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The Fractional Quantum Hall Effect

Properties of an Incompressible Quantum Fluid

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BOOKS

The Fractional Quantum Hall Effect: Properties of an Incompressible Quantum Fluid

T. Chakraborty and P. Pietiläinen

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The quantum Hall effect occurs when a two-dimensional electron gas becomes incompressible, provided that the density at which the incompressibility occurs is magnetic field dependent. For the integer quantum Hall effect, the incompressibility is easy to understand. The quantization of the electron's cyclotron motion means that only a discrete set of kinetic energies is allowed, and both the separation between allowed energies and the number of states of a given energy are proportional to magnetic field. (The set of states with a given kinetic energy is called a Landau level.) The incompressibility responsible for the integer QHE occurs at densities where an integer number of Landau levels are filled and the chemical potential jumps from one allowed kinetic energy to another.

The incompressibility responsible for the fractional QHE is due to electron–electron interactions. As the density is increased above certain fractional Landau-level fillings, it becomes impossible for electrons to avoid states of a lower relative angular momentum. Strengthened repulsive interactions cause the chemical potential to jump. At the critical filling factor there is only one many-body state in which the electrons can avoid the lower relative angular momentum. *The Fractional Quantum Hall Effect* by Tapash Chakraborty and Pekka Pietiläinen reviews the theory of these states and their elementary excitations.

Much is understood about the fractional quantum Hall effect. The incompressible states mentioned above have unique properties unlike any previously known. Their discovery has added to the small list of distinct classes of many-body systems. It is therefore not surprising that the ideas developed in trying to understand the fractional QHE are having an impact on other fields of theory, notably the theory of high-T_c superconductivity.

On the other hand, much remains to be understood about the fractional QHE. Many of the incompressible states arise in a much more subtle way than that outlined above, and little theory exists on the competition between disorder and interaction effects, despite interesting experimental results. It is important that fractional QHE theory be communicated to a broad audience both so that its notions can be applied elsewhere in physics and so that its own development will continue. That is the objective of this book.

The book succeeds especially strongly in providing the technical background necessary for those who wish to push the fractional QHE theory further ahead. The authors have gone to some trouble to make the book accessible to anyone with a solid background in statistical physics and quantum mechanics. They provide many details of derivations, omitted in the original references, and thus save many hours for those attempting to introduce themselves to the subject.

Emphasis is placed on explaining the properties of the elementary excitations of the incompressible states, including detailed accounts of their fractionally charged quasiparticles. Insights into the nature of these excitations have been achieved by the use of both small-system exact-diagonalization studies and trial wavefunctions that allow analogies to be made with classical two-dimensional plasmas. The authors describe both approaches with authority, having each made important contributions to the original research. A succinct appendix on the hypernetted-chain method, used for many of the technical calculations, adds to the completeness of the presentation and enhances the value of the book.

Readers seeking an impressionistic overview of fractional QHE theory and hoping for a lucid account of conceptual elements that they can take away and fit into another piece of nature's great puzzle are likely to be disappointed by this book. The book is intended for nonexpert researchers who want to begin investigating the fractional QHE. For these people, I believe that the book will prove invaluable and I strongly recommend it. The book can also serve as a useful reference for active researchers in either theory or experiment. I believe that this book will succeed in opening up the fractional QHE theory to a larger community. In writing it the authors have done a service to the subject, for there is much left to do.

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The Fractional Quantum Hall Effect: Properties of an Incompressible Quantum Fluid. T. Chakraborty and P. Pietiläinen. Springer-Verlag, New York, 1988. Price: \$45.00 ISBN 0-387-19111-9. (Reviewed by D. J. Thouless.)

The discovery 9 years ago by von Klitzing, Dorda, and Pepper that the Hall conductance of a silicon MOS (metal-oxide-semiconductor) device in a strong magnetic field is an integer multiple of e^2/h was dramatic and unexpected. It was almost immediately realized that it should have been predicted and could be easily understood. It led rather rapidly to technical applications and theoretical developments. On the one hand, it provides a standard of electrical resistance more reliable than those provided by national standards laboratories, and on the other hand it has led to a deeper and broader understanding of topological quantum numbers.

Two years layer, the complacency generated by these rapid developments was shattered by Störmer, Tsui, and Gossard, who found that in high-purity heterojunctions the Hall conductance could be quantized in fractional multiples of e^2/h. Although the precision of these fractions has not been tested as severely as the precision of the integer quantization, some are known to be correct to within a few parts in 10^5. The ease with which the integer quantization had been explained underlined the difficulty of explaining fractional quantization. In particular, any explanation in terms of noninteracting electrons seemed very implausible. The only explanation that has withstood careful scrutiny was that given by Laughlin, who also gave the best explanation of the integer quantum Hall effect. Laughlin proposed an approximate wavefunction for the state with simple fractional Hall conductance (such as 1/3), which seems to be remarkably accurate. It is the Coulomb repulsion between the electrons that stabilizes this particular wavefunction in the presence of a strong magnetic field. There are superficial similarities of this wavefunction to some that occur in other branches of physics, but I regard these similarities as distracting rather than illuminating. We are faced with an unfamiliar situation, and have not yet learned to deal with it at all elegantly. The excitations from this state are fractionally charged, and can themselves condense to form more complicated fractionally quantized states, such as the sequence 2/5, 3/7, 4/9, 5/11, and 6/13, all of which have been seen in experiments.

This book gives a thorough survey of the theoretical work that has been done on the Laughlin theory of the fractional quantum Hall effect. There is a very brief survey of some of the more important experimental results, but the main emphasis is on calculations and theory. The description of calculations is clear and balanced. It is shown how exact calculations for a small number of electrons on a sphere or torus, Monte Carlo simulations for larger systems, and approximate theories for infinite systems hang together. For the simple fractions, the ground state is separated by an energy gap from any excited state, and the simplest excitations have a structure rather like the roton in the theory of superfluid helium.

The book is not good at relating this problem to wider issues in physics. I noticed two errors on the first page; it is not true that the wave vector is a good quantum number in an inversion layer (because impurity scattering changes the wave vector), and it is not true that a qualitative understanding of the integer quantum Hall effect is possible only in terms of noninteracting electrons (this may be a mistake in syntax rather than in physics). Even in a brief discussion of the experiments, I thought the authors should have mentioned that one important difference between an MOS structure and a heterojunction is that in the former the electron density can be controlled by varying the gate voltage, while in the latter the electron density of a sample is usually fixed. I sympathize with their decision not to discuss the ring exchange theory in detail, but the work of Girvin and MacDonald on off-diagonal long-range order deserves careful discussion. Its importance has become more obvious in the past year when the relations with the resonating valence bond model of magnetism have become apparent; this is a topic that is barely mentioned in a footnote.

I am impressed by the achievements of the Laughlin model, but do not regard the theory as yet matching the clear-cut quality of the experiments on fractional quantization of the Hall conductance. This book gives a good account of the main features of the theory, but it does not really assess the general situation or indicate the way ahead.

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Book Reviews

Many-Body Theory

The Fractional Quantum Hall Effect. Properties of an Incompressible Quantum Fluid. T. Chakraborty and P. Pietiläinen. Springer-Verlag, New York, 1988. xii, 175 pp., illus. $45. Springer Series in Solid-State Sciences, vol. 85.

The fractional quantum Hall effect was discovered experimentally in 1982. It was immediately recognized as a fundamentally new macroscopic manifestation of the quantum laws underlying electrical conduction in solid matter. Initially puzzling to theorists, it was explained in its essentials within a year by R. B. Laughlin with his proposal of a liquid wave function. This work spawned a number of novel concepts, most notably that of fractional charge—the idea that an interacting many-electron system could appear, for example, to consist of a two-dimensional gas of particles of charge equal to one-third of the charge of the electron. Because of this and other remarkable features, the theory of the fractional quantum Hall effect is often felt to be the most significant advance in many-body theory of this decade.

These initial breakthroughs have been followed by progress in experimental techniques; in particular, steady advances in semiconductor technology have improved the quality of the transport measurements that identify the quantum fractions. Theoretical progress has been made as well, especially in developing the hierarchy picture, which extends Laughlin's original work to most (though not all) of the currently observed quantum states. Much remains to be done, however. On the experimental side, the development of other measurements to complement the transport data, the extension to multilayer structures, and the clarification of the even-denominator fractions are most pressing. On the theoretical side, the issues associated with higher-order and even denominators, finite temperatures, and multilayers still need a great deal of work.

One virtue of Chakraborty and Pietiläinen's book is therefore its timeliness. The field has completed an initial phase in which the most fundamental issues have been identified and largely clarified. This means that there is plenty of material for a full-length monograph. There has now come a realization that there are a number of interesting and rather diverse research paths to be followed. Any theoretical physicist who wants to be involved in these efforts needs a thorough grounding in the basics. This new book offers that.

The three main computational methods of the field—exact diagonalization of small systems and Monte Carlo and diagrammatic methods—are each explained in detail. Particularly welcome is the exposition of the "hypernetted chain" approach, which has never received a unified and clear treatment in the context of the Hall effect. The book explains the well-established applications to various aspects of the problem—the ground state and its excitations, both quasiparticle and collective. The authors discuss spin-reversed states at considerable length, a welcome choice of topic because of the experimental discovery of these states since the book was written. In sum, all of the topics for which there are firm results are covered, and covered in detail sufficient to equip anyone who has digested the contents for beginning research.

This book invites comparison with the only other comprehensive survey of the field, the second half of *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin. Chakraborty and Pietiläinen's focus is narrower in that they discuss theoretical issues mainly on the level of internal consistency and technical detail and mostly ignore broader implications. This is perfectly appropriate in a very specialized treatise, but the air does get a bit thin at times. Still, one could contend that this makes a good contrast and complement to the earlier book, which offers a much broader perspective. Prange and Girvin is still the best place for a beginner to start.

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