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Charged Excitons in the Quantum Hall Regime

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We review our recent optical experiments on two-dimensional electron systems at temperatures below 1 K and under high magnetic fields. The two-dimensional electron systems are realized in modulation-doped GaAs-AlGaAs single quantum wells. Via gate electrodes the carrier density of the two-dimensional electron systems can be tuned in a quite broad range between about $1 \times 10^{10} \text{ cm}^{-2}$ and $2 \times 10^{11} \text{ cm}^{-2}$. In dilute two-dimensional electron systems, at very low electron densities, we observe the formation of negatively charged excitons in photoluminescence experiments. In this contribution we report about the observation of a *dark* triplet exciton, which is observable at temperatures below 1 K and for electron filling factors $< 1/3$, i.e., in the fractional quantum Hall regime only. In experiments where we have increased the density of the two-dimensional electron systems so that a uniform two-dimensional electron system starts to form, we have found a strong energy anomaly of the charged excitons in the vicinity of filling factor $1/3$. This anomaly was found to exist in a very narrow parameter range of the density and temperature, only. We propose a model where we assume that localized charged excitons and a uniform Laughlin liquid coexist. The localized charged exciton in close proximity to the Laughlin liquid leads to the creation of a fractionally-charged quasihole in the liquid, which can account for the experimentally observed anomaly.

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1. Introduction

Besides the famous quantum Hall effects, which are observed in transport experiments, many-particle interactions in modulation-doped semiconductor heterostructures in a magnetic field give rise to a large variety of phenomena which manifest themselves in optical recombination spectra. The past decades have shown that optical experiments, like photoluminescence (PL) spectroscopy, can provide further and in some sense complementary information about the ground state of quantum Hall systems [1–9]. In fact, in optical experiments one is able to probe the bulk of a quantum Hall system, whereas transport is mostly governed by edge channels. However, one of the striking differences of optical experiments, compared to transport, is the presence of photoexcited holes in the system, which can significantly influence its properties. In samples with very low carrier concentration, the Coulomb interaction between electrons and holes leads to the formation of negatively charged excitons, which have been a subject of intense research during the past decade (see, e.g., [6–22]). For samples with relatively strong disorder, the commonly accepted picture is that at low density the two-dimensional electron system (2DES) breaks up into areas with finite density (electron puddles) and completely depleted regions, where neutral excitons (X^0) can form. On the other hand, experiments on high-quality GaAs–AlGaAs samples suggested [23] that charged and neutral excitons may also reside in the same spatial regions. This is, however, controversial since, theoretically, neutral excitons should not be stable if charged excitons exist as bound states at lower energies. Furthermore, time-resolved experiments on p -doped CdTe–CdMgZnTe quantum wells [24] revealed that both the positively charged excitons and the residual holes are localized, whereas the neutral excitons remain freely moving. We will show below that the effect of disorder is one of the main ingredients in our experiments, too.

On the other hand, at relatively large carrier concentrations, where excitonic interactions are mostly screened, the polarization of the Fermi sea by photoexcited holes can cause optical singularities in emission [25] and absorption [26, 27], known as the Fermi-edge singularity (FES). The description and understanding of the transition and the connections between both regimes is still one of the theoretical challenges in this field.

In our work we could show that the charged excitons can, under specific conditions, be used as a tool to investigate the properties of a highly correlated quantum Hall liquid [9]. We will show that the interaction between a localized charged exciton and the incompressible liquid in the $\nu = 1/3$ state leads to the creation of a fractionally-charged hole in the liquid. This low-lying charged excitation is a fundamental consequence of the incompressible nature of the ground state — the liquid state proposed by Laughlin [28].

2. Experimental details

The samples are one-sided modulation-doped GaAs–Al_{0.3}Ga_{0.7}As single quantum wells with 25 nm well width. To start with low carrier densities in the range of $2 \times 10^{11} \text{ cm}^{-2}$, a 88 nm thick AlGaAs spacer was grown to separate the quantum well from the doped barrier. The asymmetric band bending of the one-sided doped structures is schematically shown in the inset of Fig. 1a. On top of the samples, 7 nm thick titanium gates were deposited to tune the carrier densities by applying a negative gate voltage between the 2DES and the gate. In total, the metallic gate is separated from the quantum well by a 300 nm thick AlGaAs barrier. The sample was glued upside down on a glass substrate using an UV curing optical adhesive. Subsequently, the sample was thinned from the back side by a selective etching process [29] down to the superlattice to a total thickness of about 1.3 μm . By applying a negative gate voltage, V_{Gate} , we can tune the density in the range between about $1 \times 10^{10} \text{ cm}^{-2}$ and $2 \times 10^{11} \text{ cm}^{-2}$. PL and absorption measurements were performed either in an optical split-coil cryostat, or via glass fibers in a ³He/⁴He dilution cryostat at a base temperature of $T = 40 \text{ mK}$ and magnetic fields up to 16 T. In the dilution cryostat, a sensor at the sample position indicated that during illumination the temperature directly at the sample is about $T = 100 \text{ mK}$, while the base temperature is still $T = 40 \text{ mK}$. Circularly polarized light was created directly inside the mixing chamber and left and right circularly polarized spectra were measured by ramping the magnet from positive to negative magnetic fields. For the PL, the sample was excited by a Ti:sapphire laser at 750 nm. For the absorption measurement, a white light source was used.

3. Dark and bright triplet excitons

Figure 1 shows PL spectra, taken at $T = 2 \text{ K}$, for different gate voltages V_{Gate} . At $V_{\text{Gate}} = 0$ in Fig. 1a, the typical signal of a 2DES is seen with a maximum around the energy of the band gap between conduction band and valence band (1518 meV) and a cut-off at the Fermi energy (1527 meV). With applied negative V_{Gate} , the band edge shifts to higher energies, whereas the position of the Fermi energy stays approximately constant. At about $V_{\text{Gate}} = -4 \text{ V}$, the system exhibits an abrupt crossover to a regime where negatively charged excitons form, which manifest themselves in intense sharp lines. At $B = 0$, the charged excitons are singlet excitons, X_s^- , with antiparallel spin orientations of the electrons. At finite magnetic field, triplet excitons, X_t^- , become stable (see, e.g., [11, 12]). In Fig. 1b, spectra at fixed magnetic field $B = 7.4 \text{ T}$ are displayed for different gate voltages. At $V_{\text{Gate}} = -2 \text{ V}$, emission from the lowest Landau level can be observed (maximum at 1526.5 meV in Fig. 1b). Starting from $V_{\text{Gate}} = -3 \text{ V}$, the X_s^- and the triplet exciton X_t^- are found. With further negative voltages, the neutral exciton X^0 evolves and is dominant at $V_{\text{Gate}} = -9 \text{ V}$.

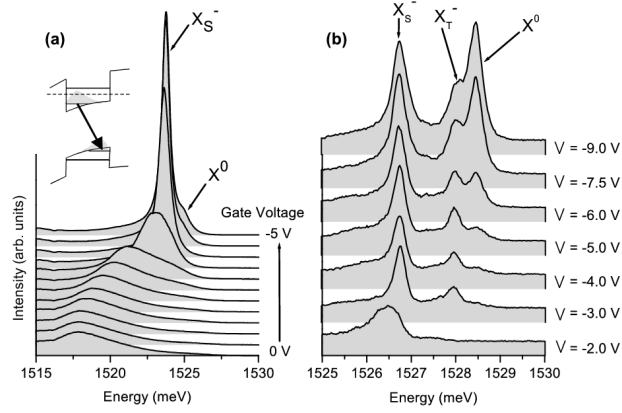


Fig. 1. (a) Photoluminescence spectra of a 25 nm quantum well for different gate voltages and $B = 0$. (b) Gate voltage dependence of spectra of charged excitons for a fixed magnetic field of $B = 7.4$ T.

It was calculated that at high magnetic fields the triplet exciton, which was assumed to be a dark state, forms the ground state of the system. Magnetic fields of 30–40 T were predicted for the singlet–triplet crossing for *symmetric* quantum-well systems [17, 18, 30]. For some time, experimental attempts to detect this singlet–triplet crossing failed. Instead, a saturation of the X_t^- binding energy was found at high fields [11, 14, 31, 32]. Recent theoretical [19] and experimental work [6, 7] solved this puzzle. It was found [19] that at high magnetic field *two different* triplet excitons exist as bound states. One of these excitons has a total angular momentum of $L = 0$, and is therefore called the *bright* triplet, X_{tb}^- . The second triplet exciton is called the *dark* triplet, X_{td}^- , because it cannot decay radiatively ($L = -1$). The calculations showed that the X_{td}^- becomes the ground state at high fields and the experimentally observed saturation of the binding energy can be attributed to the bright triplet. In a subsequent work, Szlufarska et al. found that in *asymmetric* quantum wells the singlet–triplet crossing can be at considerably lower fields [33]. In experiments at very low temperatures the dark triplet exciton was identified by Yusa et al. [6] and Schüller et al. [7]. Also the predicted singlet–triplet crossing was reported recently [34]. In this paper we summarize our results [7], which, concerning the PL experiments are consistent with the observations of Yusa et al. [6], i.e., (i) we observe a charged exciton at very low temperatures, which is not present in the spectrum at $T = 2$ K, (ii) this exciton occurs for filling factor $\nu < 1/3$ only, i.e., related to the fractional quantum Hall effect (FQHE) regime, and (iii) the PL intensity of this excitation is of similar strength to that of the singlet exciton.

Figure 2 shows a comparison of left-circularly polarized PL spectra, which were recorded at $T = 2$ K (upper spectrum), and at $T = 0.1$ K (lower spectrum). In this polarization configuration, at $T = 2$ K, the well-known lower Zeeman com-

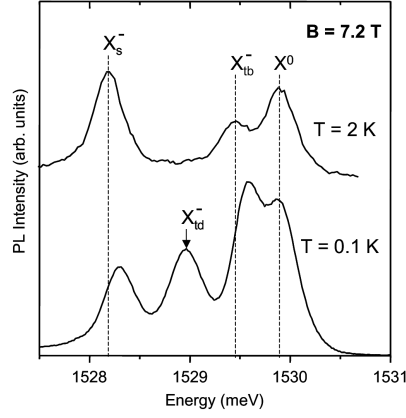


Fig. 2. Comparison of left-circularly polarized PL spectra for $T = 2$ K and $T = 0.1$ K. The electron density is around $n \approx 2 \times 10^{10} \text{ cm}^{-2}$.

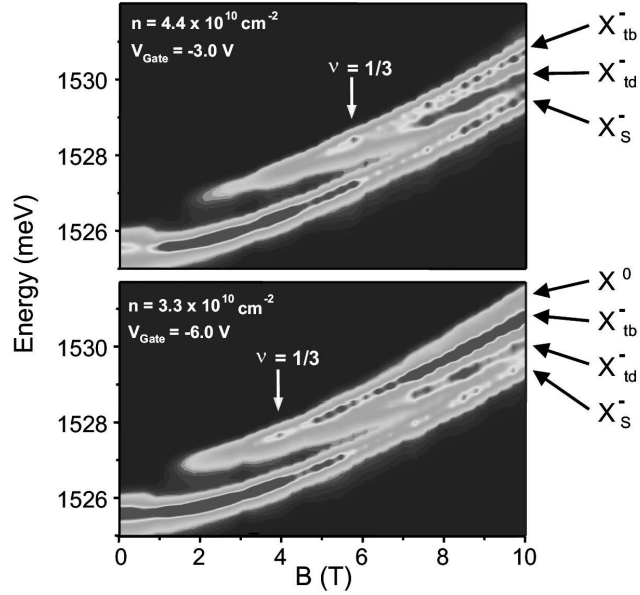


Fig. 3. Contour plots of left-circularly polarized PL spectra for different magnetic fields at $T = 0.1$ K and two different electron densities as indicated.

ponents of the singlet exciton X_s^- and the neutral exciton X^0 , and the bright triplet X_{tb}^- with parallel spin orientation of the electrons are visible. In the spectrum at $T = 0.1$ K, a strong line (labeled X_{td}^-) appears in between X_s^- and X_{tb}^- . We will argue below that this exciton is indeed a *dark* triplet exciton. For both spectra, the density is in the range $2 \times 10^{10} \text{ cm}^{-2}$. Our overall finding is that X_{td}^- is present in the spectra for filling factors $\nu < 1/3$, which is consistent with the results re-

ported in Ref. [6]. As examples, in Fig. 3, contour plots of left-circularly polarized spectra are displayed for two different gate voltages, i.e., two different densities in the range of 10^{10} cm^{-2} . One can see that X_{td}^- splits from the bright triplet X_{tb}^- for $\nu < 1/3$. Consistent with the behavior predicted in Ref. [19], the binding energy of the X_{td}^- increases with magnetic field and the PL line approaches X_{s}^- at high field. However, we note here that up to the highest field in our experiment (16 T) we did not observe the predicted singlet–triplet crossing.

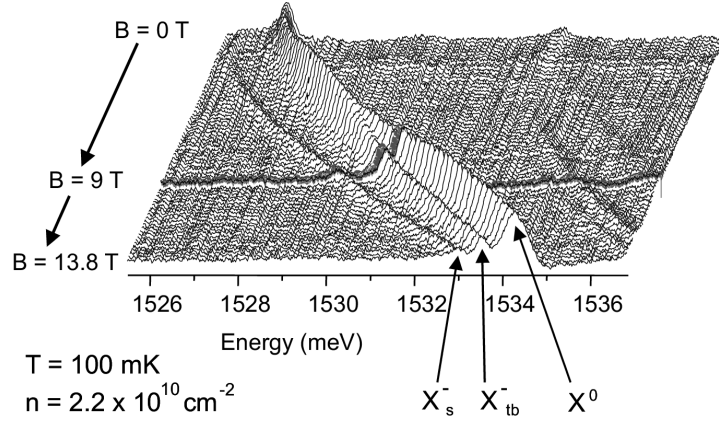


Fig. 4. Unpolarized absorption spectra of excitons in a 25 nm GaAs quantum well for different magnetic fields. The spectrum taken at $B = 9 \text{ T}$ is dark gray shaded.

In the following we demonstrate that the observed X_{td}^- is indeed a *dark* mode. Figure 4 depicts absorption spectra in dependence on magnetic field at $T = 0.1 \text{ K}$ for an electron density of $n \approx 2 \times 10^{10} \text{ cm}^{-2}$. One can clearly see absorption due to the *bright* excitons X_{s}^- and X^0 . Remarkably, in Fig. 4, the neutral exciton X^0 exhibits the highest absorption, i.e., the largest oscillator strength, which is proportional to $1/\tau_{X^0}$, where τ_{X^0} is the radiative lifetime. It is well known that the oscillator strength is not directly reflected in the PL experiments, since the intensity of a PL line also depends, via $N_{\text{X}}/\tau_{\text{X}}$, on the number of exciton, N_{X} in the respective state, and on localization effects. From Fig. 4 one can see that X_{td}^- does not appear in the absorption spectrum, though it exhibits a well developed line in the PL spectrum. From our signal-to-noise ratio we can estimate that X_{td}^- has an oscillator strength which is at least one order of magnitude smaller than the oscillator strength of X_{s}^- . This can be taken as a clear signature that X_{td}^- is indeed a *dark* mode. We suspect that localization effects at low temperature might lift the optical selection rule and allow for the observation of the X_{td}^- .

4. Optical probe of fractionally charged quasiholes

For samples with moderate electron mobility, in the range of $10^5 \text{ cm}^2/(\text{V s})$, we expect that at low electron densities residual disorder plays a significant role

and localization effects of charged excitons might be of importance. We assume that for our sample for densities around $1 \times 10^{11} \text{ cm}^{-2}$ a uniform 2DES just starts to form. In the following we will call this regime the intermediate density regime. Figure 5 shows PL spectra for an electron density of $0.9 \times 10^{11} \text{ cm}^{-2}$ at a temperature of $T = 1.8 \text{ K}$ (Fig. 5a) and $T = 0.1 \text{ K}$ (Fig. 5b). Let us note that the only difference between the experiments displayed in Figs. 5a and b is the different temperature. For filling factors $\nu < 2$, i.e., if the lowest Landau level is completely occupied, we find qualitatively the same behavior in both experiments (gray shaded spectra in Figs. 5a and b): the observed PL lines show a nearly linear magnetic-field dispersion. This can be attributed to recombinations of electrons from the occupied Landau levels with photocreated holes in the valence band. At $\nu = 2$, we find an abrupt crossover to a regime where negatively charged singlet excitons form, which we infer from the nearly quadratic dispersion of the observed PL line for $\nu < 2$. A similar behavior, i.e., a crossover between a Landau level-like and a charged exciton regime at $\nu = 2$, was reported recently [23, 35] for symmetric quantum wells, and was attributed to a breaking of the so-called hidden symmetry [36, 37] for $\nu > 2$ [35].

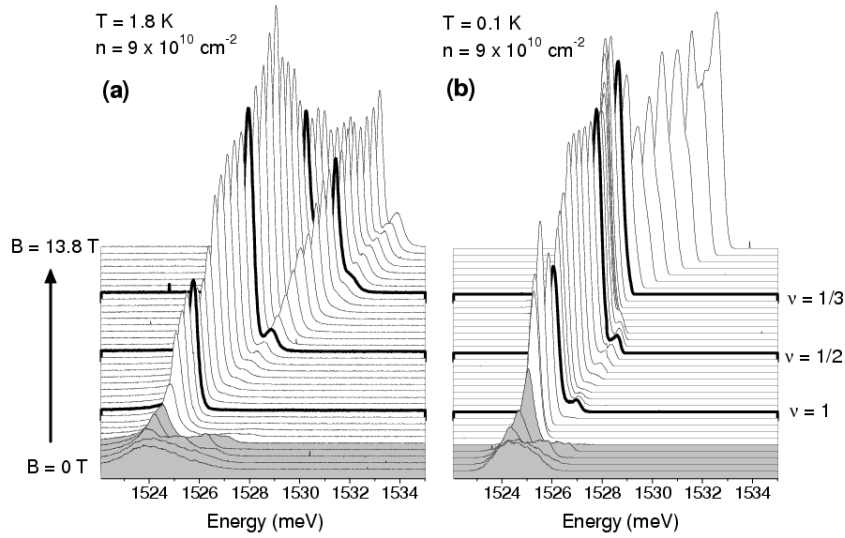


Fig. 5. PL spectra for different magnetic fields for a carrier density of $\approx 0.9 \times 10^{11} \text{ cm}^{-2}$ at a temperature of (a) $T = 1.8 \text{ K}$ and (b) $T = 0.1 \text{ K}$.

The interesting region, however, where we observe significant differences in the experiments displayed in Fig. 5, is the magnetic field range where $\nu < 1$. Here, the measurements at $T = 1.8 \text{ K}$ (Fig. 5a) exhibit the well known formation of negatively charged excitons: the excitation with lowest energy, the X_s^- , is visible over the whole magnetic field range for $\nu < 2$. For $\nu < 1$, the bright triplet exciton

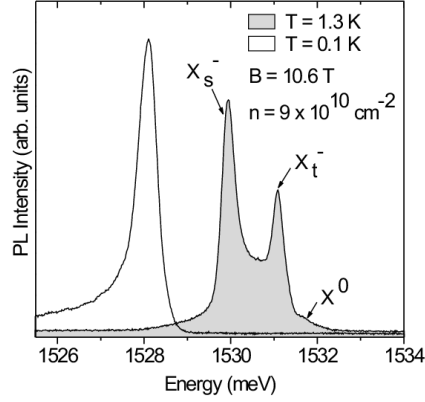


Fig. 6. Left-circularly polarized photoluminescence spectra for a carrier density of $0.9 \times 10^{11} \text{ cm}^{-2}$ and a magnetic field of $B = 10.6 \text{ T}$, corresponding to $\nu = 1/3$, at temperatures 0.1 K and 1.3 K.

X_{tb}^- gradually appears, and, at high fields, the neutral exciton X^0 shows up at the high energy flank of the X_{tb}^- . The situation changes drastically if we lower the temperature to 0.1 K (Fig. 5b). Here, the X_{tb}^- appears abruptly at $\nu = 1$, and, more strikingly, the lowest energy line, presumably the X_{s}^- , shows a strong down curvature in energy for filling factors $\nu < 1/2$. To emphasize this experimental finding, in Fig. 6, two spectra, taken at $B = 10.6 \text{ T}$, which roughly corresponds to filling factor $\nu = 1/3$, are compared. Let us note that, again, the only difference between the two spectra is the different temperature. The spectrum at $T = 1.3 \text{ K}$ shows the charged and neutral excitons, whereas at $T = 0.1 \text{ K}$ only a single line, which is strongly redshifted, is observed. To analyze this behavior in more detail, in Fig. 7 the energy positions of the observed lines at $T = 1.8 \text{ K}$ (open symbols) and $T = 0.1 \text{ K}$ (solid symbols) are plotted versus magnetic field for a density of $0.9 \times 10^{11} \text{ cm}^{-2}$. One can see that, starting at $\nu = 1$, the energies of both X_{s}^- (solid circles in Fig. 7) and X_{tb}^- (solid up-triangles in Fig. 7) are lowered in the experiment at $T = 0.1 \text{ K}$ with respect to the experiment at $T = 1.3 \text{ K}$ (open symbols in Fig. 7). The inset of Fig. 7 shows the experimentally determined anomaly ΔE versus magnetic field. The most striking result here is that a thermal energy of $\approx 2 \text{ K}$ ($\approx 0.2 \text{ meV}$) is sufficient to completely destroy the anomaly which has a strength of about 2 meV. This rules out any trivial localization effect of excitons, since one would then expect that at least a thermal energy of about 2 meV would be necessary to delocalize the excitons. In addition to PL spectroscopy, we have also performed transmission experiments [8] (not shown here) using a white light source to measure the absorption. Dispersions of the PL line and absorption lines (attributed to the X_{s}^- exciton) show the same anomaly. From these results, one can infer that the anomaly is an intrinsic effect and is not caused by localization of excitons. Interestingly, if we lower the density to the range $\approx 10^{10} \text{ cm}^{-2}$, the

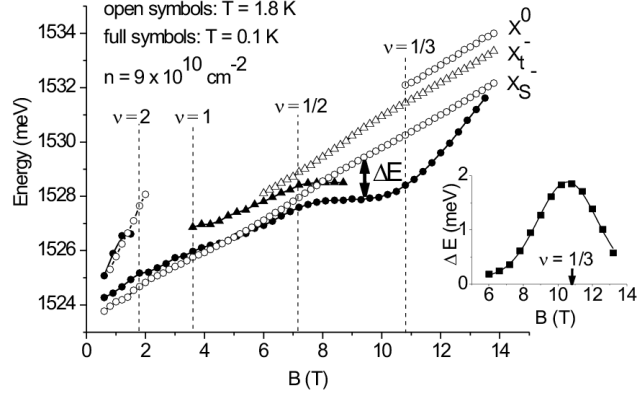


Fig. 7. Experimentally observed mode positions at $T = 1.8$ K (open symbols) and $T = 0.1$ K (solid symbols). The inset shows the anomaly ΔE versus magnetic field.

anomaly disappears completely, even in experiments at very low temperature. This can be seen in Fig. 3, in the previous section. There, in the range of filling factor $\nu = 1/3$, no anomalous dispersion of the charged excitons can be observed.

Before we give an explanation for the anomaly around $\nu = 1/3$, we want to summarize at this point the relevant facts from the experiments: (a) The anomaly is not seen at *higher* electron densities where no charged excitons but usual electrons exist, and is also not seen for *lower* electron densities where exclusively charged excitons are present [7]. Also, for higher mobility samples ($\approx 5 \times 10^6$ cm²/(V s), not shown here) it is not present. Because of the low mobility and relatively low density of the sample, excitons are expected to remain localized. (b) The anomaly appears near $1/3$, i.e., excitons are near an incompressible liquid. (c) The most intriguing observation is that *a very small thermal energy ($\ll 2$ meV) is required to destroy the anomaly*. (d) The anomaly does not appear near $\nu = 1, 2$ which is therefore an indication that the lowest-energy charged excitations, the quasiholes (for reasons to be discussed below) are perhaps involved in the process. The quasielectrons are predicted to have higher energies [38].

5. Theoretical model

In our explanation of the observed anomaly [9] we assume that, as a result of potential fluctuations due to impurities in the system, excitons remain localized but they are in close proximity to the incompressible liquid at $\nu = 1/3$. Two of us recently investigated a system [39] where a parabolic quantum dot (QD) [40] is coupled (via the Coulomb force) to a 2DES which is in a $\nu = 1/3$ Laughlin state. Electrons in the dot are confined by a parabolic potential [40], $V_{\text{conf}}(x, y) = (1/2)m^*\omega_0^2(x^2 + y^2)$, where ω_0 is the confinement potential strength and the corresponding oscillator length is $l_{\text{dot}} = (\hbar/m_0^*)^{1/2}$. Calculating the low-

-energy excitations of that quantum dot–liquid system (named a *qd-liquid*), we found that for a single electron in the dot the physics is somewhat similar to that of a point impurity in a $\nu = 1/3$ liquid state investigated earlier [41]. In this case, the QD emits a fractionally-charged *quasihole* ($e/3$) that orbits around the QD, as evidenced from the charge-density calculations [39, 41]. Here we propose that the observed anomaly is related to the qd-liquid where the QD contains a charged exciton [9]. The QD in our model of Ref. [39] represents a localized exciton (charged or neutral) in the present case, and perturbs the incompressible fluid due to its close proximity by creating fractionally-charged defects. Details of the formal aspects of our theory can be found in Ref. [39]. We model the incompressible state at $\nu = 1/3$ filling using the spherical geometry [42] for six electrons. Electrons are treated as spinless particles corresponding to the state described by the Laughlin wave function [28]. We consider the QD size $l_{\text{dot}} = 15$ nm and the liquid–dot separation $d = 1.5l_0$. The QD contains either a pair of electrons and hole (e,h) (charge-neutral QD), or (2e,h) (charged QD). Figure 8 shows the energy spectra

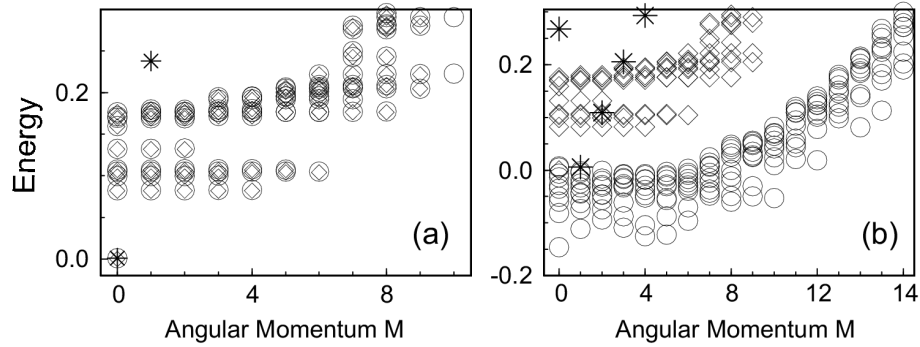


Fig. 8. Energy (in units of Coulomb energy) versus the azimuthal rotational quantum number M for an isolated quantum dot (*), a two-dimensional electron liquid (\diamond), and a qd-liquid (\circ). The QD of the qd-liquid either contains (1e,h) [in (a)], or (2e,h) [in (b)].

for the qd-liquid where the QD contains either (e,h) [in (a)] or (2e,h) [in (b)]. In the figures, the energy spectra of isolated dots (*), an incompressible liquid in the $\nu = 1/3$ state (\diamond) and the binding energy of the QD to the incompressible liquid (\circ) are plotted for comparison. From Fig. 8a we can see that for a charge-neutral dot there is no dispersion of the energy as a function of M , and most important, the incompressible liquid is not influenced by the dot at all. On the other hand, the energy of the qd-liquid is significantly lowered for a charged QD, as compared to the isolated QD or the incompressible liquid without the dot (Fig. 8b). This is in line with the experimental observation where only the charged excitons show the anomaly by a lowering of their energy.

Figure 9 exhibits the electron density distribution in the liquid (L) and in the dot (QD) for the lowest states and for a given angular momentum of the

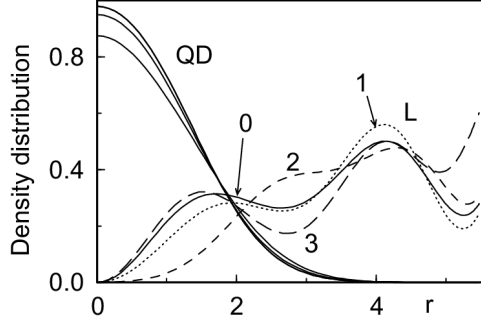


Fig. 9. Charge-density profile of the ground state and low-lying excitations of electrons in the dot (QD) and the liquid (L) of a qd-liquid at $\nu = 1/3$ (for $M = 0-3$, as indicated in the figure) corresponding to Fig. 8b.

qd-liquid system. The electron (or hole) system in the dot is close to the ground state of an isolated dot, i.e., the influence of the incompressible liquid on it is very small. On the other hand, the low-lying excited states of the qd-liquid can be described by the ionization process [39, 41] as emission of a quasihole: since the net charge of a QD is negative, the ground state of the qd-liquid can be considered as a QD plus three quasiholes. If we increase the angular momentum of the 2D electrons, one of the quasiholes moves away from the QD. This is inferred from the calculated charge distribution of electrons around the QD, where a local minimum corresponds to the quasihole moving away from the QD as the angular momentum is increased. The position of the local minimum at different angular momenta of the charge density corresponds to the orbit radius of the quasihole [39, 41]. We have evaluated the quasihole creation energy in a qd-liquid. For the Laughlin state in a pure 2DES it is $0.0276e^2/\epsilon l_0$ [43]. For the qd-liquid, the corresponding value is 0.32 meV and is expected to decrease a little further with increasing number of electrons in the system representing the incompressible liquid [42]. This result indicates that the small thermal energy of about 0.2 meV required to destroy the anomaly is in fact, the quasihole energy gap.

6. Conclusion

We have performed PL and absorption experiments on the dilute 2DES at low temperatures and high magnetic fields. Comparing PL and absorption spectra at very low temperatures, we could identify a dark triplet exciton X_{td}^- by showing that, in spite of a relatively high PL intensity, its oscillator strength is very small and justifies the assignment as a *dark* mode. We find that the X_{td}^- occurs at filling factors $\nu < 1/3$ only and is not visible at $T = 2$ K. Furthermore, PL experiments on a 2DES in an intermediate density range exhibit a significant lowering of exciton energies at and around $\nu = 1/3$. This anomalous dispersion is explained as due

to the perturbation of the incompressible liquid at $\nu = 1/3$ by localized charged excitons, which results in the creation of fractionally-charged quasiholes in the liquid.

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