Predicting narrow states in the spectrum of a nucleus beyond the proton drip line

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Properties of particle-unstable nuclei lying beyond the proton drip line can be ascertained by considering those (usually known) properties of its mirror neutron-rich system. We have used a multi-channel algebraic scattering theory to map the known properties of the neutron-^{14}C system to those of the proton-^{14}O one from which we deduce that the particle-unstable ^{15}F will have a spectrum of two low lying broad resonances of positive parity and, at higher excitation, three narrow negative parity ones. A key feature is to use coupling to Pauli-hindered states in the target.

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There is much current interest in the properties of, and reactions with, nuclei that lie out of the valley of stability. The masses of hundreds of such nuclei that lie between the nucleon drip lines are now known. So also are some spectral properties of those that can be formed with sufficient intensity for a radioactive ion beam (RIB) to be made. Besides the inherent interest in studying the properties of weakly-bound many-nucleon systems, these radioactive nuclei are crucial in current investigations of energy and mass production in astrophysics. But little is known about nuclear systems at and, especially, beyond the drip lines. Those particle-unstable systems are difficult to study as they may only be formed during nuclear reaction processes.

Recently, data have been obtained [1, 2] from elastic scattering of radioactive ^{14}O ions from hydrogen which reveal two states in the proton-rich nucleus ^{15}F; a nucleus that lies beyond the proton drip line. Those data indicate that besides the resonant ground state, ^{15}F has a narrower first excited resonant state 1.3 MeV above the (broad resonance) ground. Herein we report on our analysis of those data and predict that there should be even narrower resonances in ^{15}F lying in an energy range just above the limit of the reported data.

As method of analysis we use the multi-channel algebraic scattering theory (MCAS) [3]. It has the distinctive capacity to embrace in the scattering equations single-particle aspects, collective-type coupled-channel dynamics, and the Pauli principle. The Pauli principle is taken into account using the Orthogonalizing-Pseudo-Potential (OPP) method. Past studies [3–5] used that OPP scheme to deal only with Pauli-blocked and Pauli-allowed states. In this letter we use the OPP scheme to consider also Pauli-hindered states, namely states where the Pauli-blocking is partially relaxed (Pauli-hindrance). With that new feature, and with the instructive property of considering results in the limit of zero deformation [5], our analyses of the p+^{14}O system and of its mirror, the n+^{14}C system, infer new spectroscopy of the exotic nucleus, ^{15}F.

The concept of Pauli-hindrance relates to levels that are neither Pauli-forbidden nor Pauli-allowed but are somewhere in between. This concept naturally arises in cluster-dynamics formulations based, for example, on the Resonating Group Method (RGM). Therein, such conditions can be studied in detail, even analytically, starting from the properties of the eigenvalues of the RGM norm kernel [6]. The technique based on the introduction of the OPP method, which we have adopted and generalized to multichannel dynamics in the MCAS formulation, is particularly suited for treating such intermediate situations. For reference, Pauli-allowed states relate to zero coupling in the OPP term and complete Pauli-blocking is the limit of infinite OPP couplings. In practice, blocking-effects can be obtained numerically by having large (of order GeV) values to the OPP couplings, while for Pauli hindrance couplings of the order of a few MeV are required in the strength of the corresponding OPP term. In our current formulation of the MCAS approach, we had to include this concept of Pauli-hindrance in the OPP scheme to deal with breaking effects in shell closures, particularly of $0p_{\frac{1}{2}}$ proton orbits, which is a physical phenomenon to be expected in weakly-bound light exotic nuclei.

Shell-closure aspects represent not only a fundamental question in current research in nuclear structure and reactions involving exotic nuclei, but are also of great relevance for atomic and molecular physics in general. In addition, breaking signals in the full occupancy of deep and well-packed orbits are the subject of a new proposal of studies in atomic physics [7], specifically regarding possible upper limits in the violation of the Pauli principle (VIP). We stress, in this respect, that the shell-breaking phenomena in weakly-bound (or unbound) nuclei that we consider in this Letter, and the related concept of Pauli-hindrance, are entirely consistent with the validity of the Pauli principle.

Use of the MCAS approach in the analysis of scattering data (of nucleons and nuclei) has the advantage that such nontrivial effects of the Pauli principle can be incorporated with the OPP method in the multichannel scattering equations. Sturmian expansions of the nuclear interactions are used to obtain an algebraic form for the multichannel scattering matrices. The method treats bound as well as continuum regimes of the compound system equally. It also incorporates a resonance-finding procedure by which all bound states and all resonances up to the limit energy considered will be defined (spin, parity, centroid energy, and width). That is so no matter how narrow or broad any resonance may be. Importantly, use of the OPP method in the construction of the Sturmian functions ensures that the Pauli principle is not violated even when a collective-model prescription of the nucleon-nucleus interactions is used.

Low-excitation bound states and resonances in the spectra of nuclei in the mass region $A \sim 13 - 31$ include many that are expected to be due to weak coupling of a nucleon in the sd shell to the $(A-1)$ nucleon core. Such states have been found in the spectrum of ^{15}C. The ground and first excited states are bound and have spin-parities of $\frac{1}{2}^+$ and $\frac{5}{2}^+$ and they have energies lying below the n+^{14}C threshold by 1.218 and 0.478 MeV respectively [8]. On the other hand, the observed two resonances in ^{15}F are centered about 1.47 and 2.78 MeV above the p+^{14}O threshold. They have the same spin-parity values of the two bound states in ^{15}C and so are considered as analogues. Hence we consider the n+^{14}C system and the states in ^{15}C first and match the result by adding a Coulomb field in the calculations to specify the spectrum and scattering cross section for p+^{14}O. That spectroscopy is determined from MCAS evaluations, input to which are interaction potentials for the channels coupled in the systems. Both mass-14 nuclei have a 0^+ ground state and then a cluster of excited states ~ 6 MeV away. In that cluster there are a second 0_2^+, a 2^+, a 1^- and a 3^- state. Of those, for simplicity in calculations, we consider coupling to the ground only with the 0_2^+ and the 2^+ states.

We use a standard collective model prescription for the interaction (coupling) potentials with deformation limited to the quadrupole term but taken to second order in expansion of the deformed surface radius. Details are given in Ref. [3]. We take the quadrupole coupling strength, β_2, to be -0.5. The choice was made solely because a similar value was used for the n+^{12}C system in previous MCAS analyses [3–5]. We have no compelling reason as yet to use an alternate value. However, note that the ground state interaction is a mixture of zeroth and second order terms and so is akin to having an interaction very like that of a spherical ground state density. The precise β_2 value then most strongly defines the transition interaction for excitation/deexcitation of the 2^+ state.

To consider Pauli effects, we need to interpret the structure of the target in terms of shell orbits and their occupancies. We presume that the ground states are described dominantly by two holes in an otherwise closed ^{16}O. Thus for ^{14}C (neutrons) and ^{14}O (protons), the $0s_{\frac{1}{2}}, 0p_{\frac{3}{2}}$, and $0p_{\frac{1}{2}}$ relevant nucleon orbits in the ground states are considered full. For the ground-state channels in the nucleon-nucleus systems those orbits (of the relevant 8 nucleons) are Pauli blocked while all other orbits are treated as Pauli allowed. However, we presume that the excited states have considerable 2p$-$4h (and higher) configurations with the occupancies of the $0p_{\frac{1}{2}}$ orbits most affected. Thus we treat that orbit, for the relevant nucleon type and in the channels involving the excited states, as Pauli hindered. All such Pauli principle effects are generated using the OPP scheme by which the Sturmians are orthogonal to any Pauli-blocked state and affected by any that are Pauli hindered. Those Sturmians are used to expand the interaction matrix of potentials and then the scattering matrices.

The Sturmians are solutions of homogeneous Schrödinger equations for the chosen matrix of interaction potentials. In coordinate space, with those potentials designated by local forms $V_{cc'}(r)\delta(r - r')$, the OPP method uses Sturmians that are solutions for nonlocal potentials

$$\mathcal{V}_{cc'}(r, r') = V_{cc'}(r)\delta(r - r') + \lambda_c A_c(r) A_c(r')\delta_{cc'}, \quad (1)$$

where $A_c(r)$ is the normalized radial part of the single-particle bound-state wave function in channel c spanning the phase space excluded by the Pauli principle. The channel indices c designate all relevant quantum numbers. The OPP method for treating Pauli-blocked state effects holds in the limit $\lambda_c \to \infty$, but use of $\lambda_c = 1000$ MeV suffices. For Pauli-allowed states, of course, $\lambda_c = 0$. But for Pauli-hindered states specific values of 1000 MeV $>> \lambda_c > 0$ are required and which, for the single orbit of relevance, are presently treated as adjustable parameters.

We use the same collective model prescription for the matrix of interaction potentials that we have used previously [3–5] but in this case with the mix of central (0), spin-orbit (so), and l^2 (ll) deformed potential terms,

$$V_{c'c}(r) = V_0 v_{cc'}^{(0)}(r, \beta_2) + V_{so} v_{cc'}^{(so)}(r, \beta_2) + V_{ll} v_{cc'}^{(ll)}(r, \beta_2) \; . \quad (2)$$

The $V_{c'c}(r)$ are derived from Woods-Saxon forms in which the dependence on the deformed radius parameter is expanded through second order in β_2 [3]. The undeformed radius parameter is 3.1 fm and the diffuseness is 0.65 fm. The successful calculations of the neutron-^{14}C system required potential strengths of $V_0 = -45.0$ MeV, of $V_{so} = 7.0$ MeV, and of $V_{ll} = 0.42$ MeV. The same interaction was used to determine the positive and negative parity results so that the only parity dependence arises from use of the OPP term in Eq. (1). The Coulomb radius used in the p+^{14}O calculations was 3.1 fm and, with respect to the n+^{14}C system, the results we show were

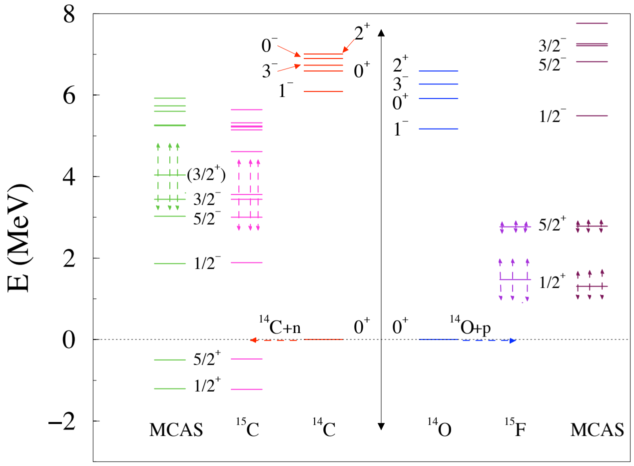

FIG. 1: (Color online) Low energy spectra of ^{14,15}C and of ^{14,15}O, and from the results of our MCAS calculations. The zero of the energy scale is set to that of the relevant mass 14 ground states.

obtained by reducing the central potential strength V_0 slightly to −44.2 MeV.

The spectra, known and calculated using MCAS, are shown in Fig. 1. The specific cases are as indicated in the diagram. Consider the experimental information on the ^{14,15}C nuclei [8], and the results we have obtained for the n+^{14}C system. The excited states of the ^{14}C are clustered and well separated by $\sim$ 6 MeV from the ground. The spectrum of ^{15}C has two bound states of spin-parities $\frac{1}{2}^+$ (ground) and $\frac{5}{2}^+$ and which are dominantly described by a single *sd* shell neutron on the ^{14}C ground state. Then there are three quite narrow resonances, all having negative parity, which lie within the spread of a broad $\frac{3}{2}^+$ resonant state. That broad $\frac{3}{2}^+$ was seen very clearly in the cross section from a measurement [9] of the ^{14}C(d, p) reaction. The MCAS result matches all of those features well. In the zero deformation limit ($\beta_2 \to 0$), the MCAS results reveal that the bound ($\frac{1}{2}^+$ and $\frac{5}{2}^+$) and resonant $\frac{3}{2}^+$ states are due to the coupling of a $1s_{\frac{1}{2}}$, of a $0d_{\frac{5}{2}}$, and of a $0d_{\frac{3}{2}}$ neutron to the ground state of ^{14}C. It is noteworthy that there are no other bound states; in particular none having negative parity. Such would occur if in the n+^{14}C system the $0p_{\frac{1}{2}}$ neutron orbit were not Pauli blocked. The negative parity states have as their progenitor a $0p_{\frac{1}{2}}$ coupled to the 0_2^+ state (for the $\frac{1}{2}^-$ state) and to the 2^+ state (for the $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states). To find these states at this excitation in ^{15}C required that the Pauli-hindrance of the neutron $0p_{\frac{1}{2}}$ orbit in the 0_2^+ and 2^+ states of ^{14}C target be generated with $\lambda_c(0p_{\frac{1}{2}})$ values of 3.11 and 3.87 MeV, respectively.

Scattering cross-section results are shown in Fig.2. In the top panel the cross sections from ^{14}O scattering from hydrogen (in inverse scattering of protons from ^{14}O) at 180° in the center of mass are given. Therein our MCAS result (solid curve) is compared with the recent data of both Goldberg *et al.* [1] (open circles) and Guo *et al.* [2] (filled squares). The Guo data were in arbitrary units and so we normalized them to the $\frac{5}{2}^+$ resonance values of Ref. [1]. Though the more recent experiment obtained results to 6 MeV, the authors indicate that such are reliable to about 5 MeV. In the bottom panel of Fig. 2 we show our prediction of the total scattering cross section of neutrons from ^{14}C for energies to 5 MeV. The zero of the energy scale has been placed to optimally match the $\frac{5}{2}^+$ bound state in ^{14}C to the centroid of the analogous resonance state in ^{15}F. The experimental values [8] of states in the two mass-15 systems are indicated by the arrows with the relevant spin-parities given alongside.

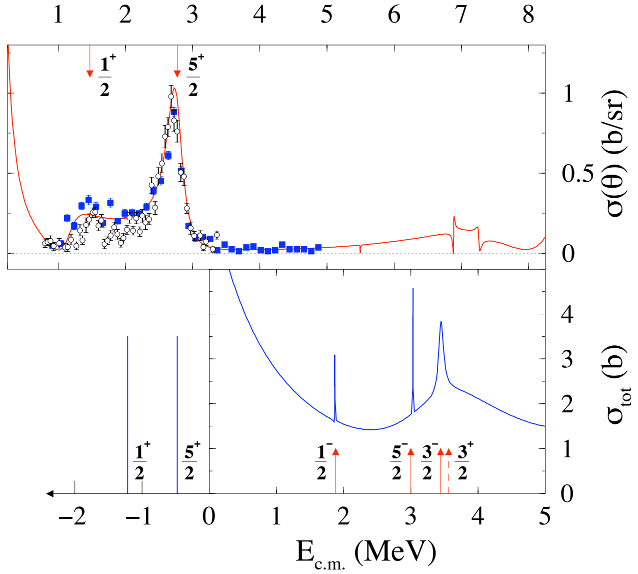

FIG. 2: (Color online) The elastic cross sections from scattering of ^{14}O ions from hydrogen at 180° in the center of mass (top) and our predicted total cross section for the scattering of neutrons from ^{14}C (bottom). In both cases the known spectral values are indicated by the arrows.

Consider the results for the neutron total cross section from ^{14}C. That cross section has four obvious resonances, three quite narrow (the negative parity resonances) and one, a $\frac{3}{2}^+$ resonance, very broad. That broad resonance agrees with one such found in the cross section from the stripping reaction, ^{14}C(d, p) [9]. All of these features have a partner in the 180° cross section for the p+^{14}O system that is shown in the top panel. Clearly the MCAS fit to the available data is good and as good as has been found with other analyses [1, 10]. Noteworthy is that the ground state of the particle-unstable ^{15}F is an s-wave resonance. That is so only because of the Coulomb barrier in the p+^{14}O system. Without the Coulomb barrier there would be no s-wave resonance, only a virtual bound state [11]. That criticality was the reason we needed a small reduction in the central interaction strength (of but

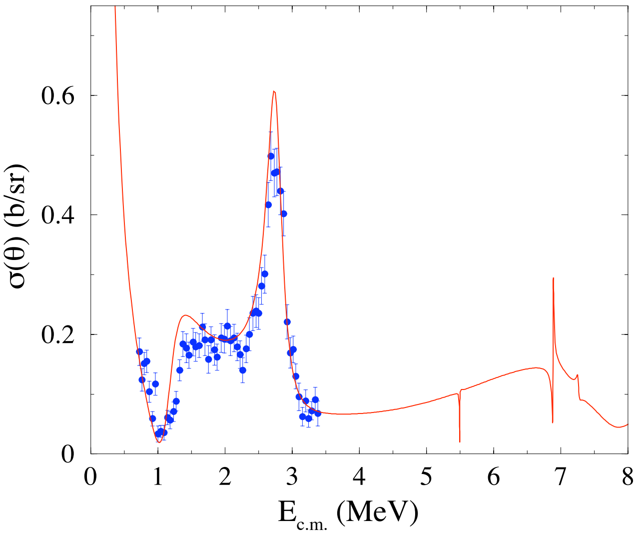

FIG. 3: (Color online) The elastic cross sections from scattering of ^{14}O ions from hydrogen at 147° in the center of mass. The data was taken from Ref. [1].

0.8 MeV) to place this resonance at the desired centroid. Otherwise the interactions used were exactly those determined by our study of the n+^{14}C system. The two bound states found for ^{15}C have become resonances and are of single-particle-like nature. There are other resonance features in our calculated results lying just above the highest energy at which experimental results are known to date. These have negative parities and are analogues of the negative parity resonances seen in ^{15}C. Thus the origin of these new, narrow negative parity resonances in ^{15}F differ from those of the observed low-lying ones. They are compound resonances and, as with those identified in ^{15}C, are due to the Pauli-hindrance of the proton $0p_{\frac{1}{2}}$ orbit in the 0_2^+ and 2^+ excited states of ^{14}O. Finally, we note that these new resonances persist and are relatively more noticeable in cross-sections at other scattering angles. As an example, we show in Fig. 3 results from our MCAS calculation compared with data [1] taken at 147°. Again the two low lying, broad resonances are predicted well (location, width and magnitude) and now the higher, narrow, negative parity resonances are clearly seen to reside on a broad $(\frac{3}{2}^+)$ resonance; the analogue of that one suggested by the ^{14}C(d,p) experiment [9].

In conclusion, the MCAS approach has been used with mirror mass-15 systems to define the spectroscopy of the particle-unstable nucleus, ^{15}F. The procedure involved first making an analysis of the neutron-mirror mass ^{15}C system for which experimental information is known. Crucial to the description of the experimental spectrum was the concept of Pauli-hindrance of single-particle orbits coupled to the collective 0_2^+ and 2^+ excitations in the mass 14 nuclei. It leads to an appropriate description of the observed three low-lying negative-parity resonances. Then, by incorporating Coulomb interactions, the same nuclear force was used to analyze the proton-^{14}O case and thus to predict the spectroscopy of ^{15}F up to 8 MeV excitation. We clearly see three narrow negative-parity resonances in the calculated cross section. This demands further experiments to test the theoretical interpretation.

The MCAS scheme may be used to estimate spectroscopy of other nuclei that are just outside of the proton drip line given that the numbers of neutron-rich isotopes within the neutron drip line usually exceed those on the proton-rich side. Thus the mirror system against which the proton-rich, unstable, system spectroscopy is to be compared will not be particle unstable and may possibly have experimentally known and detailed properties.

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