ELECTRONS AND EXCITONS IN A QUANTUM DOT IN MAGNETIC FIELDS

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The electronic properties of a quantum dot in magnetic fields are discussed. We describe the effect of electron-electron interaction on the energy spectrum of a single dot and a Coulomb-coupled dot-pair. For a two-dimensional hydrogenic exciton in a parabolic dot, the excitation energies at low fields divide into two sets: one set behaving like center-of-mass excitations with no apparent magnetic field dependence while the other is an excitation due to relative motion. We estimate that at low confinement energy ($< 10\ meV$) and low magnetic fields ($< 10\ T$), Coulomb interaction dominates the behavior of the exciton.

I. INTRODUCTION

Recent experimental work on quasi-zero-dimensional electron systems at a semiconductor interface (*quantum dots*) has generated considerable interest on the physics of few-electron systems in low dimensions[1–5]. Of particular interest is the study of the electronic properties of quantum dots in a magnetic field. In this paper, we present a brief discussion of our current understanding of electron dots in a magnetic field and some puzzling features of a two-dimensional hydrogenic exciton in a parabolic dot subjected to a perpendicular magnetic field. In both systems, we find that the energy spectrum reflects the important role played by the electron-electron interaction.

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II. ELECTRONS IN A QUANTUM DOT

Far infra-red (FIR) spectroscopic measurements on quantum dot structures in InSb by Merkt et al.[1] showed that the measured resonant frequency is independent of electron number within the experimental error. These resonances are related to single-particle transition energies in a bare confinement potential. Demel et al.[2] recently created quantum dot arrays in AlGaAs/GaAs heterostructures. The FIR resonance absorption structures they observed show similar dispersion with magnetic field but a resonant *anticrossing* in the energy levels was also resolved.

In the absence of a magnetic field, Bryant[3] has shown that the electron-electron interaction has a significant influence on the energy spectrum of a quantum dot. In a magnetic field, the nature of the many-electron states is a challenging problem. One consequence is that different ground states are expected to occur at different magnetic fields[4]. From self-consistent numerical solutions of the Poisson and Schrödinger equations for a quantum dot in the Hartree approximation, Kumar et al.[5] showed that the confining potential has nearly circular symmetry and therefore angular momentum is approximately a good quantum number. Also with increasing magnetic field, the evolution of energy levels was found to be similar to that for a parabolic potential. We have shown earlier[4] that for a parabolic confinement of the electrons, FIR spectroscopy is sensitive only to the center of mass motion (CM) of the electrons which has exactly the single-electron excitation energy of a bare confinement potential. This is in line with the experimental observation by Merkt et al.[1].

The ground state and low-lying excited states of a quantum dot with three electrons vs the total angular momentum L is shown in Fig. 1. The allowed dipole transitions are also indicated, which are the transitions to the CM energy levels (dashed lines) as explained above. In that figure we also present the case of a pair of dots which are (a) uncoupled and (b) coupled by inter-dot Coulomb interaction. In the last case, the circular symmetry is broken and the angular momentum is no longer a good quantum number. This is one possible way to study the effect of many-electron interaction on optical excitations. We consider the Hamiltonian for the Coulomb-coupled dot-pair in a parabolic confinement to be of the form $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\rm ee}$, where

$$\mathcal{H}_0 = \sum_{\alpha=1,2} \sum_i \left[\frac{1}{2m^*} \left(\mathbf{p}_{\alpha i} - \frac{e}{c} \mathbf{A}(\mathbf{r}_{\alpha i}) \right)^2 + \tfrac{1}{2} m^* \Omega^2 (\mathbf{r}_{\alpha i} - \mathbf{R}_\alpha)^2 \right]$$

$$\mathcal{H}_{\rm ee} = \frac{1}{2} \frac{e^2}{\epsilon} \sum_{\alpha=1,2} \sum_{i \neq j} \frac{1}{|\mathbf{r}_{\alpha i} - \mathbf{r}_{\alpha j}|} + \frac{e^2}{\epsilon} \sum_{i,j} \frac{1}{|\mathbf{r}_{1i} - \mathbf{r}_{2j}|}.$$

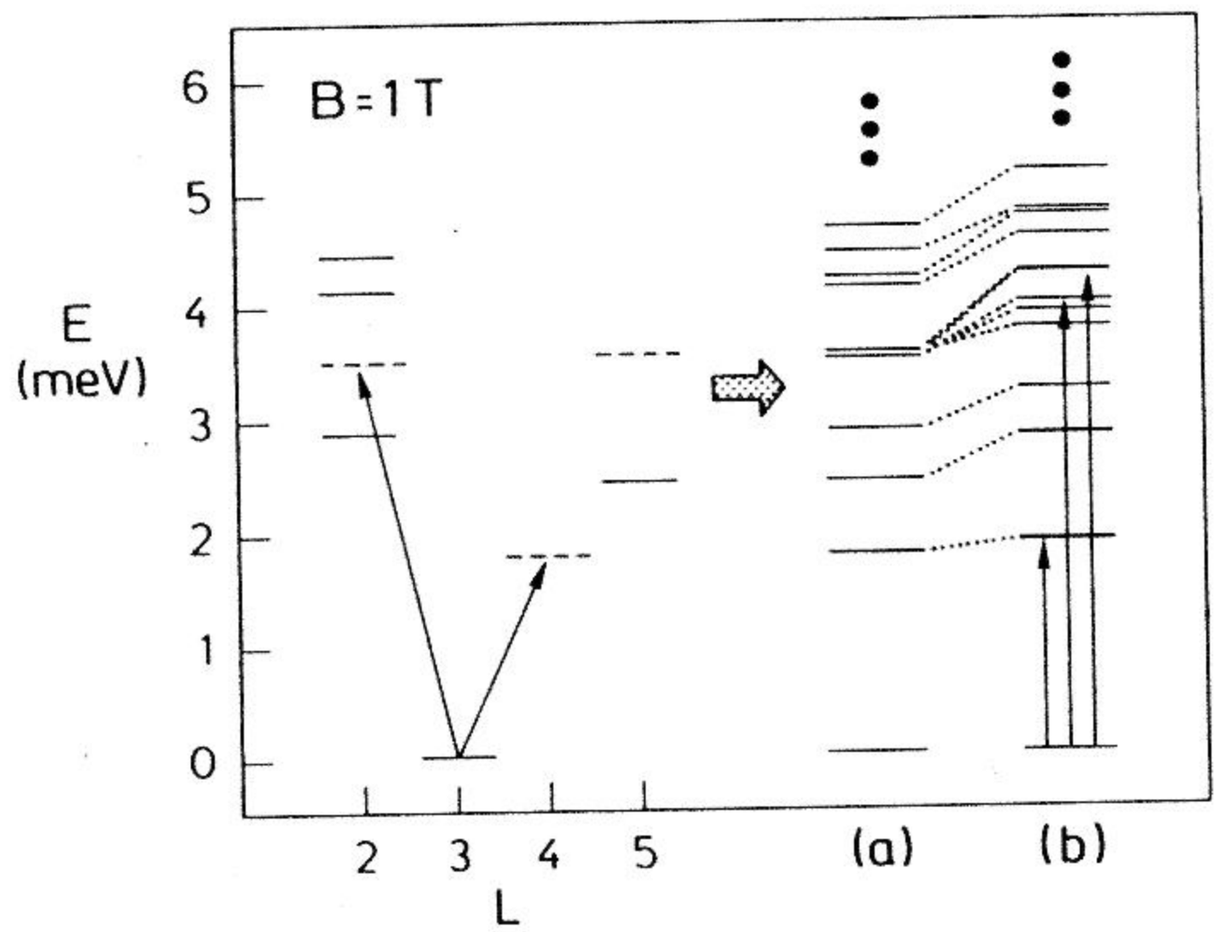

Figure 1: Energy spectrum for a single dot and the dot-pair (a) without and (b) with inter-dot coulomb coupling.

Here the sums over i and j run over the number of electrons in a single dot, m^* is the effective mass, Ω is the *strength* of the confining potential, ϵ is the background dielectric constant and $\mathbf{R}_\alpha$ is the position of the center of the α-th dot. No attempt is made to include any single-particle interaction between the dots, i.e., the electrons feel the other dot only through the Coulomb interaction $\mathcal{H}_{ee}$.

We show in Fig. 1(b) how the Coulomb interaction between the two dots couples the excitations with $L = 2$ (CM) and $L = 5$ (CM) of an individual dot. The lower mode of the dipole transition is caused by the transition to $L = 4$ (CM) level which has no other level to couple to. The coupling of the CM and relative motions causes interesting structures in the dipole transition energies as a function of magnetic field (Fig. 2). As explained above, the lower mode of the transition energies for the electron-dot pair is always close to the single-particle mode. On the other hand, the upper mode is seen to exhibit interesting anticrossing behavior due to the coupling described above.

To conclude this part, we have compared the energy spectrum of a single quantum dot and a Coulomb-coupled dot-pair. In the latter case, the circular symmetry is broken and as a result the radiation couples to the internal motion of the electrons. The effect of many-electron interaction then manifests itself in the anticrossing behavior of the transition energies akin to that observed by Demel et al[2]. We have demonstrated above how the different energy levels couple (at a given magnetic field) to cause the anticrossing behavior in the dipole transition energy as a function of the

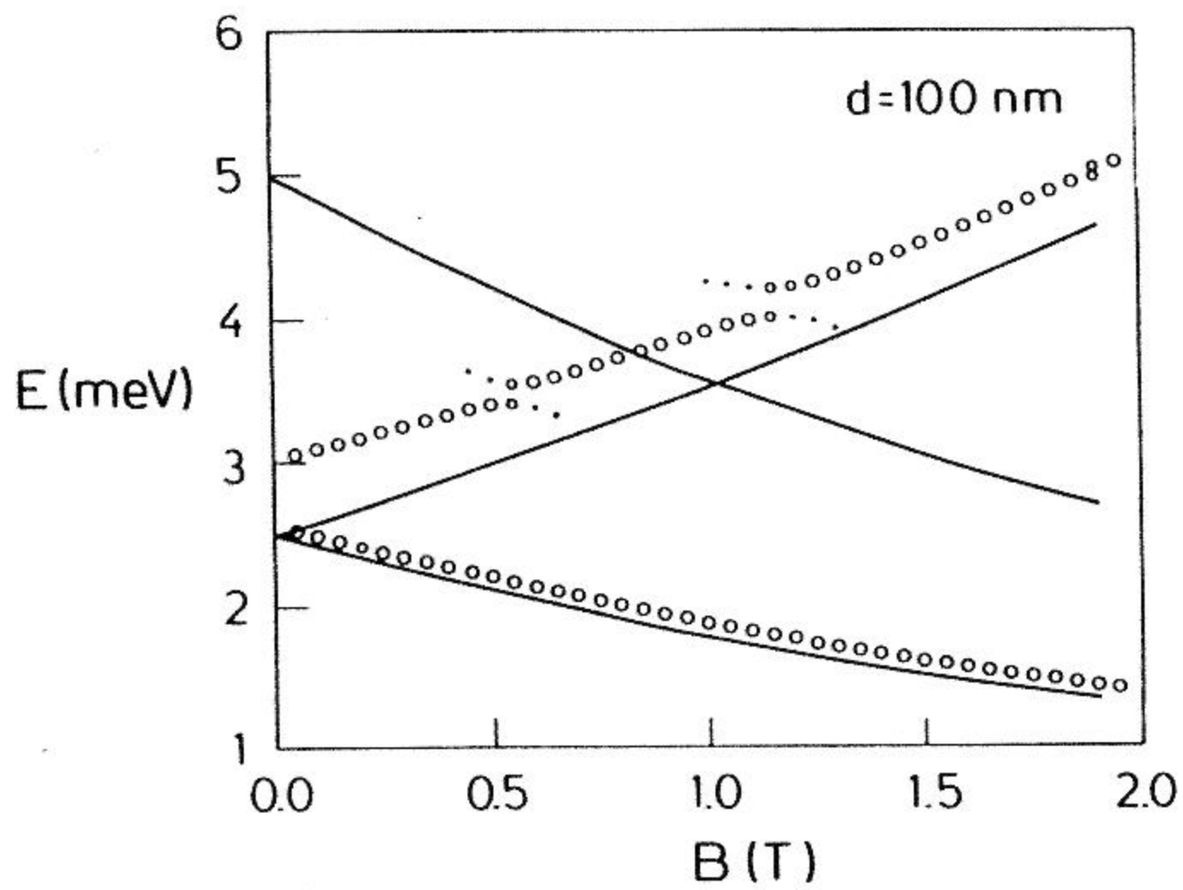

Figure 2: Dipole transition energies and intensities of a three-electron per dot pair. Dot separation 100 nm, confinement energy $\hbar\Omega = 2.5\ meV$. Solid lines are one-particle transition energies. Diameters of the circles are proportional to the *intensity* of the transition.

magnetic field.

III. AN EXCITON IN A QUANTUM DOT

We now consider an *ideal* two-dimensional hydrogenic exciton in a quantum dot. For simplicity, we consider the confinement potentials for both the electrons and holes to be parabolic which might be difficult to achieve experimentally. Other forms of confinement potentials for the excitons in a quantum dot have been studied previously by some authors[6]. The Hamiltonian in the present case is written as

$$\mathcal{H} = \frac{1}{2m_e}\left(-i\hbar\nabla_e - \frac{e}{c}\mathbf{A}_e\right)^2 + \frac{1}{2m_h}\left(-i\hbar\nabla_h + \frac{e}{c}\mathbf{A}_h\right)^2 + \frac{1}{2}m_e\Omega_e^2 r_e^2 + \frac{1}{2}m_h\Omega_h^2 r_h^2 - \frac{e^2}{\varepsilon}\frac{1}{|\mathbf{r}_e - \mathbf{r}_h|}.$$

Let us now introduce the CM and relative coordinates in the usual manner, $\mathbf{R} = \frac{1}{M}\left(m_e\mathbf{r}_e + m_h\mathbf{r}_h\right)$, $\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h$, where, $M = m_e + m_h$, $\mu = m_e m_h/M$, and $\gamma = (m_h - m_e)/M$. Considering the symmetric gauge, $\mathbf{A} = \frac{1}{2}\mathbf{B}\times\mathbf{r}$,

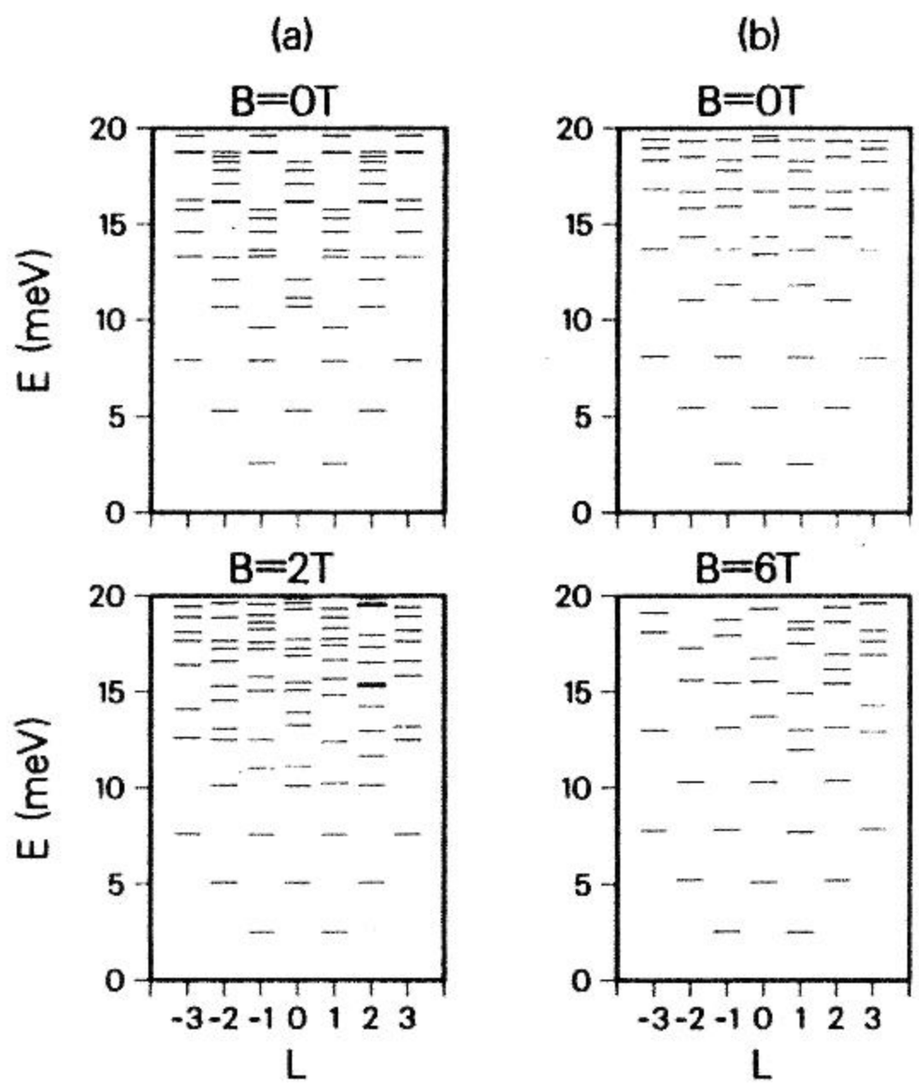

Figure 3: Low-lying excitation energies for the (a) light-hole ($m_h = m_e = 0.067m_0$) exciton and (b) heavy-hole ($m_h = 0.377m_0$) exciton. The confinement energy is $\hbar\Omega_e = \hbar\Omega_h = 2.5\ meV$.

the Hamiltonian is then rewritten as, $\mathcal{H} = \mathcal{H}_{\rm cm} + \mathcal{H}_{\rm rel} + \mathcal{H}_{\rm x}$, where

$$\mathcal{H}_{\rm cm} = -\frac{\hbar^2}{2M}\nabla^2_{\rm cm} + \frac{1}{2}M\left[\frac{1}{M}\left(m_e\Omega_e^2 + m_h\Omega_h^2\right)\right]R^2$$

$$\mathcal{H}_{\rm rel} = -\frac{\hbar^2}{2\mu}\nabla^2_{\rm rel} + \frac{i\hbar e}{2\mu c}\gamma\mathbf{B}\cdot\mathbf{r}\times\nabla_{\rm rel} + \frac{e^2B^2}{8\mu c^2}r^2 + \frac{1}{2}\mu\left[\frac{1}{M}\left(m_h\Omega_e^2 + m_e\Omega_h^2\right)\right]r^2 - \frac{e^2}{\varepsilon}\frac{1}{r}$$

$$\mathcal{H}_{\rm x} = \frac{i\hbar eB}{Mc}\mathbf{r}\times\nabla_{\rm cm} + \mu\left(\Omega_e^2 - \Omega_h^2\right)\mathbf{R}\cdot\mathbf{r}\ .$$

Here $\mathcal{H}_{\rm cm}$ is a Hamiltonian for a two-dimensional harmonic oscillator with energy spectrum

$$E_{\rm cm} = (2n_{\rm cm} + |\ell_{\rm cm}| + 1)\,\hbar\Omega_{\rm cm}$$

$$\Omega_{\rm cm} = \left[\frac{1}{M}\left(m_e\Omega_e^2 + m_h\Omega_h^2\right)\right]^{\frac{1}{2}}\ .$$

The other two terms $\mathcal{H}_{\rm rel}$ and $\mathcal{H}_{\rm x}$ are the Hamiltonian for the relative motion and the cross term respectively.

The problem of calculating the ground state and excited state propertie of a two-dimensional hydrogenic exciton which is in a parabolic confinemen potential and in a static perpendicular magnetic field can be solved accu rately using the method of numerical diagonalization of the Hamiltonian The basis functions are constructed such that they are eigenfunction o $\mathcal{H}_0 = \mathcal{H}_{\rm cm} + \mathcal{H}_{\rm rel}$. As the Coulomb interaction conserves the total angula momentum L, we can separate the calculation for different L. It is importan that the method of selecting the basis states includes the specified numbe of all the lowest eigenstates of $\mathcal{H}_0$ which are connected to each other by th Coulomb interaction. Calculated excitation energies with various confine ment energies and magnetic fields exhibit the behavior of a two-dimensiona harmonic oscillator (Fig. 3). This is surprising, given the fact that in th presence of a parabolic confinement and the magnetic field the Hamiltonia does not separate into CM and relative terms. The explanation is tha the cross term $\mathcal{H}_{\rm x}$ is a small correction to $\mathcal{H}_0$, at least for low confinemen energy and low magnetic fields.

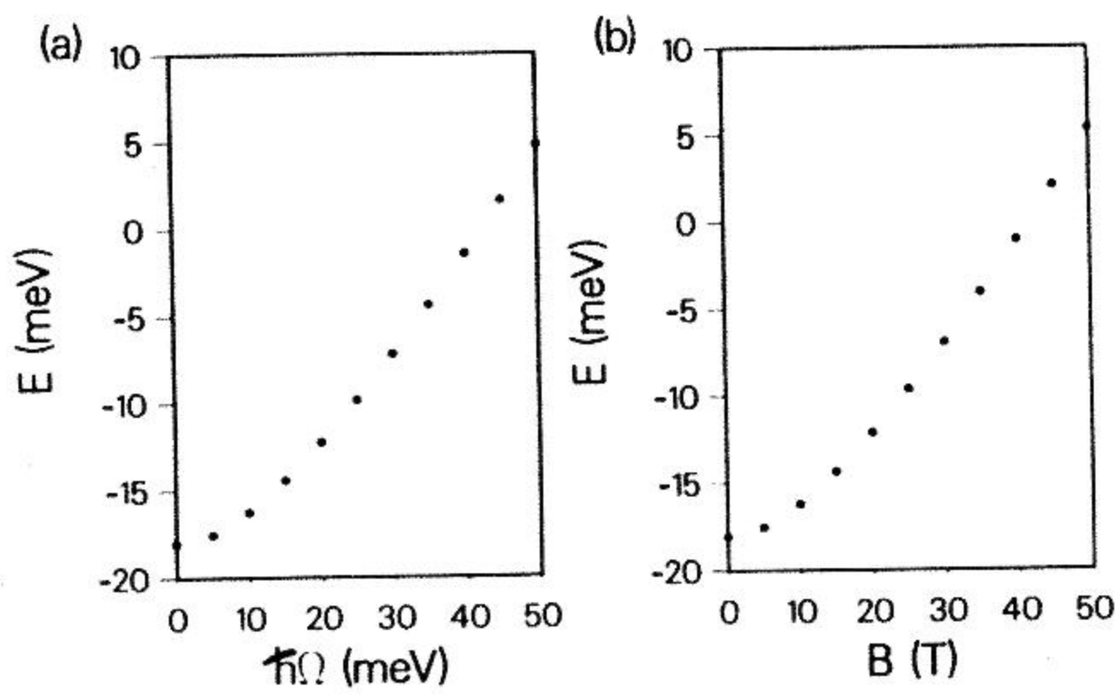

Figure 4: Ground state energy of the heavy-hole exciton vs (a) the confinement potential energy (center of mass energy $\hbar\Omega$ is subtracted) at $B = 0T$ and (b) the magnetic field (no confinement).

It should be noted that $\mathcal{H}_{\rm rel}$ is the Hamiltonian of a two-dimensional charged particle in a magnetic field and in a parabolic and Coulomb potentials. The Schrödinger equation for the radial part of this Hamiltonian can be solved numerically. The results for the ground state energy of (a) an exciton in a parabolic confinement and (b) a free exciton in a magnetic field

are presented in Fig. 4. It should be mentioned that there is no essential difference between the two cases because if one separates the Hamiltonian into CM and relative terms, the latter term is the same in both cases so that the parabolic confinement can be scaled into the magnetic field. We could infer from this figure that when the confinement energy is smaller than about 10 *meV* and the magnetic field is smaller than 10 *tesla* the ground state energy is close to that at zero confinement and at zero magnetic field. Therefore, the dominating term in the present system is the Coulomb interaction which is in line with the results of Fig. 3.

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