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Interaction of a quantum dot with an incompressible two-dimensional electron gas

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Abstract

We consider a system of a incompressible quantum Hall liquid in close proximity to a parabolic quantum dot containing a few electrons. We observe a significant influence of the interacting electrons in the dot on the excitation spectrum of the incompressible state in the electron plane. Our calculated charge density indicates that unlike in the case of an impurity, interacting electrons in the dot seem to confine the fractionally charged excitations in the incompressible liquid. © 2002 Elsevier Science B.V. All rights reserved.

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A two-dimensional electron gas under the influence of a strong, external magnetic field exhibits the celebrated fractional quantum Hall effect (FQHE) [1,2], that has been the subject of intense investigation for almost two decades. One of the most successful description of the quantum state in such a system is the Laughlin state [3], which describes the ground state of an electron system with $\frac{1}{3}$-filled lowest Landau level, as a highly correlated incompressible liquid. Incompressibility of the state implies the existence of a gap in the excitation spectrum and that the low-lying elementary excitations are fractionally-charged quasiparticles and quasiholes [1–3]. A large body of theoretical and experimental work has already established many of these properties of the unique quantum state. Dispersion of various collective modes in the FQHE has also been well established [2].

Quantum Hall (QH) properties of the electron gas, such as the ones mentioned above, are however for electron motion in a plane. On the other hand, when the electron motion in the plane is further quantized in the planar two dimensions, we get what is known as a quantum dot (QD) [4,5]. Quantum dots represent the ultimate reduction in the dimensionality of a semiconductor device where electrons have no kinetic energy and as a consequence, they have sharp energy levels like in atoms. These zero-dimensional electron systems have enjoyed enormous popularity because of their importance in understanding fundamental concepts of nanostructures and at the same time, for their application potentials. Development of extremely small self-assembled quantum dots (only a few nanometers across) containing only a few

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electrons that can be inserted into the dot in a controlled manner, have led to important new device applications. In this letter, we propose a system where a parabolic QD [4] containing only a few electrons is brought in close proximity to a two-dimensional electron gas (2DEG) that was initially in the Laughlin state with $\frac{1}{3}$ filling of the lowest Landau level. We investigate the influence of interacting electrons in the dot on the Laughlin state that exists in the absence of the quantum dot.

Why (and how) should one bring a quantum dot in the vicinity of a 2DEG? The answer is, a device similar to the type of system described here already exists as a single-photon detector [6]. In this device, a layer of nanometer-sized InAs quantum dots is placed near a 2DEG, separated by a thin barrier. Charge carriers photoexcited by incident light (visible or near-infrared wavelengths) are trapped by the dots which results in a depletion of the electron density adjacent to the dots. As a consequence, the conductivity of the 2DEG is altered. This change in conductivity is in fact, measurable. Observation of step-like rise of conductivity, each step is due to discharge of a QD by a single photon, has been associated with the detection of a single photoexcited carrier in a single dot [6]. We propose here that, placed in a strong magnetic field, incident light on such a device would perhaps light up the quasiparticles of the incompressible state.

Formally, our system is similar to a double-layer FQHE system [7,8], except that in one of the two layers electrons are confined by a harmonic potential

$$V_{\mathrm{conf}}(x,y)=\tfrac{1}{2}m\omega_0^2(x^2+y^2),$$

where ω_0 is the confinement potential strength and the corresponding oscillator length is $l_{\mathrm{dot}}=(\hbar/m\omega_0)^{1/2}$. We consider Coulomb interaction between the electrons in the dot and in the plane. In our calculations presented below, the 2DEG is kept at the filling factor $\nu=\frac{1}{3}$ (Laughlin state) and the QD is filled with $N_{\mathrm{D}}=1$ or 2 interacting electrons.

We evaluate the Laughlin state numerically in a spherical surface geometry which is a well-established method to describe the ground state and low-lying excitations at $\frac{1}{3}$ filling factor [9]. This geometry is more appropriate in our present case due to the circular symmetry of the problem. For the proposed system, the single-particle wave function of the electron in the layer has the form:

$$\varphi_m=\left[\frac{2S+1}{4\pi}\binom{2S}{S+m}\right]^{1/2}u^{S+m}v^{S-m},$$

where $m=-S,\dots,S$ is z-component of the angular momentum of the electron and $2S$ is the number of flux quanta throughout the sphere in units of the elementary flux quantum; $u=\cos(\theta/2)\mathrm{e}^{i\phi/2}$, $v=\sin(\theta/2)\mathrm{e}^{-i\phi/2}$ and $\theta,\ \phi$ are polar angles; $\binom{2S}{S+m}$ is the binomial coefficient.

The single particle wave functions in the quantum dot have the usual form:

$$\psi_{n,l}(x,\phi)=\left(\frac{b}{2\pi l_0^2}\frac{n!}{(n+|l|)!}\right)^{1/2}\times\sum_{j=0}^{n}C(n,l,j)\mathrm{e}^{-\mathrm{i}l\phi}\mathrm{e}^{-(x^2/2)}x^{2j+|l|},$$

where $x=(b/2l_0)^{1/2}r,\ b=(1+4\omega_0^2/\omega_c^2)^{1/2}$ and

$$C(n,l,j)=(-1)^j\frac{(n+|l|)!}{(n-j)!(|l|+j)!j!},$$

where $n=0,1\dots$ is the radial quantum number and l is the azimuthal quantum number. In our case of a large quantum dot (15 nm) only the single particle states with $n=0$ and $l=0,1,\dots$ are important. Then the single particle energy spectrum has the one-dimensional oscillator form: $l\hbar[(\omega_c^2+\omega_0^2)^{1/2}-\omega_c]/2$.

We also study the electron density distribution for electrons in the dot (ρ_{D}) and for the electrons in the layer (ρ_{L}):

$$\rho_{\mathrm{D}}(r)=\int\cdots\int \mathrm{d}\vec{r}_{\mathrm{D},1}\dots\mathrm{d}\vec{r}_{\mathrm{L},1}\dots\times\sum_{i=1}^{N_{\mathrm{D}}}\delta(\vec{r}-\vec{r}_{\mathrm{D},i})\,|\,\Psi_M(\vec{r}_{\mathrm{D},1},\dots|\vec{r}_{\mathrm{L},1}\dots)|^2$$

$$\rho_{\mathrm{L}}(r)=\int\cdots\int \mathrm{d}\vec{r}_{\mathrm{D},1}\dots\mathrm{d}\vec{r}_{\mathrm{L},1}\dots\times\sum_{i=1}^{N_{\mathrm{L}}}\delta(\vec{r}-\vec{r}_{\mathrm{L},i})\,|\,\Psi_M(\vec{r}_{\mathrm{D},1},\dots|\vec{r}_{\mathrm{L},1}\dots)|^2$$

where N_{D} and N_{L} are the numbers of the electrons in the dot and in the layer, respectively. The integration over $\vec{r}_{\mathrm{L},i}$ is restricted to the sphere.

The interaction part of the Hamiltonian is given by the expression

$$H_{\text{int}} = \frac{e^2}{\varepsilon}\sum_{i<j}\frac{1}{|\vec{r}_{D,j}-\vec{r}_{D,i}|} + \frac{e^2}{\varepsilon}\sum_{i<j}\frac{1}{|\vec{r}_{L,j}-\vec{r}_{L,i}|} + \frac{e^2}{\varepsilon}\sum_{i,j}\frac{1}{[d^2+|\vec{r}_{D,j}-\vec{r}_{L,i}|^2]^{1/2}}$$

where $\vec{r}_D = (r_D\cos\phi_D, r_D\sin\phi_D)$ is the two-dimensional vector corresponding to the electron in the quantum dot, $\vec{r}_L = (2R\sin(\theta/2)\cos\phi_L, 2R\sin(\theta/2)\sin\phi_L)$ is the two-dimensional vector corresponding to the electron in the sphere with sphere radius $R = S^{1/2} l_0$ and polar angle θ, and d is the separation between quantum dot and the layer.

All computations are done for six electrons in the layer (sphere) which form the incompressible liquid with filling factor $\nu = \frac{1}{3}$. In this case, the sphere radius is $R = \sqrt{7.5}\, l_0$. For electrons in the quantum dot we take 10 lowest single particle states. Under such conditions we can consider only one and two electrons in the dot. Any additional number of electrons in the dot (three and more) requires a larger sphere for electrons in the layer which results in a much larger matrix that has to be diagonalized numerically. All electrons are treated as spinless particles. In what follows we use the magnetic length l_0 as the unit of length and the Coulomb energy $E_c = e^2/\varepsilon l_0$ as the unit of energy, where ε is the background dielectric constant. In all our calculations presented here, the magnetic length is taken to be 6.6 nm, which corresponds to the magnetic field of 15 T. For the quantum dots we consider parameters appropriate to GaAs and the dot size, $l_{\text{dot}} = 15$ nm. The size of the dot is dictated by the fact that for smaller dots, the difference energy (energy difference between the ground state and the lowest excited state) is much larger than the lowest energy excitations of the incompressible state and therefore, has no noticeable effect on the spectrum. The interlayer separation d, and as a result, the interlayer interaction [7] has also been varied in our calculations. Here we consider $d = 1.5, 2.0\, l_0$ separations. The latter separation was found to be optimum in the double-layer FQHE systems [7]. Smaller separations tend to close the energy gap, the hallmark of the incompressible state.

In the absence of the quantum dot, states in the spherical geometry appear as multiplets characterized by the rotational quantum number L. However, if suppose we place an impurity near the north pole of the sphere, then the states can be classified only by the azimuthal rotational quantum number $M = L_z$, and changes in M indicate charge redistribution [10]. In this geometry, the minima in the charge density were identified with the center of a quasiparticle defect (fractionally charged) emitted by the impurity. With increasing values of M, that defect was found to progress outward [10]. Further, due to the incompressibility of the Laughlin state, there is no screening of the impurity. Laughlin state was found to be stable regardless of the strength of the impurity.

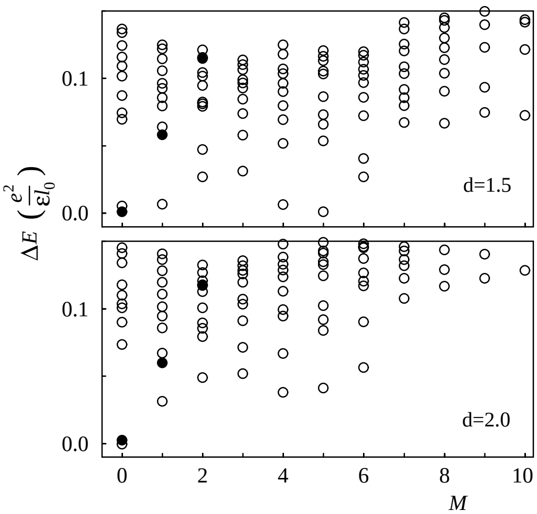

Fig. 1. Energy spectrum of a Coulomb-coupled quantum dot-quantum Hall system at $\nu = \frac{1}{3}$ (open circles). The quantum dot contains a single electron and is separated from the 2DEG by $d = 1.5, 2.0\, l_0$. The filled circles are the energies of an isolated QD, presented here as a reference.

In Fig. 1 the energy spectrum of the system with one electron in the dot is shown by open circles. This case is closely related to the system of incompressible liquid in the field of a charged impurity discussed above. However, in our case there is an additional type of collective excitation due to the additional degree-of-freedom of the electron in the dot. For small separation ($d = 1.5$), the perturbation of incompressible liquid by the electron in the dot is strong and the collective excitation is gapless. For a larger separation ($d = 2.0$) there is a well defined branch of lowest excitations at $M > 0$ (Fig. 1). To understand more about this branch we plot the electron density

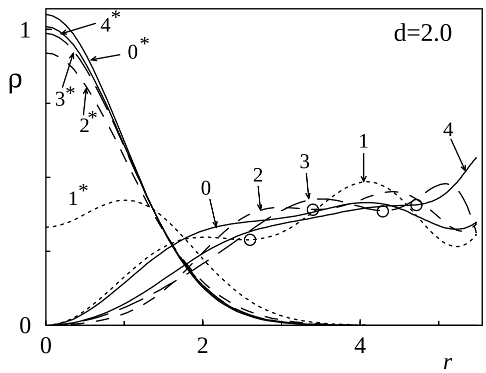

Fig. 2. Charge-density profile $\rho_L(r)$ of the ground state and low-lying excitations of a QD+QH system at $\frac{1}{3}$ filling factor, corresponding to Fig. 1, (for $M = 0$–4, as indicated in the figure) and for the QD, $\rho_D(r)$ with a single electron (with $M = 0^*$–4^*). Open circles indicate the minima in the charge density.

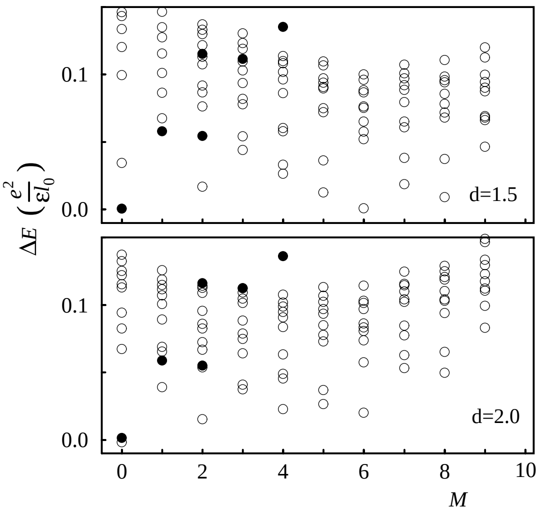

Fig. 3. Energy spectrum of a Coulomb-coupled quantum dot-quantum Hall system at $\nu = \frac{1}{3}$ (open circles). The quantum dot contains two interacting electrons and is separated from the 2DEG by $d = 1.5, 2.0 l_0$. The filled circles are the energies of the isolated QD.

distribution in the layer and in the quantum dot for the lowest states with the given M (Fig. 2). We notice that in the states with $M = 0, 2, 3, 4$, the electron in the quantum dot is almost in the ground state of the dot. The excited states at $M = 2, 3, 4$ can be described by the process of ionization as an emission of the fractionally charged quasihole: the quasihole is moving away from the quantum dot with increasing M. The positions of the quasihole are shown by open circles. This picture is the same as for the charged impurity near incompressible liquid [10]. At the same time the state at $M = 1$ has different a nature. It is the collective excitation of the electron in the dot and the electrons of incompressible liquid. Such low-energy excitations can be observed only when the separation between the energy levels in the quantum dot ($\hbar[(\omega_c^2 + \omega_0^2)^{1/2} - \omega_c]/2$) is about the incompressible gap ($0.1E_c$). This type of excitation gives rise to the linear dependance of excitation spectra as a function of M for small M.

A more interesting situation occurs when there are more than one electron in the dot. In this case the interaction between the electrons in the dot makes the quantum dot an impurity center with non-trivial charge distribution and with internal degree-of-freedom. In Fig. 3 the energy spectra of the system with two electrons in the quantum dot is shown by open circles. The states of the pure electron system in the dot are shown by filled circles. The angular momentum M is counted from the angular momentum of the ground state. For a pure two electron system in the quantum dot the angular momentum of the ground state is equal to 3. For small separation ($d = 1.5$), the incompressible liquid is strongly perturbed by the electrons in the dot. The perturbation is stronger than that of the sin-

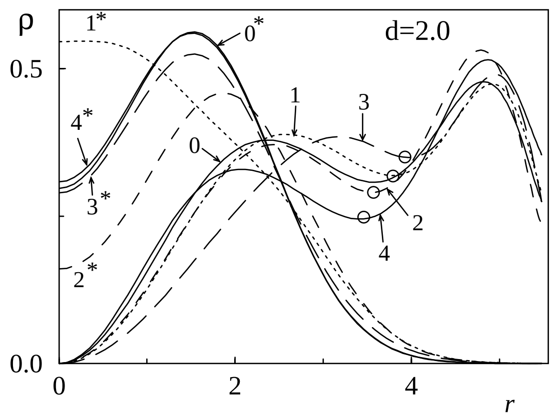

Fig. 4. Charge-density profile $\rho_L(r)$ of the ground state and low-lying excitations of a QD+QH system at $\frac{1}{3}$ filling factor, corresponding to Fig. 3, (for $M = 0$–4, as indicated in the figure) and for the QD, $\rho_D(r)$ with two interacting electrons (with $M = 0^*$–4^*). Open circles indicate the minima in the charge density.

gle electron case due to the larger charge of the dot. At a larger separation ($d = 2.0$) the lowest branch of the excitation spectra develops an oscillatory structure as a function of M. The electron density distribution (Fig. 4) shows that the state at $M = 1$ has different distribution compared to the states at $M > 1$, and can be described in the same manner as for the one electron system, that is the collective excitation of the electrons in the dot and an incompressible liquid. The excited states at $M = 2, 3, 4$, however, cannot be considered simply as the process of ionization. In this figure, the position of the minimum of charge distribution, shown by open circles exhibit oscillatory behavior with M and the quasihole remains almost confined in the same region. These oscillations are correlated with oscillations in the energy spectra (Fig. 3). Confinement of the quasihole in the incompressible state by the quantum dot is purely due to the interaction between the electrons in the quantum dot which results in the specific charge distribution in the quantum dot and an additional interaction of a qusihole of the incompressible liquid with the local excitation of the dot.

In closing, we have explored a system of a quantum dot placed in close proximity to a two-dimensional electron gas that is in the incompressible liquid state. Our results indicate that for a single electron in the dot, the physics is somewhat like that of an impurity which emits a fractionally-charged quasihole that moves away from the dot with increasing M. For small M, we notice a linear behavior of the excitation spectrum. Most importantly, however, we find that for two interacting electrons in the quantum dot, the collective excitation exhibits an oscillatory behavior which is due to confinement of the fractionally-charged quasihole excitations by the quantum dot. This is purely a consequence of interelectron interaction in the dot. Therefore, in a suitable set up, the single-photon detector might in fact, be a detector for the fractionally-charged excitations of the incompressible Laughlin state. On the other hand, investigations of charge density, for example, via STM imaging might also reveal features associated with the confinement of a quasihole as described here. Finally, we would like to point out that there has been a recent proposal to localize a $\frac{1}{2}$-Laughlin-quasihole in a $\frac{1}{2}$-Laughlin state that is created in a rotating Bose–Einstein condensate [11] consisting of a small number of atoms. Some of the signatures of that confined fractional-statistics object [12] might be similar to those of the proposed confined quasihole in our present system.

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