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Correlated electrons in narrow channels

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Abstract

We investigate the properties of a system of interacting electrons in a narrow channel in the quantum Hall effect regime. We find that an increase in the strength of the Coulomb interaction causes abrupt changes in the width of the charge-density profile of translationally invariant states. The resulting phase diagram includes many of the stable odd-denominator states and also a novel fractional quantum Hall state at lowest half-filled Landau level. The collective modes evaluated at $\nu = \frac{1}{2}$ and at $\nu = \frac{1}{3}$ reveal soft modes in between the translationally invariant states in the phase diagram.

Keywords: Quantum Hall effect; Half-filled Landau level; Narrow channel

The fractional quantum Hall effect (FQHE) was discovered in a two-dimensional electron system created in a semiconductor heterostructure subjected to a strong perpendicular magnetic field [1]. It is characterized by Hall plateaus at Landau level filling factors $\nu = N_e/N_s$ (N_e is the number of electrons and N_s is the Landau level degeneracy) corresponding to certain simple fractions with mostly odd denominators. The understanding of the FQHE is essentially due to Laughlin [2], who proposed that electrons condense into incompressible quantum fluid states at certain filling factors. The FQHE occurs when the chemical potential of the electron system is discontinuous at certain magnetic field-dependent densities leading to incompressibility [3]. The energy gap structures in the excitation spectra are also well established for the incompressible states [4, 5]. Interestingly, despite intense activities at the half-filled Landau level [6–18], such a level of understanding has not yet been achieved for the simplest even-denominator state.

In recent years there has been a great deal of interest in systems where electrons are confined in even lower dimensions [19]. For example, ballistic quantum wires fabricated [20, 21] with long transport mean-free path and very large subband spacings enable one to study effects of electron–electron interactions in one-dimensional systems. At high magnetic fields when the radius of the cyclotron orbit – the length scale in the system – is smaller than the width of the channel it is plausible that the system behaves like a two-dimensional electron gas (2DEG) in the sense that it exhibits the QH states. The boundaries of the sample are, of course, still important in that regime, and therefore, in theoretical models one should be able to treat the edges and bulk on the same footing.

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There have been experimental attempts to study the QHE in one dimension (1D-QHE). Following a theoretical prediction that in a one-dimensional system (hard-wall boundary condition) the size of the FQHE gap is comparable for the filling factors $\nu = \frac{1}{2}$ and $\nu = \frac{1}{3}$ [22], observation of incompressible states at specific filling factors in a narrow-channel electron system was reported a few years ago [23, 24]. In that experiment, in addition to various odd-denominator fractions, Hall resistance also showed signatures of quantization at the half-filled Landau level. That observation brought into focus the question about the existence and nature of even-denominator states together with the role of boundaries of the sample. Theoretical interest on 1D-FQHE has been revived by a recent work of Yoshioka [25, 26] where a parabolic confinement (see below) was used and incompressible states were found at $\nu = \frac{1}{3}$ and $\frac{2}{3}$, but that approach was somewhat different in spirit than our work described below.

There were several attempts to explain the origin of the non-existence of a stable half-filled quantum Hall state in two dimensions. In Laughlin's theory the even denominator states are symmetric with respect to particle exchange, and therefore, describes the properties of bosons rather than fermions. One other explanation for the non-existence of a stable half-filled state in two dimension was suggested in Refs. [27, 28] where it was shown that in order to stabilize the $\nu = \frac{1}{2}$ state one must reduce the short-range part of the electron–electron interaction. The popular explanation of the non-existence of QHE states at $\nu = \frac{1}{2}$ involves fermion Chern–Simons theory [13–15] where the ground state and the low-energy excitations can be described by a modified Fermi-liquid theory, more precisely, by a theory of fermions in zero magnetic field.

In our model for the 1D-QHE, electrons (spin polarized) are confined by an electrostatic potential which is parabolic in the lateral direction and flat along the channel, and are also subjected to a strong perpendicular magnetic field. The electrons are considered to be in a cell of length a. The width of the cell is determined by the lateral electron density profile which is finite due to the presence of the confinement potential. We impose a periodicity condition along the length of the cell. The total Hamiltonian of the system is

$$\mathscr{H} = \mathscr{H}_0 + \mathscr{H}_{\text{int}}, \tag{1}$$

where $\mathscr{H}_0$ includes the kinetic energy of N_e electrons of effective mass m^* and the electrostatic confining potential

$$\mathscr{H}_0 = \sum_{i=1}^{N_e} \left[\frac{1}{2m^*}(\boldsymbol{p}_i - e\boldsymbol{A}_i)^2 + \frac{1}{2}m^*\omega_0^2 y_i^2 \right] \tag{2}$$

with $\boldsymbol{A}$ being the vector potential in Landau gauge. The interaction term of the Hamiltonian consists of the Coulomb interactions and also the terms corresponding to the neutralizing background

$$\mathscr{H}_{\text{ee}} = \frac{1}{2}\sum_{i\neq j}\frac{e^2}{\varepsilon|\boldsymbol{r}_i - \boldsymbol{r}_j|},$$

$$\mathscr{H}_{\text{bb}} = \frac{1}{2}\int \mathrm{d}\boldsymbol{R}\,\mathrm{d}\boldsymbol{R}'\frac{e^2\rho(\boldsymbol{R})\rho(\boldsymbol{R}')}{\varepsilon|\boldsymbol{R} - \boldsymbol{R}'|},$$

$$\mathscr{H}_{\text{eb}} = -\sum_i \int \mathrm{d}\boldsymbol{R}\frac{e^2\rho(\boldsymbol{R})}{\varepsilon|\boldsymbol{R} - \boldsymbol{r}_i|}, \tag{3}$$

where ε is the background dielectric constant and $\rho(\boldsymbol{R})$ is the background charge density.

The single-electron wave functions labeled by $\kappa = \{m, n\}$ are

$$\psi_\kappa = \left(\frac{1}{a\sqrt{\pi}\lambda}\right)^{1/2}\exp\left(\mathrm{i}kx - \frac{\hat{y}^2}{2\lambda^2}\right)H_n\left(\frac{\hat{y}}{\lambda}\right), \tag{4}$$

where $k = (2\pi/a)m$ is the quantized wave vector and m is the momentum quantum number. The effective Larmor radius for the cyclotron motion is $\lambda = (\hbar/m^*\Omega)^{1/2}$, where $\Omega = \sqrt{\omega_0^2 + \omega_c^2}$ and $\omega_c = eB/m^*$ is the cyclotron frequency. The single-electron states, Eq. (4) are localized around $\hat{y} \equiv y + 2\pi\lambda^2 m/\gamma a = 0$. Here $\gamma = \sqrt{1 + (\omega_0/\omega_c)^2}$ is a dimensionless quantity. The single-electron states for m and m' are therefore separated spatially in lateral direction by an amount $\Delta y_{mm'} = 2\pi(m - m')\lambda^2/\gamma a$. For a very strong confinement energy with respect to cyclotron energy, spatial separation of the single-electron states vanishes – the system becomes truly one-dimensional. On the other hand, when the confinement becomes weaker with respect to cyclotron energy, the states begin to separate spatially and the system becomes quasi-one-dimensional. In Eq. (4), H_n is the Hermite polynomial of order n. In what follows we set $n = 0$

(lowest Landau level). Ignoring the constant Landau level energy the single-electron energies are given by

$$\mathscr{E}_m = \frac{\hbar^2 k^2}{2m^*}\frac{\omega_0^2}{\Omega^2}. \tag{5}$$

In the non-interacting ground state, N_e electrons occupy the lowest available single-particle states. It is then reasonable to require that the electron density is symmetric around the $y = 0$ axis, i.e., $M = \sum_j m_j = 0$. This symmetry condition holds for *odd* number of electrons if m is an *integer*. In that case the boundary condition is periodic: $\psi_k(\boldsymbol{r} + a\hat{x}) = \psi_k(\boldsymbol{r})$. For *even* number of electrons m has to be a *half-odd integer*. The boundary condition is then anti-periodic: $\psi_k(\boldsymbol{r} + a\hat{x}) = -\psi_k(\boldsymbol{r})$. As we turn on the inter-electron interaction electrons start to avoid each other. Increasing the interaction strength with respect to the kinetic energy it becomes energetically favorable for the electrons to occupy also the higher-energy single-particle levels in order to reduce the Coulomb repulsion.

Given the single-particle states the Hamiltonian of the many-electron system, projected to the lowest Landau level, is now written as

$$\mathscr{H} = \sum_m \mathscr{E}_m a_m^\dagger a_m + \sum_{m_1,\ldots,m_4} \mathscr{A}_{m_1,m_2,m_3,m_4} a_{m_1}^\dagger a_{m_2}^\dagger a_{m_3} a_{m_4}, \tag{6}$$

where $a_m^\dagger(a_m)$ is the cration (annihilation) operator for the state m. For the Coulomb potential

$$v(\boldsymbol{r}) = \frac{1}{(2\pi)^2}\int \mathrm{d}\boldsymbol{q}\frac{2\pi e^2}{\varepsilon q}\mathrm{e}^{-\mathrm{i}\boldsymbol{r}\cdot\boldsymbol{q}}, \tag{7}$$

where $\boldsymbol{q} = (q_x, q_y)$, the interaction matrix element is given by

$$\begin{aligned}
&\mathscr{A}_{m_1,m_2,m_3,m_4} \\
&= \frac{1}{2}\int \mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2 \psi_{m_1}^*(\boldsymbol{r}_1)\psi_{m_2}^*(\boldsymbol{r}_2)v(\boldsymbol{r})\psi_{m_3}(\boldsymbol{r}_2)\psi_{m_4}(\boldsymbol{r}_1) \\
&= \frac{1}{2}\frac{e^2}{\varepsilon\lambda}\exp\left[-\frac{1}{2}\left(\frac{2\pi}{\gamma a}\right)^2 (m_1 - m_4)^2\right] \\
&\quad\times\int \mathrm{d}q_y' \frac{\exp[\mathrm{i}2\pi(m_3 - m_1)q_y']\exp[-\frac{1}{2}(\gamma a q_y')^2]}{\sqrt{[(2\pi(m_1 - m_4))/\gamma a]^2 + (aq_y')^2}} \\
&\quad\times \delta_{m_1+m_2,m_3+m_4},
\end{aligned} \tag{8}$$

where $q_y' = q_y\lambda^2/(\gamma a)$ is the dimensionless integration variable, λ is the unit of length and $\mathscr{E}_c = e^2/\varepsilon\lambda$ is a measure of the interaction strength. It is to be noted that the diagonal term $m_1 = m_4$ ($m_2 = m_3$) diverges as expected for the long-range Coulomb potential. To cancel out this divergence we neutralize the system by embedding the quantum wire into a positively charged background with a density

$$\rho(\boldsymbol{R}) = \frac{N_e}{a\sqrt{\pi}d'}\exp\left[-\frac{Y^2}{d'^2}\right]. \tag{9}$$

The width of the background density profile is denoted by d' and in actual calculations we set it equal to the length of the cell. The electrostatic energy of the background charge is

$$\begin{aligned}
\mathscr{H}_b &= \frac{1}{2}\int \mathrm{d}\boldsymbol{R}\,\mathrm{d}\boldsymbol{R}'\frac{e^2\rho(\boldsymbol{R})\rho(\boldsymbol{R}')}{\varepsilon|\boldsymbol{R} - \boldsymbol{R}'|} \\
&= \frac{1}{2}N_e^2\frac{e^2}{\varepsilon\lambda}\frac{1}{a}\int \mathrm{d}q_y'\frac{1}{|q_y'|}\exp\left[-\frac{1}{2}(\gamma a d' q_y')^2\right].
\end{aligned} \tag{10}$$

The interaction of electrons with the positive background charge is taken into account by the potential $\mathscr{H}_{eb}$. In general, the electron–background interaction is a one-body operator with the diagonal matrix element

$$\begin{aligned}
\mathscr{T}_{m_1,m_2} &= \int \mathrm{d}\boldsymbol{r}\psi_{m_1}^*(\boldsymbol{r})v_{eb}(\boldsymbol{r})\psi_{m_2}(\boldsymbol{r}) \\
&= -\frac{e^2}{\varepsilon\lambda}\frac{N_e}{a}\int \mathrm{d}q_y'\frac{1}{|q_y'|} \\
&\quad\times\exp\left[\mathrm{i}2\pi m_1 q_y' - \frac{1}{4}(\gamma a q_y')^2(1 + d'^2)\right] \\
&\quad\times\delta_{m_1,m_2}.
\end{aligned} \tag{11}$$

Using the above relations, the final form of the diagonal two-body term is

$$\begin{aligned}
\mathscr{A}_{m,m',m',m} &= \frac{1}{2}\frac{e^2}{\varepsilon\lambda}\frac{1}{a} \\
&\quad\times\int \mathrm{d}q_y'\frac{1}{|q_y'|}\left\{\exp\left[\mathrm{i}2\pi(m - m')q_y' - \frac{1}{2}(\gamma a q_y')^2\right]\right. \\
&\quad -2\exp\left[\mathrm{i}2\pi m q_y' - \frac{1}{4}(1 + d'^2)(\gamma a q_y')^2\right] \\
&\quad\left.+\exp\left[-\frac{1}{2}(\gamma a d' q_y')^2\right]\right\},
\end{aligned}$$

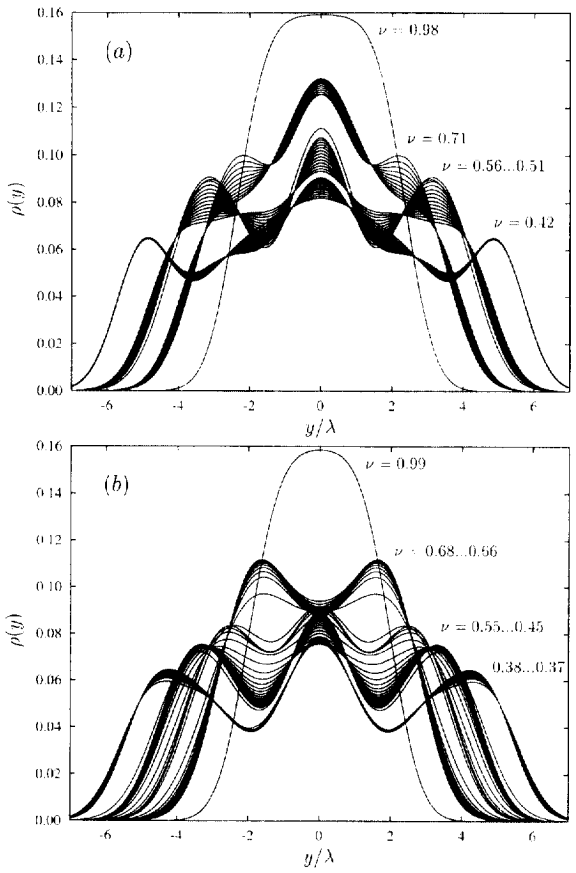

Fig. 1. (a) Calculated electon densities in the lateral direction for $a = 8$ and the $M = 0$ states. (b) The results at $a = 9.5$. The effective filling factors are also indicated.

where the divergent part of the electron–electron term is now cancelled by the positive background charge.

We now examine the change in the density profile of the translationally invariant state ($M = 0$) when we increase $\mathscr{E}_c/E_0$ for a fixed value of a. Electron density at r is calculated numerically from

$$\rho(r) = \sum_{i,j=1}^{\infty} \psi_i^*(r)\psi_j(r)a_i^\dagger a_j. \tag{12}$$

In the x-direction the charge density is constant while in the lateral y-direction it is modified because of the finite width of the system. Interestingly, as $\mathscr{E}_c/E_0$ increases the width of the charge-density profile changes abruptly from one value to another (Fig. 1). The effective filling factors, calculated via $\nu = 2\pi\lambda^2 n$ (where

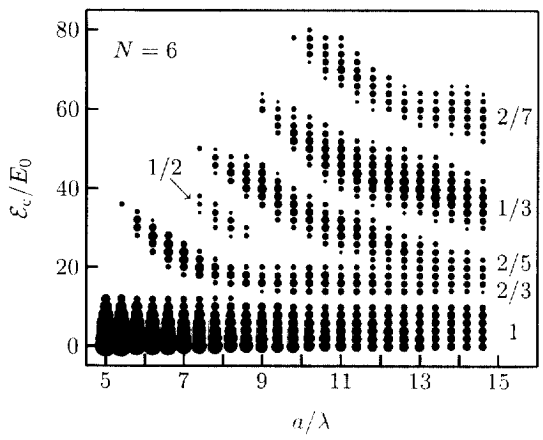

Fig. 2. Phase diagram for the FQHE states derived at the effective filling factors $\nu = 1, \frac{1}{3}, \frac{2}{3}, \frac{2}{5}, \frac{1}{2}, \frac{2}{7}$.

n is the number of electrons per unit area and the width of the wire is the full-width at half-maximum) are found to be $0.98, 0.71, \ldots, 0.51$ and 0.42 which are very close to $\nu = 1, \frac{2}{3}, \frac{1}{2}$ and $\nu = \frac{2}{5}$ in Fig. 1a ($a = 8$) and 0.99 and $0.68, \ldots, 0.66$ in Fig. 1b ($a = 9.5$) which suggests that these states are $\nu = 1$ and $\nu = \frac{2}{3}$, respectively. The state which has the effective filling factor $0.38, \ldots, 0.37$ is identified as a $\nu = \frac{1}{3}$ state also by calculating the Coulomb-$\frac{1}{3}$ state and Haldane's pseudopotential state [25, 26].

A phase diagram for the 1D-QHE states are then obtained (Fig. 2) by systematically seeking those points in the parameter space spanned by a and $\mathscr{E}_c/E_0$ where the ground state has *zero total momentum*. We then plot the energy gap between this ground state and the first excited state. In Fig. 2 the area of a filled dot is directly proportional to that gap. The phase diagram consists of separate regions of several QHE states. There is also a distinct region for the *even-denominator* state $\nu = \frac{1}{2}$ [29]. The phase diagram is dominated by two QHE states, viz. $\nu = 1$ and $\nu = \frac{1}{3}$ which are also the predominant QH states observed experimentally in a 2DEG.

The energy spectrum for various 1D-FQHE states were presented in our earlier publication [29] and they showed features very similar to what one expects in a two-dimensional incompressible systems [4, 5]. In order to explore the areas of the phase diagram where we do not have an incompressible state (between any two

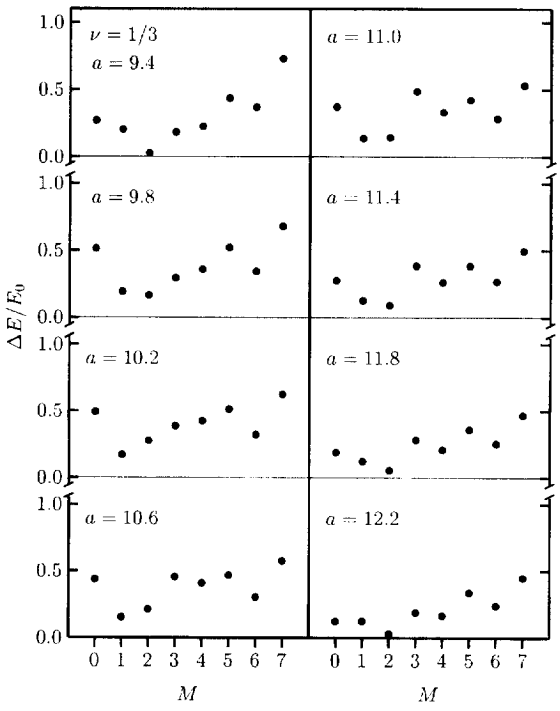

Fig. 3. The lowest excitation energy at $\nu = \frac{1}{3}$ as a function of the total momentum M for various values of a.

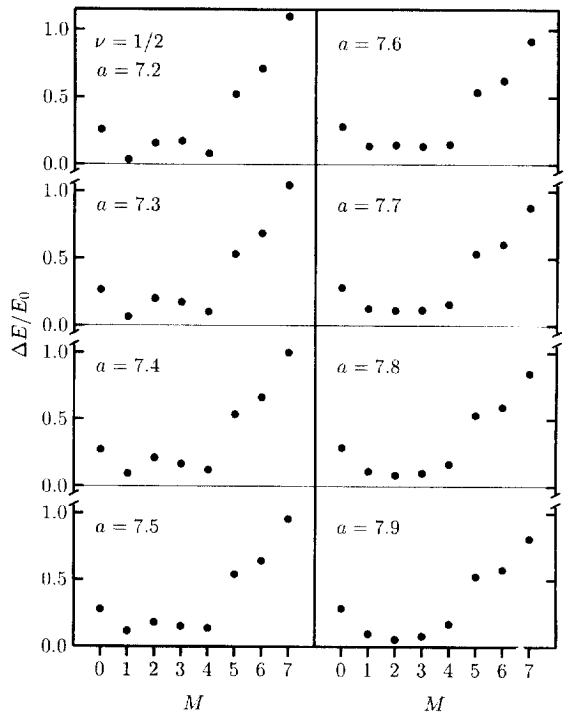

Fig. 4. Same as in Fig. 3, but for the filling factor $\nu = \frac{1}{2}$.

stable QH strips in Fig. 2), we looked at the energy spectra of various QH states when a is varied. In Figs. 3 and 4, we present such results for $\nu = \frac{1}{3}$ and $\nu = \frac{1}{2}$, respectively. Evidently, the gap structure changes dramatically from one a value to another and shows indications of soft modes at certain values of a. It therefore indicates that the symmetry changes between the ground states in different regions of the phase diagram. The precise nature of the states with $M \neq 0$ is as yet, unclear. Further work in this direction, and in particular, investigations of the nature of correlation functions near the transition regions is in progress. The earliest proposal to describe the even-denominator QH states [3] was to generalize the Laughlin state [2] where one first makes charge-2 bosons out of electron pairs and then forms a Laughlin state for bosons. In subsequent investigations by various authors, several QH states were proposed. Most important among them are the paired QH states. These include the Pfaffian state [27, 30] which is the simplest paired state for spinless particles and might be useful in the present narrow-channel system. These studies will be reported elsewhere.

To summarize our results, we have studied the properties of incompressible states that are expected to be present in a narrow-channel electron system. In between the incompressible states there are states in the phase diagram where the excitation spectra go soft. This indicates a phase transition in going from one QH state to another. The precise nature of the states in the compressible region is, as yet, unclear.

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