WileyPLUS Assignment 5 is available

Chapters 28, 29, 30

Due Friday, April 9 at 11 pm

Friday, April 2

Easter Holiday

No lectures, labs, tests...!

PHYS 1030 Final Exam

Friday, April 23, 1:30-4:30 pm

Frank Kennedy Brown Gym,

30 questions, the whole course, formula sheet provided

Bohr Model for H and H-like Atoms

- A planetary model in which the electron defies the normal laws of electromagnetism and does not radiate when in certain "stationary" states.

- The angular momentum of the electron in the stationary states is:

$$L_n = mvr = \frac{nh}{2\pi}, \quad n = 1, 2, 3 \ldots$$

  n is a "quantum number".

- The energy of the n'th quantum state is: $E_n = -13.6 \frac{Z^2}{n^2}$ eV
  (for a H-like atom of atomic number Z)
- Light is emitted or absorbed when the atom changes energy state.

- The energy of the photon is the difference in energy of the two states of the atom.

(Handwaving) justification of Bohr's condition on the angular momentum

$$L_n = mvr = \frac{nh}{2\pi} \qquad n = 1, 2, 3...$$

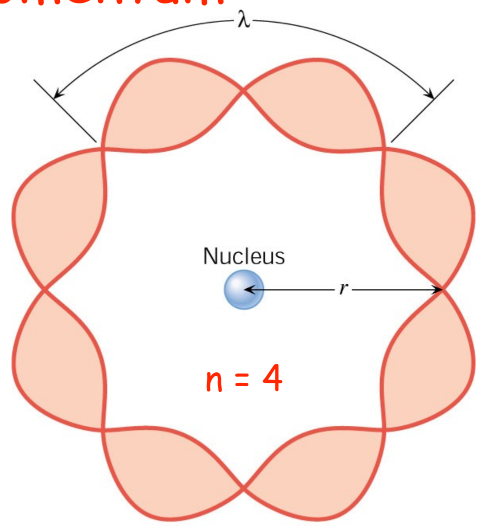
Substitute for the de Broglie wavelength of the electron:

$$mv = p = \frac{h}{\lambda}$$

$$\text{So, } mvr = \frac{hr}{\lambda} = \frac{nh}{2\pi}$$

$$\text{Therefore, } 2\pi r = n\lambda$$

This does **NOT** show the path of the electron, but indicates there is a wave moving around the orbit - take with a pinch of salt!



⇒ The circumference of the orbit is an integer number of wavelengths.

Condition for a "standing wave" that reinforces itself.

Prob. 30.13/12: A hydrogen atom is in the ground state. It absorbs energy and makes a transition to the n = 3 excited state. The atom returns to the ground state emitting two photons. What are their wavelengths?

658 nm, 122 nm

Bohr's theory applies to hydrogen-like atoms and ions – that have just one electron in orbit around the nucleus.

Examples:

H atom,	$Z = 1$,	1 electron present
He^{+} ion,	$Z = 2$,	1 electron removed
Li^{2+} ion,	$Z = 3$,	2 electrons removed
Be^{3+} ion,	$Z = 4$,	3 electrons removed, etc

Prob. 30.20/50: Doubly ionized lithium Li^{2+} ($Z = 3$) and triply ionized beryllium Be^{3+} ($Z = 4$) each emit a line spectrum. For a certain series of lines in the lithium spectrum, the shortest wavelength if 40.5 nm. For the same series of lines in the beryllium spectrum, what is the shortest wavelength?

Both are hydrogen-like atoms, with all electrons removed but one, so Bohr's formula applies for energies of stationary states:

$E_n = -13.6\ Z^2/n^2$ eV 22.8 nm

Prob. 30.1: The nucleus of the hydrogen atom has a radius of about 10^{-15} m. The electron is normally at a distance of about 5.3×10^{-11} m from the nucleus.

Assuming the hydrogen atom is a sphere of radius 5.3×10^{-11} m, find:

a) the volume of the atom,

b) the percentage of the volume of the atom that is occupied by the nucleus.

a) 6.24×10^{-31} m^3,
b) 6.7×10^{-13} %

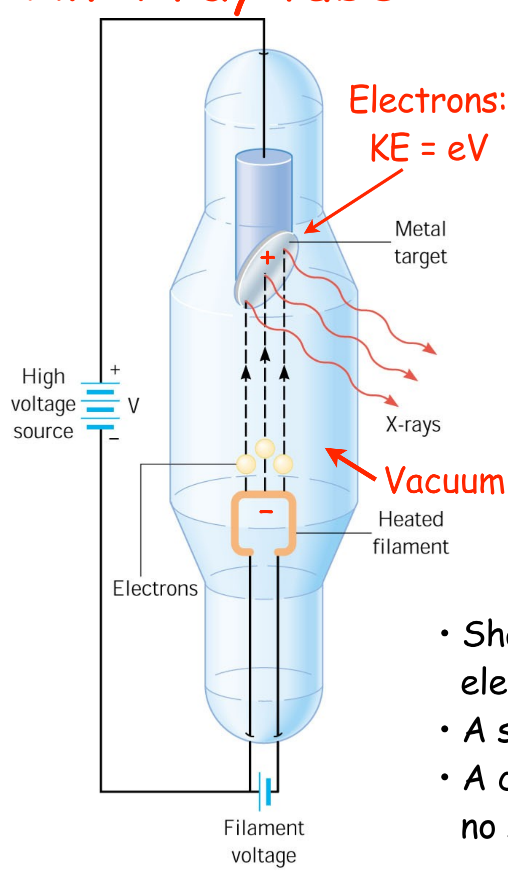
Prob. 30.5/6: There are Z protons in the nucleus of an atom, where Z is the atomic number of the element. An α particle (nucleus of He atom) carries a charge +2e.

In a scattering experiment, an α particle, heading directly toward a nucleus in a metal foil, will come to a halt when all the particle's kinetic energy is converted to electric potential energy.

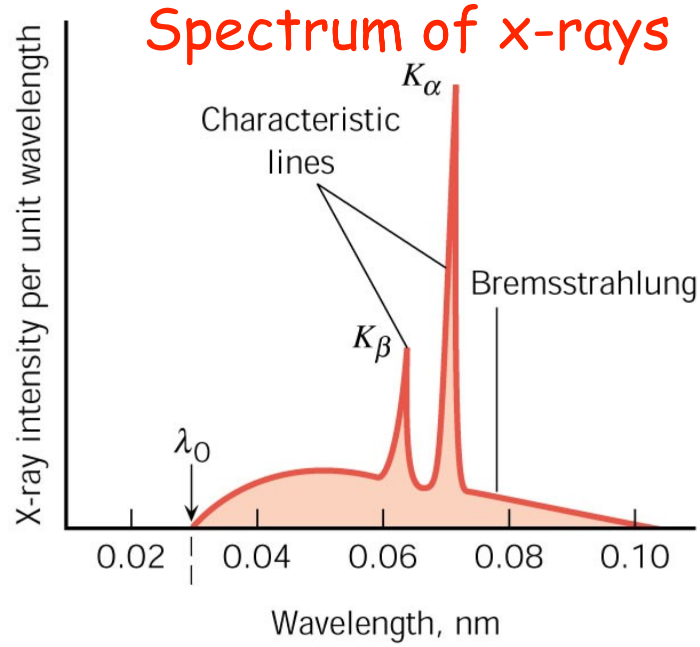
How close will an α particle with kinetic energy of 5×10^{-13} J come to a gold nucleus (Z = 79)?

7.3×10^{-14} m

An x-ray tube



Spectrum of x-rays



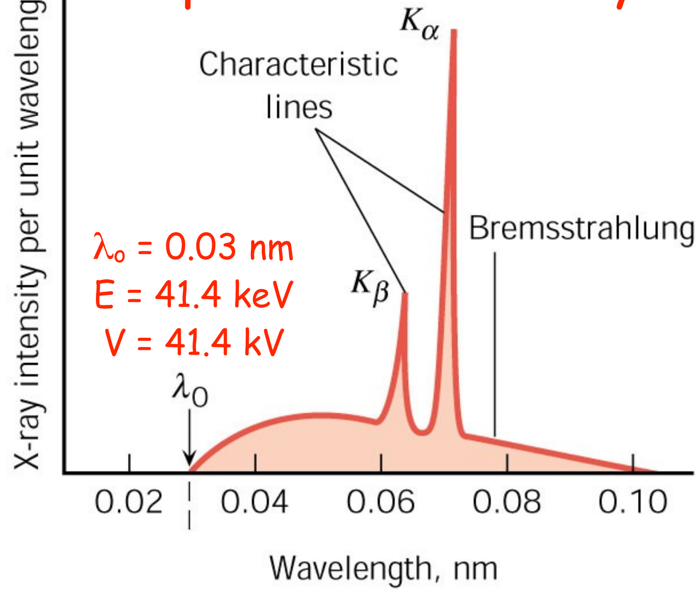
- Sharp lines characteristic of the chemical element of the target.
- A smooth background ("Bremsstrahlung")
- A cutoff wavelength, λ_0, below which there are no x-rays

Spectrum of x-rays

- Bremsstrahlung: due to the electrons being stopped rapidly on hitting the target – accelerated charges emit electromagnetic radiation.

- The shortest wavelength x-rays are produced when all of the kinetic energy of the electrons is converted into x-rays:

  KE = $eV = hf = hc/\lambda_0$

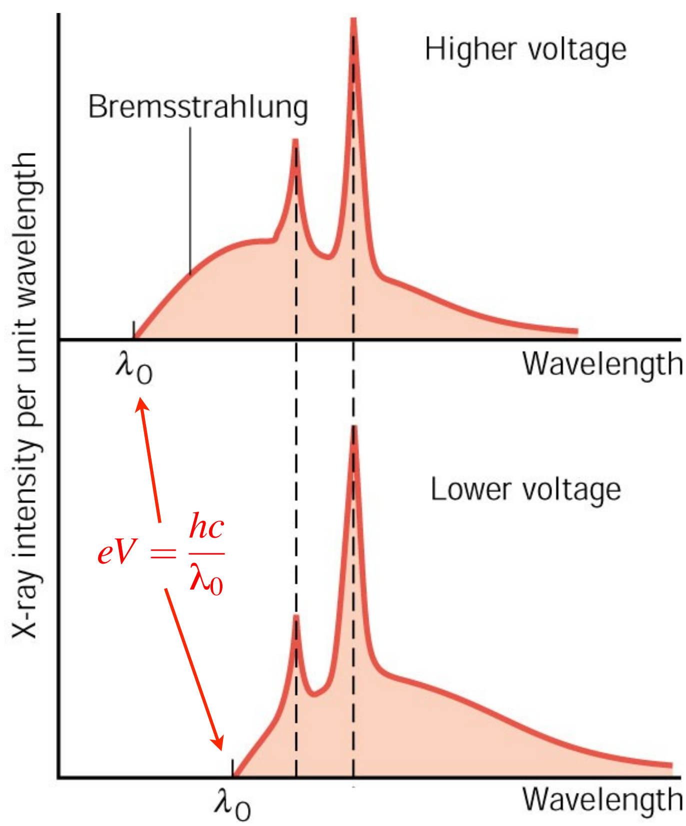


- Characteristic x-rays: from electron transitions in the target atom. Wavelengths depend on the target.

K_α: $(n = 2) \Rightarrow (n = 1)$ "K-shell" K_β: $(n = 3) \Rightarrow (n = 1)$ (Lyman)

L_α: $(n = 3) \Rightarrow (n = 2)$ "L-shell" L_β: $(n = 4) \Rightarrow (n = 2)$ (Balmer), etc

x-ray spectra at different voltages



- The characteristic x-ray peaks do not move – they are properties of the target atoms.

- The cutoff wavelength, λ_0, decreases as the accelerating voltage increases:

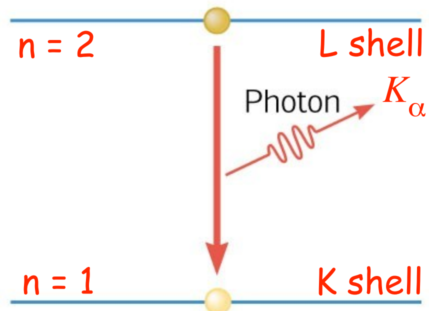
  $\lambda_0 \propto 1/V$

- The Bremsstrahlung component also shifts as the voltage is changed.

Wavelength of K_α x-rays

Moseley found a quite accurate approximate result **for K_α x-rays for atoms with one electron removed from the K shell**:

$$E_{K_\alpha} = \frac{hc}{\lambda} = 13.6(Z-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)\ \text{eV}$$



The same as the Bohr formula, only with Z replaced by Z – 1.
(there is a second electron in the K shell that partly shields the charge of the nucleus)

Found that some elements were in the wrong place in the periodic table. Identified other previously unknown elements.

Prob. 30.-/46: The atomic number of lead is Z = 82.

According to Moseley and the Bohr model, what is the energy of a K_α x-ray photon?

E = 1.1×10^{-14} J = 68.75 keV, wavelength = 0.019 nm

Prob. 30.-/36: What is the minimum potential difference that must be applied to an x-ray tube to knock a K-shell electron completely out of an atom of copper (Z = 29) target?

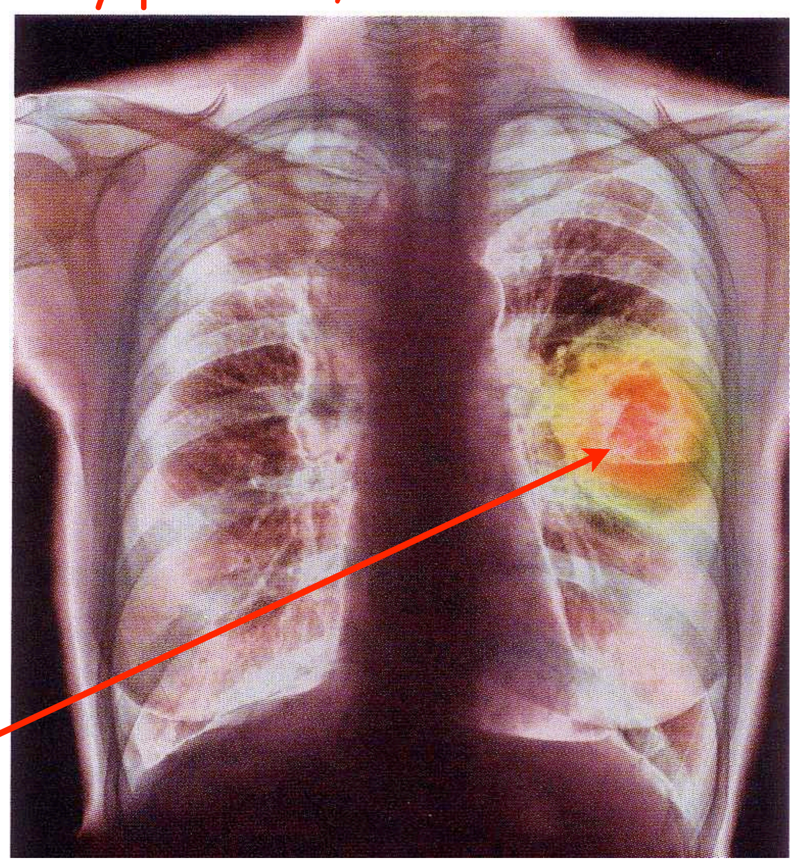
10,700 V

A conventional x-ray picture, colour-enhanced

A picture of the shadows cast by bones and tissue.

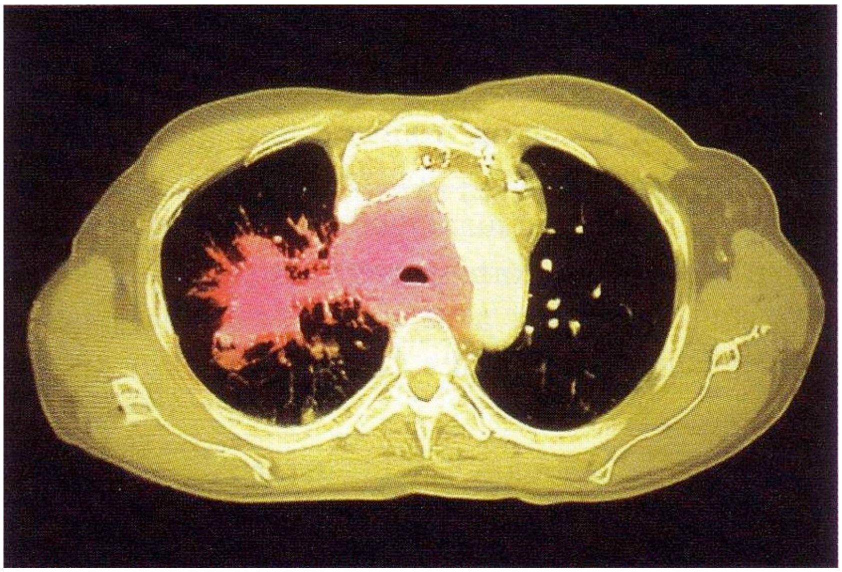
The higher the accelerating voltage in the x-ray tube, the shorter the average wavelength of the x-rays and the more penetrating they are.

There is a trade-off between penetrating power of the x-rays and contrast of the image.

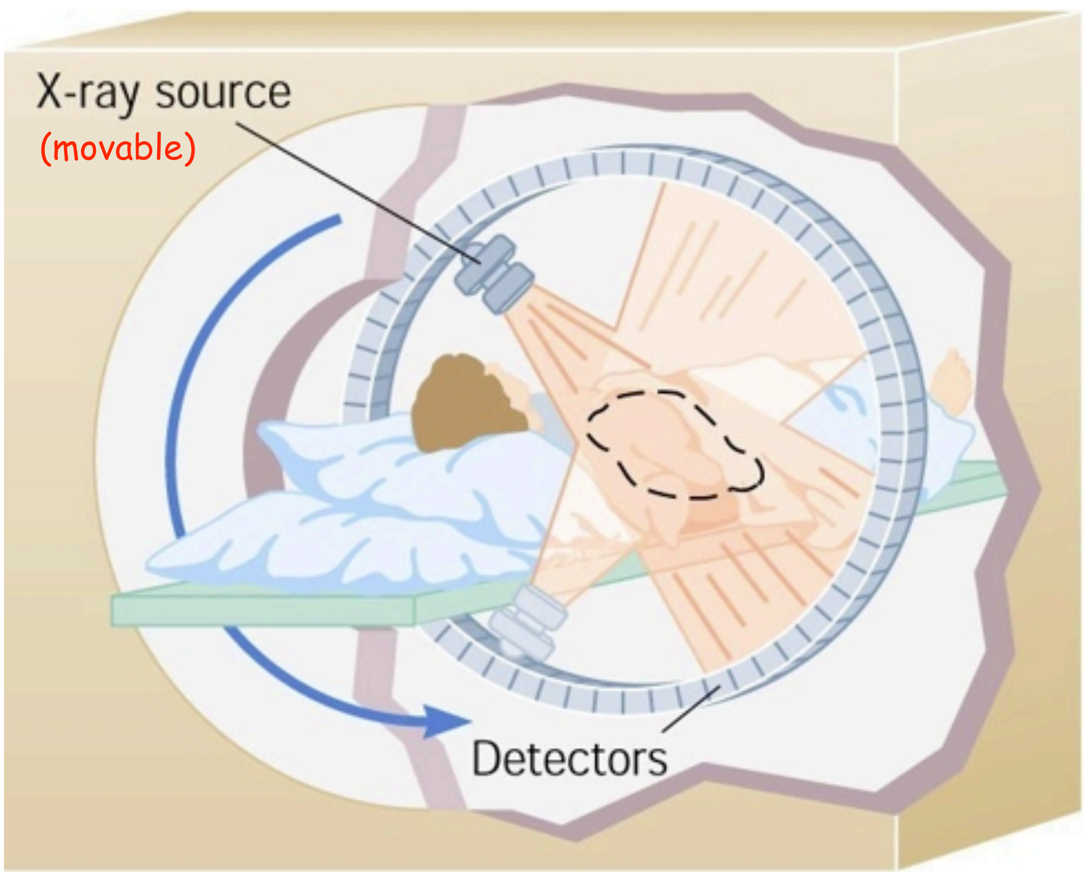


CT scans (Computerized Tomography) fires x-rays into the subject and creates x-ray shadow pictures from many directions. Computer analysis is able to reconstruct three-dimensional images from the many pictures. X-ray detectors are used in place of film.

In CAT, or CT, scans (Computerized Axial Tomography) a narrow fan of x-rays is used to image a slice through the body.

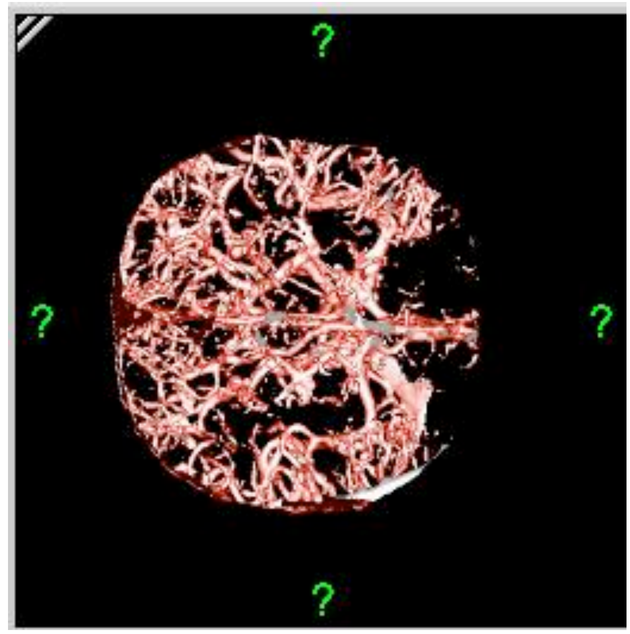


CAT Scan



"Segmentation": bone appears dense white in the CT images, which can be removed from the image by computer manipulation

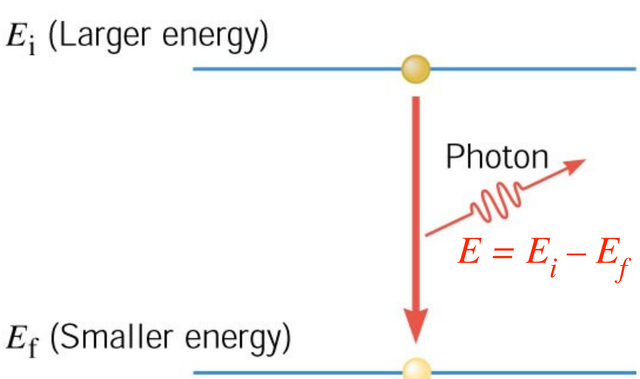
Bone reconstructed in 3D



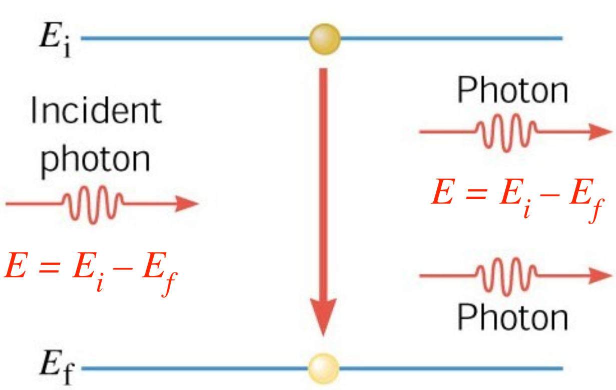
Brain vessels reconstructed in 3D after bone has been removed by segmentation

The Laser

Light Amplification by Stimulated Emission of Radiation



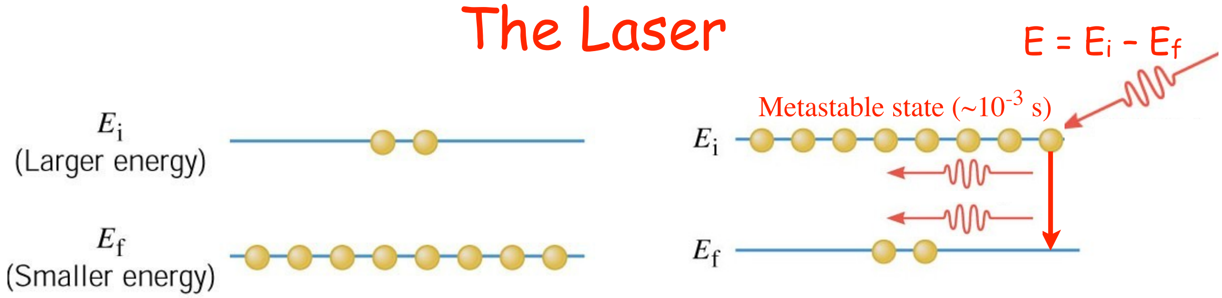
Spontaneous emission – the electron in an excited state falls back to a lower state at a random time, emitting a photon of energy $E_i - E_f$ in a random direction.



Stimulated emission – a photon of energy $E_i - E_f$ induces the emission of a photon of the same energy, resulting in two photons of the same energy travelling in phase in the same direction

⇒ amplification of the light.

The Laser



(a) Normal population (b) Population inversion

For light amplification to occur:

- There must be a large number of atoms in the excited state – a "population inversion".
- The excited state must be long-lived – a "metastable state".

Then atoms in the metastable state stay there long enough for a photon of the right energy to come along and cause stimulated emission.

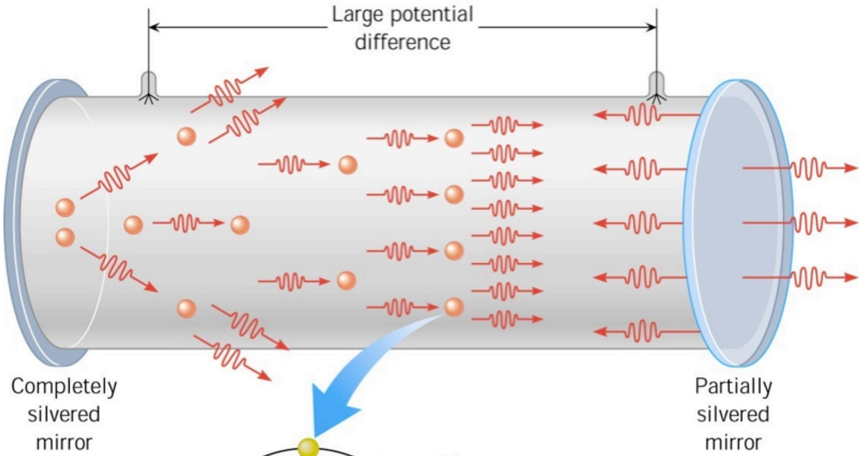
The result is a build up of photons of the same wavelength that are in phase – coherent light.

The Laser

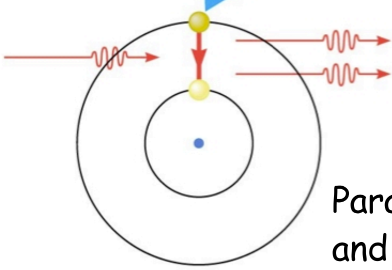
Laser light is:

- Highly coherent. As the light waves are in phase, the combined amplitude is large, and the intensity much greater than a source of incoherent light, such as a lamp, that may emit the same number of photons per second.
- Very sharply defined in wavelength, whereas an incandescent lamp emits blackbody radiation, which is over a wide wavelength range, including the invisible infra-red.
- Emitted in a narrow beam with divergence limited by diffraction.

Helium-Neon Laser (eg checkout scanner)



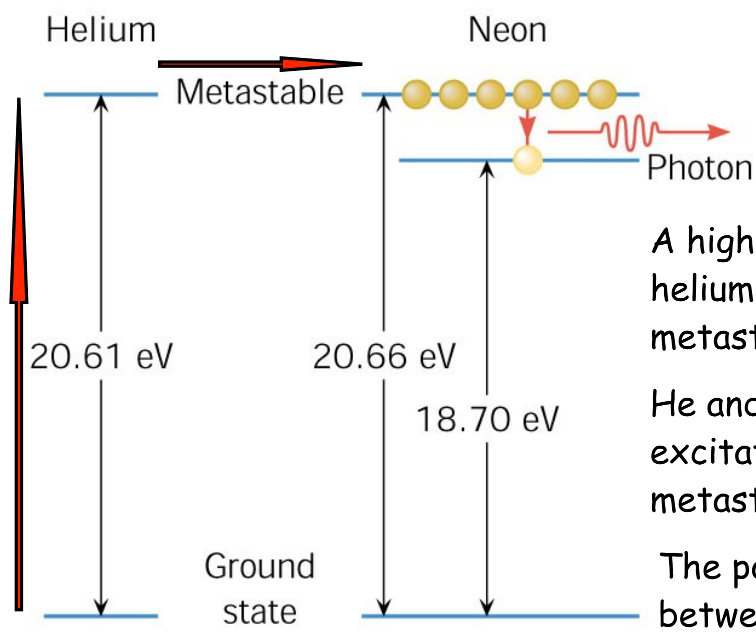
Low pressure He (15%) - Ne (85%) in a glass tube



A high voltage discharge in the gas produces a population inversion.

Parallel mirrors at the ends reflect light back and forth. Light angled too far from the axis of the laser escapes through the sides. Only light travelling parallel to the axis is amplified.

Helium-Neon Laser



A high voltage discharge excites helium atoms to the 20.61 eV metastable state.

He and Ne atoms collide, the excitation is transferred to the Ne metastable level.

The population inversion is between the 20.66 and 18.70 eV levels of Ne.

The energy of the photons is 20.66 – 18.70 eV = hc/λ

λ = 633 nm, red.

Lasers

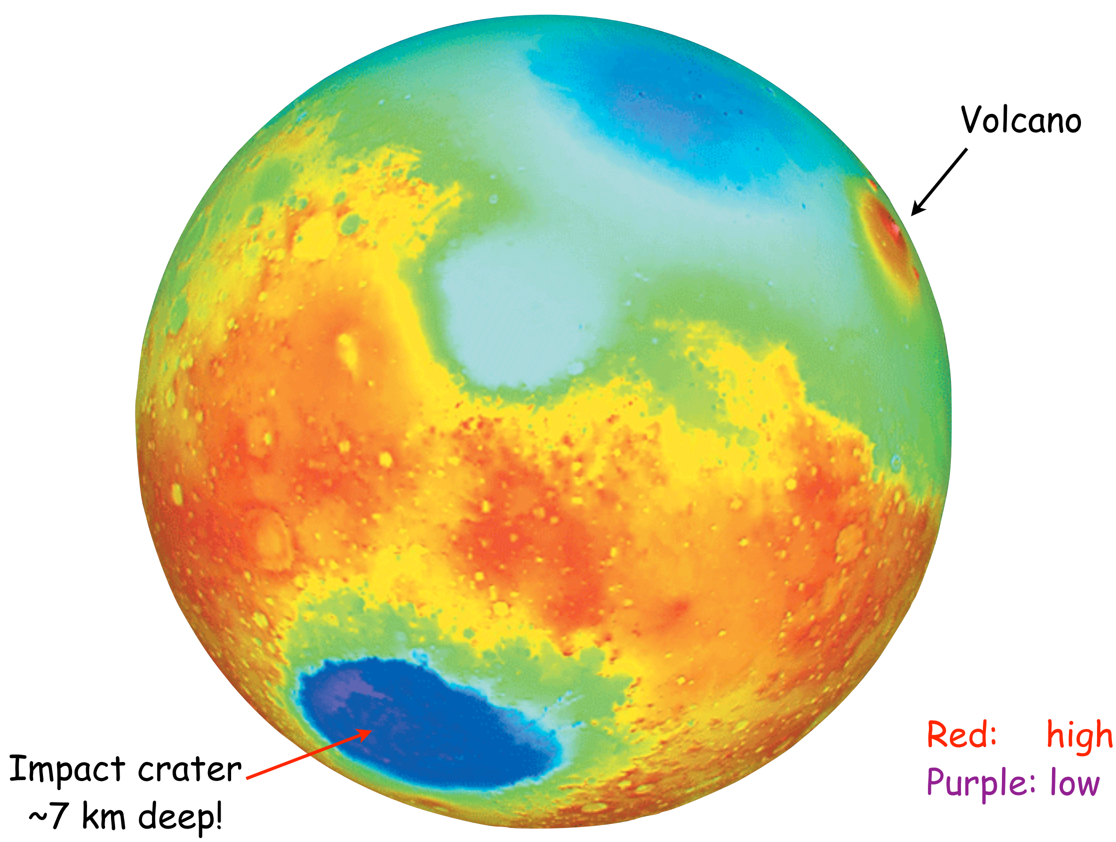
There are now many types of laser in power ranges from mW to MW, continuous beam and pulsed:

He-Ne, ruby, argon-ion, CO_2, gallium arsenide solid state, chemical dye...

Uses:

- playback/recording of CDs, DVDs
- checkout scanners
- welding metal parts
- accurate distance measurement
- telecommunications
- study of molecular structure
- medicine
  - selective removal of tissue, example, shaping the cornea of the eye
  - removal of "port wine stain" birth marks
  - destroying tumours with light-activated drugs
  - reattaching detached retinas

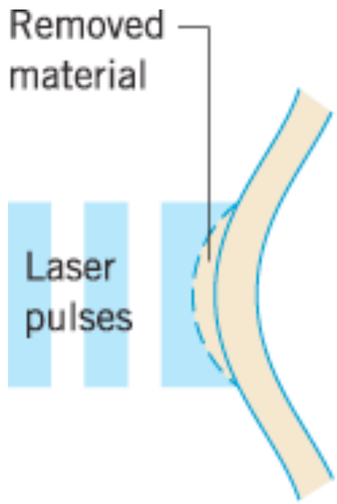
Laser Altimetry of Mars



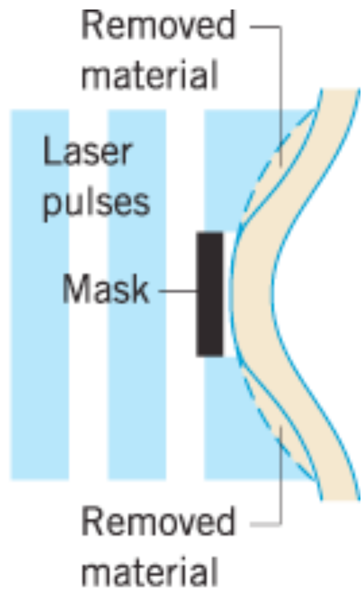
Photorefractive keratectomy (PRK)

Offers an alternative treatment for nearsightedness and farsightedness that does not rely on lenses

UV laser - light absorbed by cornea, causing vaporization of material



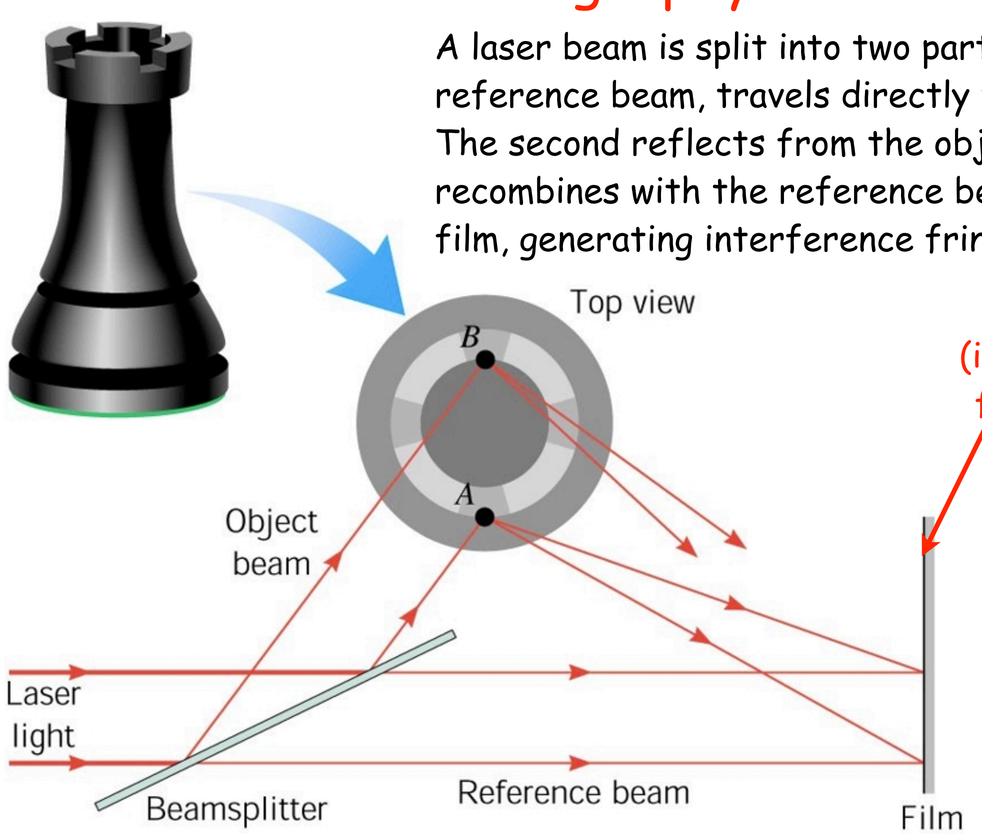
Correcting nearsighted



Correcting farsighted

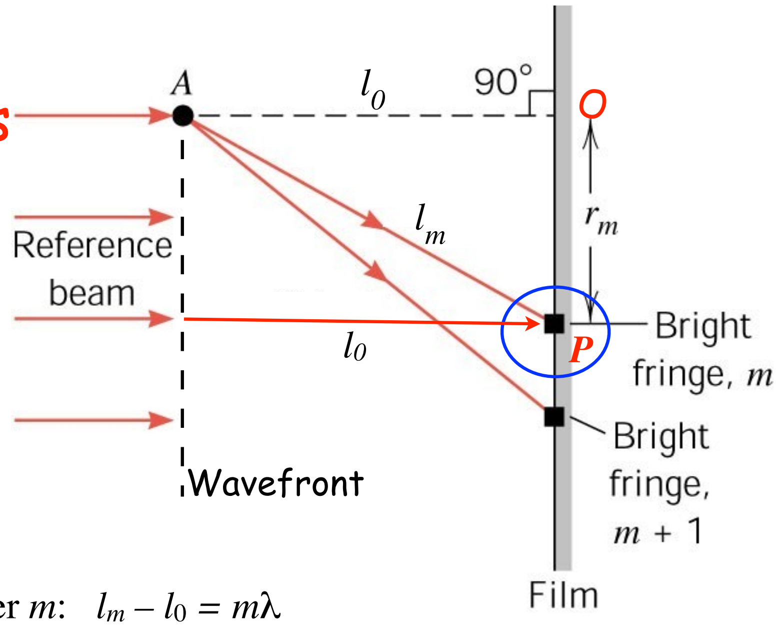
Holography

A laser beam is split into two parts. The first, a reference beam, travels directly to the film. The second reflects from the object and recombines with the reference beam at the film, generating interference fringes there.



Hologram (interference fringes, not an image)

Holography – the interference fringes



For constructive interference at P, the path difference between the reference beam and the beam that scatters from A should be an integer multiple of the wavelength of the light.

For the bright fringe of order m: $l_m - l_0 = m\lambda$

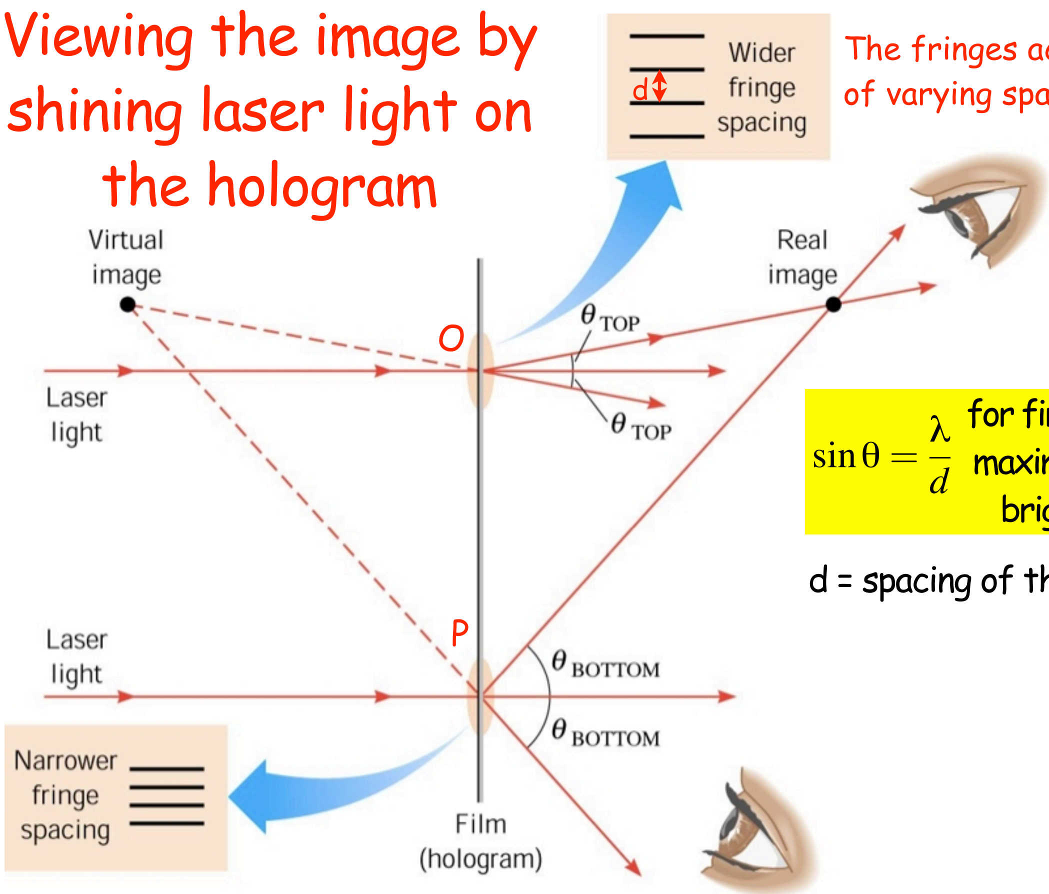
$$r_m = \sqrt{l_m^2 - l_0^2} = \sqrt{(l_0 + m\lambda)^2 - l_0^2}$$

$$r_m = \sqrt{m\lambda(m\lambda + 2l_0)} \simeq \sqrt{2ml_0\lambda} \propto \sqrt{m}$$

$\Rightarrow$ the fringes are close together for large m $\left[\Delta r_m \propto \dfrac{\Delta m}{2\sqrt{m}}\right]$

Viewing the image by shining laser light on the hologram

The fringes act as slits of varying spacing



$\sin\theta = \dfrac{\lambda}{d}$ for first-order maximum (the brightest)

d = spacing of the fringes

Prob. 30.46/41

A pulsed laser emits light in a series of short pulses, each having a duration of 25.0 ms. The average power of each pulse is 5.00 mW, and the wavelength of the light is 633 nm.

Find:

(a) the energy of each pulse and

(b) the number of photons in each pulse.

a) 3.14×10^{-19} J/photon, 1.25×10^{-4} J/pulse,

b) 3.98×10^{14}

Prob. 30.54/34

By using Moseley's adaptation of the Bohr model, decide which element is likely to emit a K_α X-ray with a wavelength of 4.5 nm.

Z = 6, carbon

Summary of Chapter 30

- The positive charge of the the atom is concentrated in a nucleus that is ~10^{-15} - 10^{-14} m in radius.

- The angular momentum of the electron in an atom is quantized, leading to quantized energy levels in hydrogen-like atoms:

  $E_n = -13.6\ Z^2/n^2$ eV

- Photons are emitted and absorbed by atoms at the same wavelength
  → identification of elements in the "atmosphere" of stars,
  discovery of helium.

- The energies of K_α x-rays can be calculated by replacing Z by Z–1
  → identification of some chemical elements for the first time.

- Stimulated emission, the laser.