



Effect of the spin–orbit coupling on the Raman spectra of a GaAs quantum ring with few electrons



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ABSTRACT

The resonant inelastic light scattering from a GaAs quantum ring containing a few electrons is investigated theoretically in the presence of Rashba spin–orbit coupling. The spin–orbit interaction generates additional Raman transitions in the ring, whose amplitudes can be controlled externally by varying the spin–orbit coupling parameter. We also find that with an increase of the external magnetic field, there is a remarkable change in the Raman spectrum of a quantum ring due to the Aharonov–Bohm oscillations.

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1. Introduction

Quantum dots (QDs) or the artificial atoms [1] containing few interacting electrons have received considerable attention over a decade because of the rich physics they exhibit. Just as the QDs, quantum rings (QRs) are also nanometre-sized structures that confine electrons in all three directions. The observation of the Aharonov–Bohm oscillations [2] and the persistent current [3] in small conducting rings, and recent experimental realization of QRs with only a few electrons [4,5], have made QRs an attractive topic of experimental research and a new playground for the many-particle theory in quasi-one-dimensional systems [6,7]. The pioneering experiment of Lorke et al. [5] demonstrated that most of their rings contained just two electrons. Hence, QRs containing two interacting electrons have received considerable attention in recent years [8–11]. Together with the InGaAs QRs obtained by self-organizing technology, strain-free GaAs QRs have also been fabricated by several groups using the molecular beam droplet epitaxy technique [12,13].

A very useful mechanism for coherent spin manipulation in quantum nanostructures is via the Rashba spin–orbit interaction (SOI) [14,15]. The SOI can arise in a QD and a QR due to the confinement and lack of inversion symmetry of the nanostructure, which creates a local electric field perpendicular to the electron plane. The SOI strength can be varied by changing the asymmetry of the quantum structure with an external electric field. There

were a few recent reports on the tunability of the SOI in few-electron QDs [16]. The Rashba SOI is also the driving mechanism for making futuristic devices based on controlled spin transport, such as spin transistors and spin filters [17].

Our earlier work involving the Rashba effect on the electron and hole states in QDs [18,19] revealed that the SOI is responsible for multiple level crossings and level repulsions in the energy spectrum that was due to the interesting interplay between the Zeeman effect and the SOI. QRs made up of semiconducting materials exhibiting Rashba-type SOI have attracted considerable attention due to fundamental spin-dependent quantum interference phenomena that are observable in these systems [20–22]. Since the strength of the SOI can be tuned with external gate voltages, QRs, or systems involving those will also have possible spintronic applications [23].

The SOI can also have a huge impact on the optical properties of these nanostructures. The influence of Rashba and Dresselhaus SOI on the energy levels and optical absorption spectrum for a two-electron QD and QR was investigated earlier [11,24]. Raman spectroscopy is a very useful tool for the investigation of electronic and other type of elementary excitations. In recent years, these experiments are intensively used for investigation of semiconductor nanostructures [25]. Theoretically, Raman scattering of the QDs containing two electrons in the presence of the Rashba SOI was previously investigated in [19], while the experiments were reported by several authors [25,26]. No such experiments are reported as yet for the QRs.

Here we report on the Raman scattering in GaAs QRs containing one or two electrons for the polarized configuration. We found that in QRs, similar to QDs [19], due to the polarization selection

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rules, only singlet–singlet transitions are observed for the zero magnetic field, while triplet–triplet transitions are possible in a magnetic field. In this paper, we report that, similar to QDs [19], in the presence of the Rashba SOI there are additional singlet–triplet and triplet–singlet Raman transitions that are forbidden without the SOI. We also show that for increasing magnetic field it is possible to completely change the Raman spectrum of the QR due to the Aharonov–Bohm oscillations.

2. Theory

We consider a two-dimensional QR with internal radius R_1 and external radius R_2 . We chose the confinement potential of the QR with infinitely high borders: $V_{\text{conf}}(\rho) = 0$, if $R_1 \leq \rho \leq R_2$ and infinity outside of the QR. The Hamiltonian of the N -electron system in the ring can be written as

$$\mathcal{H} = \sum_i^N \mathcal{H}_i^e + \frac{1}{2} \sum_{ij}^N \frac{e^2}{\varepsilon |\mathbf{r}_i - \mathbf{r}_j|}, \quad (1)$$

where the second term describes the Coulomb interaction between the electrons and \mathcal{H}_i^e is the single-electron Hamiltonian in the presence of an external perpendicular magnetic field and with the SOI included

$$\mathcal{H}_i^e = \frac{1}{2m_e} \Pi_i^2 + V_{\text{conf}}(\rho) + \frac{1}{2} g \mu_B B \sigma_z + H_{\text{SO}}, \quad (2)$$

where $\Pi = \mathbf{p} - (e/c)\mathbf{A}$ and \mathbf{A} is the vector potential of the magnetic field. The third term on the right hand side of (2) is the Zeeman splitting. The last term describes the Rashba SOI [14]

$$H_{\text{SO}} = \frac{\alpha}{\hbar} \left[\boldsymbol{\sigma} \times \left(\mathbf{p} - \frac{e}{c}\mathbf{A} \right) \right]_z, \quad (3)$$

with α being the SOI parameter, which is sample dependent and is proportional to the interface electric field that confines the electrons in the xy plane. In (2) and (3), $\boldsymbol{\sigma}$ is the electron spin operator and σ_x, σ_y and σ_z are the Pauli spin matrices.

For $\alpha = 0$ (no SOI) the eigenfunctions of the single-electron Hamiltonian (2) are

$$\psi(r, \varphi) = \frac{1}{2\pi} e^{il\varphi} f_{nl}(r), \quad (4)$$

where n and l are the quantum numbers and $f_{nl}(r)$ is the radial wave function which in the presence of a magnetic field can be expressed in terms of the confluent hypergeometric functions [27]

$$f_{nl}(r) = C_{nl} e^{-r^2/4a_c^2} r^{|l|} \left[F \left(-k_e - \frac{l-|l|-1}{2}, |l|+1, \frac{r^2}{4a_c^2} \right) + D_{nl} U \left(-k_e - \frac{l-|l|-1}{2}, |l|+1, \frac{r^2}{4a_c^2} \right) \right], \quad (5)$$

here $k_e = E_{nl}/\hbar\omega_c$, ω_c is the cyclotron frequency, a_c is the magnetic length, C_{nl} is the normalization constant and

$$D_{nl} = - \frac{F \left(-k_e - \frac{l-|l|-1}{2}, |l|+1, \frac{R_1^2}{4a_c^2} \right)}{U \left(-k_e - \frac{l-|l|-1}{2}, |l|+1, \frac{R_1^2}{4a_c^2} \right)}. \quad (6)$$

The single-electron eigenvalues E_{nl} can be obtained numerically from the boundary conditions of the wave function.

The eigenfunctions of the single-electron Hamiltonian (2) with the SOI included can be presented as a linear expansion of the wave functions (4). The single-electron energies and the wave functions are then found by the exact diagonalization of the Hamiltonian matrix (2). In order to calculate the energy spectrum of the few-electron system we need to diagonalize the

Hamiltonian (1) in a basis of the Slater determinants constructed from the single-electron wave functions.

To evaluate the Raman transition amplitudes, first we have to define the initial, final and the intermediate states. Let us consider the resonant inelastic light-scattering process in a backscattering configuration with the incident photon energy just above the effective band gap of the QR and with the wave vector transfer in the lateral dimension [26] $q = 2 \times 10^4 \text{ cm}^{-1}$. Then the initial states of the N -electron system will be the ground state, and the final states will be the intraband excitations of the N -electron system with the total momentum projection difference $\Delta J_z = 0, \pm 2$, because the Raman scattering is a two photon process. For the intermediate states, we have $N+1$ electrons in the conduction band and one additional hole in the valence band. The inclusion of band mixing effects and the Rashba effect for the hole states introduces considerable complications but does not bring in substantial changes in the results [19,28]. Therefore, we consider here only the heavy-hole states without the SOI. Under this approximation the single-hole Hamiltonian for the envelope function is similar to the one for the electron. We need to change only the value of the effective mass. The basis functions of the intermediate states can be constructed as products of the Slater determinants of the electrons and the single-hole wave functions.

The Raman scattering transition amplitude from the initial state $|i\rangle$ to the final state $|f\rangle$ is obtained from [29]

$$A_{fi} \sim \sum_{\text{int}} \frac{\langle f | H^{(+)} | \text{int} \rangle \langle \text{int} | H^{(-)} | i \rangle}{\hbar\omega_i - (E_{\text{int}} - E_i) + i\Gamma_{\text{int}}}, \quad (7)$$

where $\hbar\omega_i$ is the incident photon energy. In (7) $H^{(-)}$ and $H^{(+)}$ are the single-particle operators describing the photon absorption ($-$) and emission ($+$) processes respectively [19,29].

Two cases can be considered: (i) the polarized geometry, i.e., when the polarization vectors of incident and scattered photons are in the same direction, and (ii) the depolarized geometry, when the polarization vector of the scattered photon is perpendicular to that of the incident one. The differential cross section of the Raman scattering can be calculated using the following expression:

$$d\sigma \sim \sum_f |A_{fi}|^2 \delta(\Delta E - (E_f - E_i)), \quad (8)$$

where $\Delta E = \hbar\omega_i - \hbar\omega_s$ is the Raman energy shift. To take into account the level width of the final states [19] we use a Lorentzian instead of the Dirac delta function.

3. Discussion of results

Our numerical studies were carried out for the GaAs QR using the following parameters: $m_e = 0.067m_0$, $m_h = 0.33m_0$, $E_g = 1450 \text{ meV}$, $g_e = -0.44$, $g_h = 1.2$, $\varepsilon = 12.9$ [5,30–32]. In our previous works on Raman scattering in the QD [19] we used the level widths for initial, final and intermediate states in order to compare our results with the experimental data [26]. In the absence of relevant experiments for the QRs, here we use the same values $\Gamma_{\text{int}} = \Gamma_f = 0.5 \text{ meV}$ of level widths for the QR which might provide insights into future experiments. In Fig. 1, the magnetic field dependence of the low-lying energy levels of a QR with one (a, b, c) and two (d, e, f) electrons is presented for different values of the total momentum J_z and for three values of the SOI parameter α . As in the case of the QD [19] the outstanding features in the energy levels of QR with SO coupling are again lifting of degeneracy at a vanishing magnetic field, rearrangement of some of the levels at small fields, and the level repulsion at higher magnetic fields. It should be noted that the effect of level repulsion is more pronounced in the case of two electrons

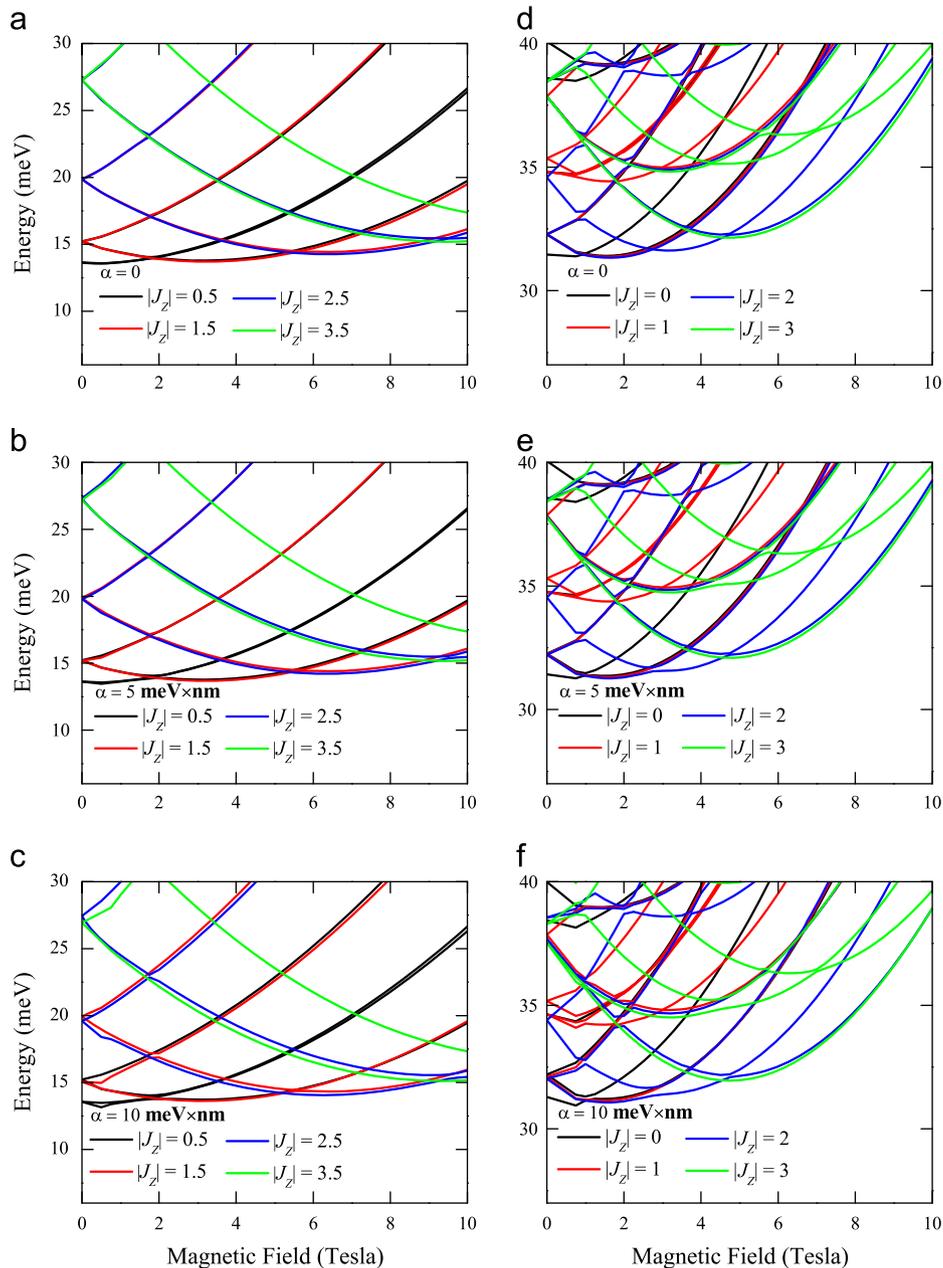


Fig. 1. (Color online) Magnetic-field dependence of the low lying energy levels in a one-electron (a–c) and two-electron (d–f) quantum ring for various values of the SOI strength α (in meV nm) and total momentum projection J_z .

and especially for the first and second excited levels with total momentum $J_z = 3$ (Fig. 1(e) and (f)).

The most outstanding feature in the energy spectra of the QR as compared to that in a QD is the observation of the Aharonov–Bohm oscillations [2]. For the ground state of the one-electron QR, several oscillations can be observed (Fig. 1(a)–(c)) which manifests themselves by the change of total momentum J_z of the ground state. In the case of two electrons, due to the interplay between Aharonov–Bohm effect and Coulomb interaction the oscillations of the ground state have a mixed nature. Again there are Aharonov–Bohm oscillations, but here due to the Coulomb interaction not all the values of total momentum J_z for the ground state are energetically favorable, so for the magnetic field range up to 10 Tesla the ground state never acquires total momentum $J_z = 1$ and this effect was observed for the QD earlier [1]. Besides that for the region of magnetic field 2–4 Tesla the triplet to singlet transition can be observed but with the same total momentum

$J_z = 2$ and this is again the result of interaction between particles indicated earlier for the QD [18].

The Rashba SO coupling can play an important role in Raman spectroscopy of the QRs. The effect of Rashba SO coupling on the Raman excitations for quantum rings containing one or two electrons is shown in Fig. 2. In both cases we consider Raman excitations in the polarized geometry with selection rules on the total angular momentum projection $\Delta J_z = 0, \pm 2$. It should be mentioned here that the large values of α used in the figures are perhaps not realistic for GaAs QRs, but we keep those values for understanding the limiting behaviors which can be useful for other QRs.

We begin with the Raman scattering for one and two electron quantum rings without the SO coupling (Fig. 2(a) and (d)). It is easy to see that for the case of the one-electron QR, we have only one peak with the Raman energy shift of $\Delta E = 6.2$ meV. That peak corresponds to the excitation of the system from the ground state

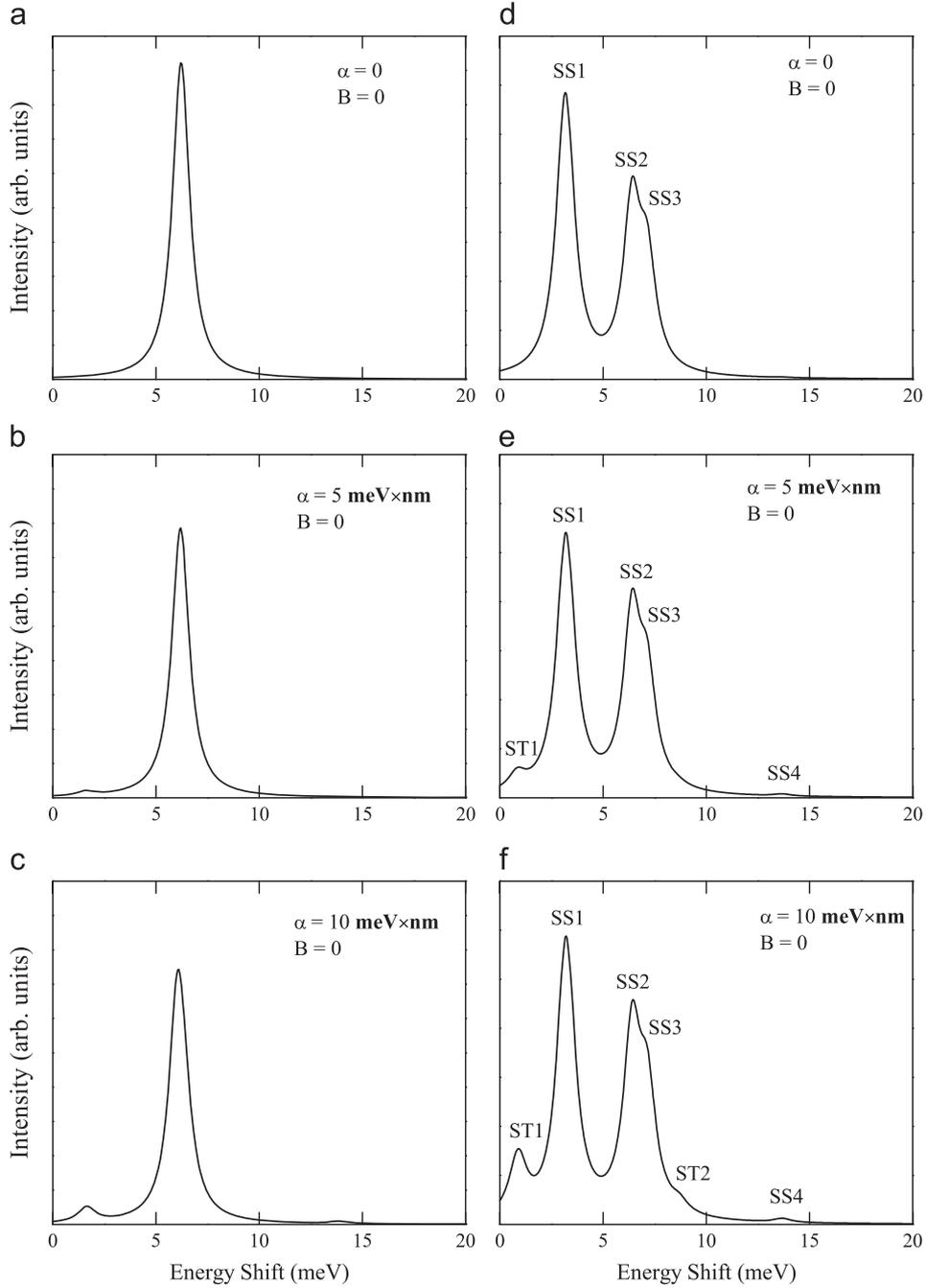


Fig. 2. Raman scattering amplitudes for the polarized geometry and for various values of SO coupling parameter α for single-electron (a–c) and for two-electron (d–f) quantum rings.

$|n=1, l=0, s=1/2\rangle$ with total momentum projection $J_z=1/2$ to the excited state $|1, 2, 1/2\rangle$ with $J_z=5/2$ and $|1, -2, 1/2\rangle$ with $J_z=-3/2$. In the case of a quantum ring with two electrons, additional Raman modes appear. In the absence of the magnetic field the ground two-electron state is a singlet with $J_z=0$ and in polarized geometry the Raman transitions are only possible for singlet excited states [33]. We label such transitions as singlet–singlet (SS).

Clearly the Rashba SO coupling is responsible for additional Raman excitations even for the one-electron system. With the change of the SO parameter α it is possible to manipulate the amplitudes of these additional excitations. To understand this effect we consider the first three one-electron states in the absence of a magnetic field (Fig. 1(a)). Without the SO coupling

Raman excitations with higher amplitude are possible only between the states with $\Delta l=0, \pm 2$ and the transition is only from the ground state to the second excited state. However, the SOI mixes all those states that can be expressed as a linear combination of the states with different angular momenta l . Therefore, we now have the possibility of Raman transitions from the ground state to both excited states. That is why we see an additional transition with smaller energy in Fig. 2(b) and (c).

A similar situation is also observed for a two-electron QR (Fig. 2(d)–(f)). Unlike for $\alpha=0$, here we see many additional peaks in the observable energy range. With the increase of the Rashba coupling strength α the SOI will again mix the states with different total spins and therefore we cannot characterize the states as fully singlet or triplet. All the states must now be presented as

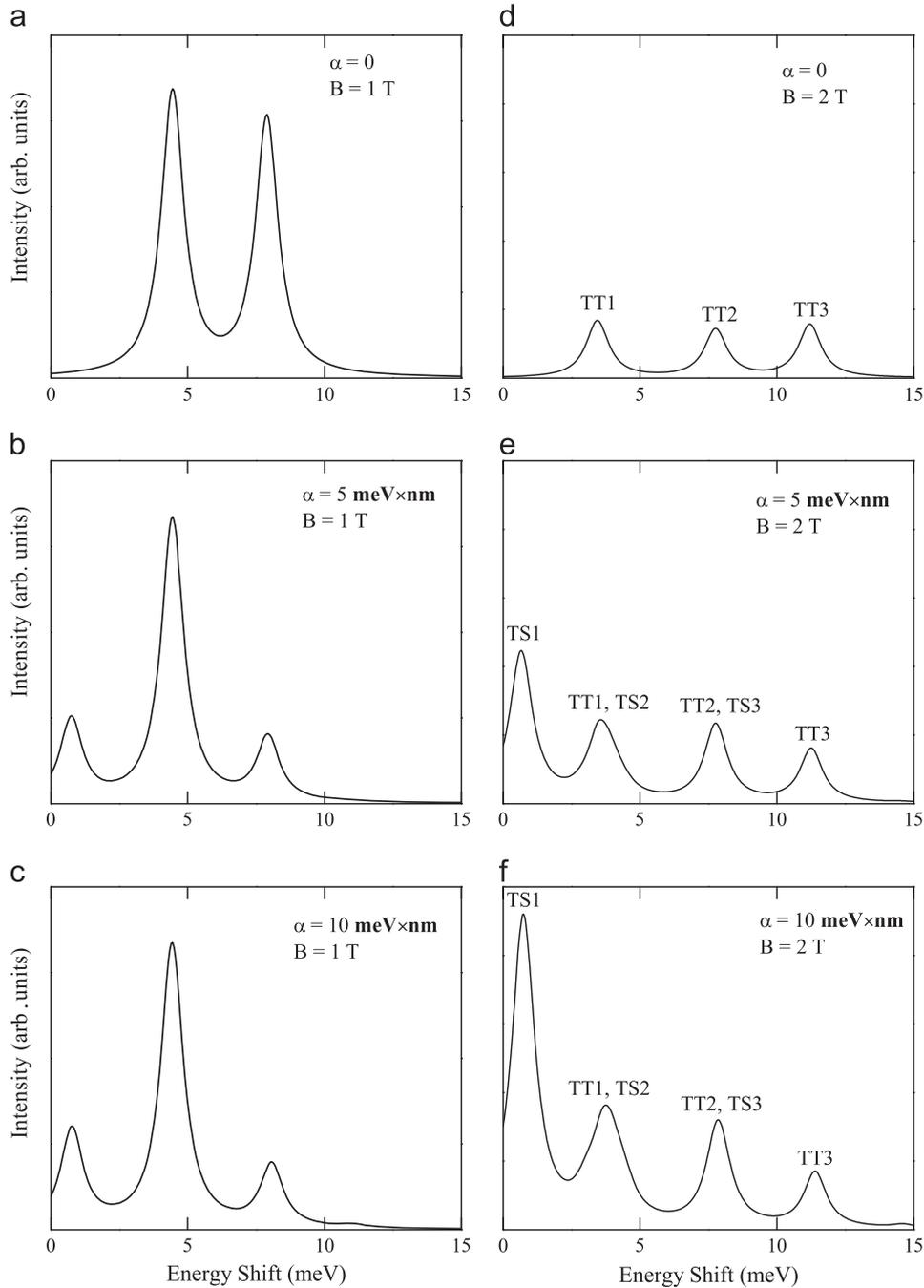


Fig. 3. Same as in Fig. 2 but for various values of the magnetic field B .

superpositions of two-component basis states $|n_1, l_1, s_1\rangle|n_2, l_2, s_2\rangle$. For $\alpha = 5$ meV nm the most important component of the ground state is the singlet state $|1, 0, 1/2\rangle|1, 0, -1/2\rangle$ with weight 73.7% and we can still characterize it as a singlet. Due to the SO mixing, the final states also will have triplet components. For example, the most important components of the two final states in the first peak in Fig. 2(e) (this peak results from the transition to two final states) are the triplet components $|1, -1, 1/2\rangle|1, 0, 1/2\rangle$ with weight 91.1% for the first and $|1, 0, -1/2\rangle|1, 1, -1/2\rangle$ with weight 91.8% for the second state. Hence we can call that the singlet-triplet (labeled as ST1) transition. The peaks SS1, SS2 and SS3 in Fig. 2(e) are very similar to the peaks for the case without the SOI and are essentially singlet-singlet transitions. In Fig. 2(e) and (f) several singlet-triplet (ST) and singlet-singlet (SS) transitions are visible.

In Figs. 3 and 4 we present the results for various values of the magnetic field B . The magnetic field significantly changes the Raman spectra. The situation is also different from the case of a QD considered in [19] due to Aharonov-Bohm oscillations in the QR. In Fig. 3(a), the Raman transitions for the one-electron QR are presented for $B = 1$ Tesla without the SOI. Here again the ground state is $|1, 0, 1/2\rangle$. Here there are two separate transitions which correspond to the final states $|1, -2, 1/2\rangle$ and $|1, 2, 1/2\rangle$. The magnetic field removes the degeneracy of these states and we see two peaks. With an increase of α (Fig. 3(b) and (c)) again some new transitions become visible. In Fig. 3(d)–(f) we consider the case of a two-electron QR in a field of $B = 2$ Tesla. In this field a singlet-triplet transition of the ground state appears and now for $\alpha = 0$ the ground state is triplet state with the most important component $|1, 0, 1/2\rangle|1, 1, 1/2\rangle$ with $J_z = 2$. That is why this case is

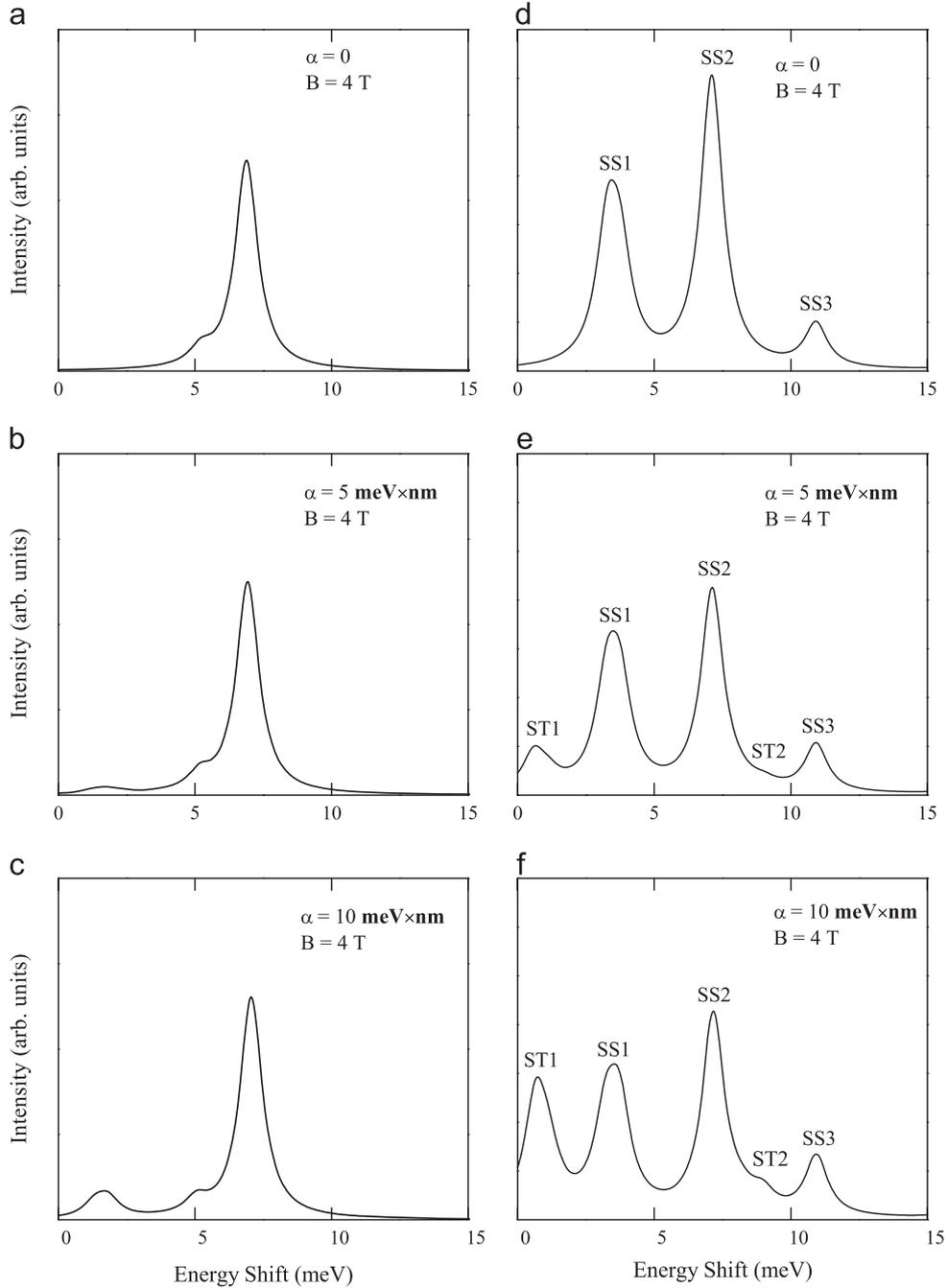


Fig. 4. Same as in Fig. 2 but for the value of the magnetic field $B=4$ Tesla.

principally different from the case presented in Fig. 2(a). Now Raman transitions are possible only to the final states with $J_z=0$ and $J_z=2$. In Fig. 3(d) three peaks are visible with approximately equal amplitudes corresponding to triplet–triplet transitions. With an increase of α (Fig. 3(e) and (f)) additional triplet–singlet transitions become visible (TS1, TS2, TS3). For the value $\alpha = 10$ meV nm the transition TS1 becomes dominant.

Fig. 4 is the same as Fig. 2 but for $B=4$ Tesla. For the one-electron QR without the SOI (Fig. 4(a)) the ground state is changed to $|1, 1, 1/2\rangle$. In this case there is one strong transition to the state $|1, -1, 1/2\rangle$ and one weaker to the state $|1, 3, 1/2\rangle$. With an increase of the SOI strength the most important components of the ground state are $|1, 1, 1/2\rangle$ and $|1, 2, -1/2\rangle$. In this case again we observe several additional transitions with spin flip, the amplitudes of which increase with the increase of α . For the

two-electron QR without the SOI the most important components of the ground state are $|1, 1, -1/2\rangle|1, 1, 1/2\rangle$, $|1, 0, 1/2\rangle|1, 2, -1/2\rangle$ and $|1, 0, -1/2\rangle|1, 2, 1/2\rangle$. Therefore the ground state again has changed to the singlet but the total momentum projection is still $J_z=2$. That is why the picture in Fig. 4(d) is principally different from Fig. 3(d). Here we observe only singlet–singlet transitions (SS1, SS2, SS3). With an increase of α we again observe several additional singlet–triplet transitions with growing amplitudes.

To summarize, we have studied the influence of the Rashba SOI on the resonant Raman electronic excitations in one- and two-electron GaAs quantum rings for the polarized configuration. The SO coupling has relatively weak effect on the energy spectrum of the system, but it can bring in additional Raman transitions, the amplitudes of which depend on the coupling parameter α . In the case of a two-electron QR, in addition to the usual singlet–singlet

and triplet–triplet Raman transitions (for polarized configuration) we also observe the singlet–triplet and triplet–singlet Raman transitions. With an increase of the external magnetic field, the Raman spectrum of the QR changes completely due to the Aharonov–Bohm oscillations.

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