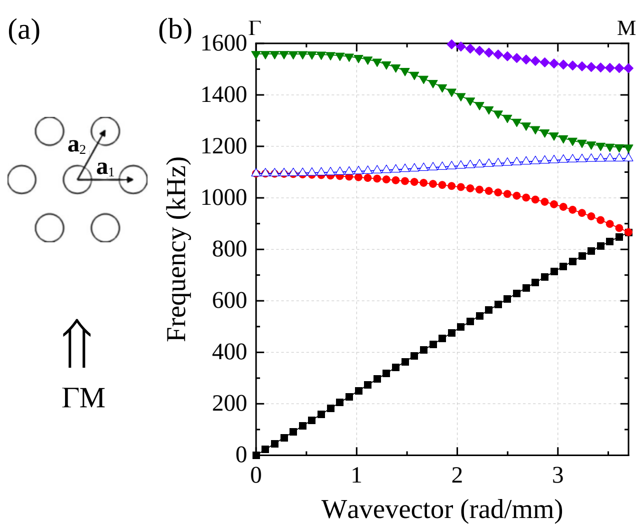

FIG. S1: (a) The triangular direct lattice structure of the 2D nylon rod phononic crystal, showing the primitive lattice vectors $\mathbf{a}_1$ and $\mathbf{a}_2$. The vertical arrow indicates the direction of propagation (ΓM). (b) The conventional band structure along the ΓM direction. The third band (open blue triangles) is "deaf" as it cannot be excited by an incident plane wave.

As an initial point of comparison with the analysis presented in the letter, we plot in Fig. S1(b) the conventional band structure along the propagation direction that was studied in this work (the ΓM direction). [To see how this propagation direction relates to the direct crystal lattice, see the vertical arrow in the lower part of Fig. S1(a)]. For this figure, the band structure was simulated using the COMSOL Finite Element Method software with nylon parameters of density ρ = 1150 kg/m^3, longitudinal velocity v_L = 2400 m/s, and shear velocity v_T = 994 m/s, and with water parameters ρ = 998.7 kg/m^3 and velocity v_L = 1475 m/s. The first and fourth branches resemble the lowest two branches for phononic crystals of steel rods,[1] where rod resonances do not occur at these frequencies,[2] implying that the second and third branches for the 20% nylon rod crystal here must be associated with the lowest frequency resonance of the nylon rods (f_{res} ~1 MHz, depending on temperature). The second branch (solid red circles) has the right symmetry to couple to an incoming plane wave, but the third one (open blue triangles) does not, so this third branch is predicted to be "deaf". Curiously, there is no

[a)] Current address: NVH Test Team 2, Research Development Division, Hyundai Motor Company, Hwaseong-si, Gyeonggi-do, Republic of Korea, 18280

[b)] Author to whom correspondence should be addressed: john.page@umanitoba.ca

FIG. S2: Picture of the 20% nylon rod crystal and support structure.

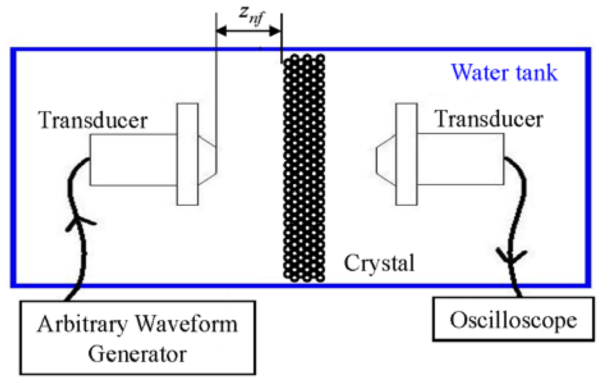

FIG. S3: Schematic representation (not to scale) of the experimental setup.

distinct band gap between the first and second branches, for which the symbols in this simulation overlap at the Brillouin zone boundary. Of greater relevance in the context of this work is the following: while it might be tempting to try to interpret the experimental data starting from this band structure diagram, it is clear that it cannot explain the complex behavior observed near 1 MHz, and in particular why switching occurs as the number of layers in the crystal is varied or as the temperature is varied. Hence, a more detailed analysis appropriate to crystals of a finite thickness is needed, as described in the main text of the letter.

To ensure excellent crystal quality for the experimental measurements, the nylon rods were accurately held in position via a support structure with regularly positioned holes, through which the nylon rods were threaded and then maintained under sufficient tension to ensure that the rods remained straight. A photograph of one of the crystals and its support structure is shown in Fig. S2. The locating holes for the nylon rods were drilled so that the orientation of the triangular lattice has its ΓM direction perpendicular to the front and back interfaces of the crystal. Thus, for a normally incident ultrasonic beam, the measured transmission corresponds to propagation along this direction.

To carry out the ultrasonic transmission experiments, the crystal was lowered into a large water tank, where it was situated between two planar ultrasonic immersion transducers, one acting as emitter and the other as receiver. The distance between the transducers and the phononic crystals was set at the transducer near field distance, z_{nf}, so as to ensure that the ultrasonic beam had a smooth transverse profile with minimal angular divergence. For the measurements reported in this letter, the central frequency of each transducer was 1.0 MHz, and a narrow broadband Gaussian input pulse was used to acquire time-dependent transmitted signals that spanned the frequency range from 0.6 to 1.5 MHz. The transmitted waveforms were signal averaged on the oscilloscope and transferred to a computer for data analysis. A reference signal was recorded with the crystal removed so that the phase and amplitude of the transmitted ultrasonic waves relative to the input pulse could be determined. Transmitted phase and amplitude information was extracted from the complex Fourier transforms of the time domain signals. Hence, measurements of the real part of the wave vector, $k(\omega) = \Delta\phi / L$, and the intensity transmission coefficient A^2_{trans}/A^2_{input}, were obtained. Here $\Delta\phi$ is the phase delay and A denotes amplitude. Because the input pulse was deliberately chosen to be broadband, these quantities were measured over a broad frequency range from a single experiment.

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[1] A. Sukhovich, Li Jing and J. H. Page, “Negative refraction and focusing of ultrasound in 2D phononic crystals”, Phys. Rev. B **77**, 014301 (2008).

[2] E.J.S. Lee, PhD Thesis, University of Manitoba (2014). See section 2.2.1 and Figs. 2.3 and 2.4 for a comparison of the total cross section of nylon and steel rods with the same diameter (0.47 mm) as that used for the current work.