Tuning of the Gap in a Laughlin-Bychkov-Rashba Incompressible Liquid

Marco Califano,[1] Tapash Chakraborty,[1,*] and Pekka Pietiläinen[1,2]

[1]*Department of Physics and Astronomy, University of Manitoba, Winnipeg, Canada R3T 2N2*

[2]*Department of Physical Sciences/Theoretical Physics, P.O. Box 3000, FIN-90014 University of Oulu, Finland*

(Received 19 November 2004; published 20 June 2005)

We report on our investigation of the influence of Bychkov-Rashba spin-orbit interaction (SOI) on the incompressible Laughlin state. We find that experimentally obtainable values of the spin-orbit coupling strength can induce as much as a 25% increase in the quasiparticle-quasihole gap E_g at low magnetic fields in InAs, thereby increasing the stability of the liquid state. The SOI-modulated enhancement of E_g is also significant for $\nu = 1/5$ and $1/7$, where the fractional quantum Hall state is usually weak. This raises the intriguing possibility of tuning, via the SO coupling strength, the liquid to solid transition to much lower densities.

DOI: 10.1103/PhysRevLett.94.246801 PACS numbers: 73.43.-f, 73.21.-b

The ground state of a two-dimensional electron gas (2DEG) in the presence of a high perpendicular magnetic field is well known to appear as a multitude of highly correlated incompressible fractional quantum Hall effect (FQHE) states at a few special values of the Landau level filling factors [1,2]. It is also well established that the strongest effect appears at the lowest Landau level filling factor ($\nu = \frac{1}{3}$) that has the largest quasiparticle-quasihole gap [1,2]. The gaps are much smaller for $\nu < \frac{1}{3}$, and, as a consequence, the nature of the electron states in that regime has remained a challenge until now. This is due to the fact that in the low density regime, the incompressible liquid is expected to undergo a phase transition to a crystalline state [3]. However, a definitive conclusion about the onset of this quantum phase transition has remained elusive because experimentally one observes weak effects for filling factors $\frac{1}{5}$, $\frac{1}{7}$, etc., and theoretically one compares two very small energies (ground state) in order to determine which phase is energetically favored. For more than two decades, investigations of the FQHE have focused largely on 2DEGs that are embedded in GaAs heterostructures, where spin-related effects are small (though important [4]) compared to other effects, because of the small value of the g factor of electrons in GaAs [5]. However, studies of the spin-orbit (SO) coupling in a 2DEG within an InAs (or InSb) quantum well with very large g values are at the cusp of a rapid advance, due largely to their relevance to spin transport in low-dimensional electron channels [6,7]. In order to investigate the influence of SO interaction on the incompressible Laughlin states, we have carried out the well established finite-size studies in a periodic rectangular geometry, but for the first time with the SO coupling included in the Hamiltonian. We find that as the SO coupling strength is increased, there is an increase in the quasiparticle-quasihole gap, indicating that the incompressible state can be rendered more stable by appropriate tuning of the SO coupling. This might prove to be particularly useful in the low-electron density regime where, as described above, the FQHE state is usually very weak for conventional systems.

For a 2DEG in the xy plane with a magnetic field $\mathbf{B}$ along the z direction, in the Landau gauge [with the choice of vector potential $\mathbf{A} = (0, Bx, 0)$], the single-particle states and the corresponding energies are obtained by solving the one-electron Hamiltonian

$$H = \frac{(\mathbf{p} + e\mathbf{A})^2}{2m^*} + \frac{\alpha}{\hbar}[\boldsymbol{\sigma} \times (\mathbf{p} + e\mathbf{A})]_z + g\mu_B B\sigma_z \quad (1)$$

that includes the Bychkov-Rashba term [8] and the Zeeman term. Here $\mathbf{p}$ is the momentum operator, α is the SO coupling strength, and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices. Experimental values of the SO coupling constant lie in the range of 5–45 meV nm [9]. The high values of α are deduced from magnetotransport experiments where the SO interaction is tuned for a fixed carrier density in a 2DEG by using gate electrodes in a square asymmetric InAs quantum well [9]. We therefore focus our investigation on InAs ($m^*/m_0 = 0.042$, $\epsilon = 14.6$, $g = -14$), as it represents the most promising material, as yet, for achieving large SO coupling.

We solve the Schrödinger equation

$$H\psi = E\psi \quad (2)$$

in a rectangular geometry with supercell sides L_x and L_y (i.e., aspect ratio $\lambda = L_x/L_y$) and expand the single-particle wave functions $\psi_{k_y}(\mathbf{r})$ as a superposition of solutions of the Hamiltonian in the absence of SO interaction [10]

$$\psi_{k_y}(\mathbf{r}) = e^{ik_y y}\sum_{n,\sigma}\phi_n(x - x_0)C_n^\sigma|\sigma\rangle/\sqrt{L_y}, \quad (3)$$

where

$$\phi_n(x - x_0) = \beta_n e^{-(x-x_0)^2/2l_0^2}H_n[(x - x_0)/l_0]/\sqrt{\sqrt{\pi}l_0}$$

is the usual solution to the harmonic oscillator problem, with $H_n(x)$ the Hermite polynomial, $l_0 = (\hbar/m^*\omega_c)^{1/2}$ the radius of the cyclotron orbit with frequency $\omega_c = eB/m^*$ and center $x_0 = k_y l_0^2$, n is the Landau level index, $\beta_n = 1/\sqrt{2^n n!}$, and

$$\sigma = up, dn = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

is the electron spinor.

Substituting Eq. (3) into the Schrödinger Eq. (2), multiplying both sides by $\phi_l(x - x_0)$ and integrating over x, we obtain a system of equations [10]

$$\begin{aligned} &i(\alpha/l_0)\sqrt{2l}C_{l-1}^{up} + [(l + 1/2)\hbar\omega_c + E_d]C_l^{dn} = 0, \\ &[(l + 1/2)\hbar\omega_c + E_u]C_l^{up} - i(\alpha/l_0)\sqrt{2(l + 1)}C_{l+1}^{dn} = 0, \\ &l = 0, 1, 2, \ldots \end{aligned} \tag{4}$$

whose solution yields [10]

$$(1/2\hbar\omega_c + E_d)C_s^{dn} = 0, \qquad s = 0,$$

$$\begin{bmatrix} (s - 1/2)\hbar\omega_c + E_u & -i(\alpha/l_0)\sqrt{2s} \\ i(\alpha/l_0)\sqrt{2s} & (s + 1/2)\hbar\omega_c + E_d \end{bmatrix}\begin{pmatrix} C_{s-1}^{up} \\ C_s^{dn} \end{pmatrix} = 0,$$

$s = 1, 2, 3, \ldots,$ where $E_u = g\mu_B B - E$ and $E_d = -g\mu_B B - E$. Corresponding to $s = 0$ there is only one level, the same as the lowest Landau level without SO interaction, with energy

$$E_0 = 1/2\hbar\omega_c - g\mu_B B$$

and wave function

$$\psi_{0,k_y} = e^{ik_y y}\phi_0(x - x_0)/\sqrt{L_y}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right].$$

For all other values of $s \neq 0$ there are two branches of levels [10]

$$\psi_{s,k_y}^{+} = \frac{e^{ik_y y}}{\sqrt{L_y A_s}}\begin{pmatrix} -iD_s\phi_{s-1}(x - x_0) \\ \phi_s(x - x_0) \end{pmatrix} \tag{5}$$

and

$$\psi_{s,k_y}^{-} = \frac{e^{ik_y y}}{\sqrt{L_y A_s}}\begin{pmatrix} \phi_{s-1}(x - x_0) \\ -iD_s\phi_s(x - x_0) \end{pmatrix} \tag{6}$$

with energies

$$E_s^{\pm} = s\hbar\omega_c \pm \sqrt{E_0^2 + 2s\alpha^2/l_0^2}. \tag{7}$$

Here

$$D_s = \frac{\sqrt{2s}\alpha/l_0}{E_0 + \sqrt{E_0^2 + 2s\alpha^2/l_0^2}} \tag{8}$$

and $A_s = 1 + D_s^2$. From Eqs. (5) and (6) we see that SO interaction couples *two* Landau levels. While previous works [10] were restricted to the study of the single-particle states, we use these equations as a starting point for our *exact* many-body treatment. Applying periodic boundary conditions, we obtain ($k_y = x_0/l_0^2$)

$$x_0 = X_j = 2\pi l_0^2 j/L_y, \qquad L_x = 2\pi l_0^2 m/L_y,$$

and, consequently,

$$\begin{aligned} \psi_{s,j}^{+}(\mathbf{r}) = \frac{1}{\sqrt{\sqrt{\pi}l_0 L_y A_s}}\sum_n \exp\left[i(X_j + nL_x)\frac{y}{l_0^2} - \frac{(X_j + nL_x - x)^2}{2l_0^2}\right] \\ \times \begin{pmatrix} -iD_s\beta_{s-1}H_{s-1}(\frac{(X_j+nL_x-x)}{l_0}) \\ \beta_s H_s(\frac{(X_j+nL_x-x)}{l_0}) \end{pmatrix}, \end{aligned}$$

$$\psi_{s,j}^{-}(\mathbf{r}) = \frac{1}{\sqrt{\sqrt{\pi}l_0 L_y A_s}}\sum_n \exp\left[i(X_j + nL_x)\frac{y}{l_0^2} - \frac{(X_j + nL_x - x)^2}{2l_0^2}\right]\begin{pmatrix} \beta_{s-1}H_{s-1}(\frac{(X_j+nL_x-x)}{l_0}) \\ -iD_s\beta_s H_s(\frac{(X_j+nL_x-x)}{l_0}) \end{pmatrix}.$$

We then build the antisymmetrized products (Slater determinants) using ψ^+ and ψ^- as a complete basis for the many-body wave function expansion

$$\Psi = \sum_{\{i_k\}}\mathcal{P}(i_1, i_2, \ldots, i_n)a_{i_1}^{\dagger}a_{i_2}^{\dagger}\cdots a_{i_n}^{\dagger}|0\rangle,$$

where $i_k = (s_k, j_k, \tilde{\sigma}_k)$, $\tilde{\sigma}_k = \pm$, and $\mathcal{P}(i_1, i_2, \ldots, i_n)$ is the antisymmetrization operator.

The many-body Schrödinger equation was then solved by performing an exact diagonalization of the many-body Hamiltonian

$$\mathcal{H} = \sum_j \mathcal{W}_j a_j^{\dagger}a_j + \sum_{j_1}\sum_{j_2}\sum_{j_3}\sum_{j_4}\mathcal{A}_{j_1 j_2 j_3 j_4}a_{j_1}^{\dagger}a_{j_2}^{\dagger}a_{j_3}a_{j_4}. \tag{9}$$

The kinetic energy term

$$\mathcal{W}_j = S + E_j \tag{10}$$

includes the effects of a neutralizing background. Here $E_j = E_s^{\pm}$,

$$S = -\frac{e^2}{\epsilon l_0}\frac{1}{\sqrt{2\pi m}}\left[2 - \sum_{k_1,k_2}{}' \sqrt{\frac{\pi}{z}}[1 - \mathrm{erf}(\sqrt{z})]\right] \tag{11}$$

is the Madelung energy, e is the electron charge, ϵ is the dielectric constant, $z = \pi(\lambda^2 k_1^2 + k_2^2)/\lambda$, and the prime in the summation means that the term $k_1 = k_2 = 0$ is excluded. The expression for the scattering matrix element $\mathcal{A}_{i_1 i_2 i_3 i_4}$ depends on the quantum numbers $i_1 i_2 i_3 i_4$, where, again, $i_k = (s_k, j_k, \tilde{\sigma}_k)$. For the case of positively polarized "spins" (i.e., $\tilde{\sigma}_k = +$ for all k) we have

$$\begin{aligned}\mathcal{A}_{i_1i_2i_3i_4}|_{\tilde{\sigma}_i=+} &= \delta'_{j_1+j_2,j_3+j_4}\frac{1}{2}\frac{e^2}{\epsilon l_0}\sqrt{\frac{\lambda}{2\pi m}}\prod_{i=1}^{4}\left(\frac{D_{s_i}\beta_{s_i}}{A_{s_i}}\right)(-1)^{s_2+s_4}s_1!\,s_2!\,s_3!\,s_4! \\ &\times \sum_{k_1=1}^{\infty}\sum_{k_2=-\infty}^{\infty}\frac{\delta'_{j_1-j_4,k_2}}{\sqrt{k_1^2+\lambda^2k_2^2}}e^{-\pi(k_1^2+\lambda^2k_2^2)/\lambda m}\left(k_2\sqrt{\frac{2\pi\lambda}{m}}\right)^{\sum_{i=1}^{4}s_i}\cos\left[\frac{2\pi}{m}k_1(j_1-j_3)\right] \\ &\times \left\{\sum_{t_1=0}^{s_1}\sum_{p_1=1}^{up_1^c}\sum_{t_2=0}^{s_3}\sum_{p_2=1}^{up_2^c}\frac{s_1!}{(s_1-t_1)!}\frac{s_3!}{(s_3-t_2)!}\binom{s_4}{t_1+2p_1}\right. \\ &\times \binom{s_2}{t_2+2p_2}(-1)^{t_1+t_2+p_1+p_2}2^{t_1+t_2-1}\left(k_2\sqrt{\frac{2\pi\lambda}{m}}\right)^{2(t_1+p_1+t_2+p_2)} \\ &\times \left.\left(k_1\sqrt{\frac{2\pi}{\lambda m}}\right)^{2(p_1+p_2)}\mathcal{L}_{t_1}^{2p_1}\left(k_1^2\frac{2\pi}{\lambda m}\right)\mathcal{L}_{t_2}^{2p_2}\left(k_1^2\frac{2\pi}{\lambda m}\right)\right\}, \end{aligned} \qquad (12)$$

where

$$\mathcal{L}_n^{\alpha}(x) = \sum_{m=0}^{n}(-1)^m\binom{n+\alpha}{n-m}\frac{x^m}{m!}$$

are the Laguerre polynomials and

$$up_1^c = \begin{cases}(s_4-t_1) & \text{if } (s_4-t_1)<0 \\ \mathrm{Int}\{(s_4-t_1)/2\} & \text{otherwise}\end{cases}$$

and the same yields for up_2^c, with $s_4 \to s_2$ and $t_1 \to t_2$, [Int$\{x\}$ is the integer part of x].

Because of the presence of the spinors σ, the two branches $\psi^{\pm}$, and the coupling of the Landau levels in pairs, the derivation of the complete expression for $\mathcal{A}_{i_1i_2i_3i_4}$ is highly nontrivial. Moreover, the Hamiltonian matrix can be very large and its diagonalization computationally very intensive. We calculated the ground state energy per particle E_0 for a system with four electrons in the lowest two Landau levels with different filling factors $\nu = 4/N_s$, where $N_s = 8, 9, \ldots, 20$. The results are shown in Fig. 1 for $B = 10$ Tesla. We see that the presence of SO coupling lowers considerably the value of E_0, compared to the result for $\alpha = 0$, which coincides with the usual results obtained for the FQHE with no SO, with both the Zeeman and the kinetic energies included.

The most intriguing effect of SO coupling in 2DEG, however, is found in connection with the magnitude of the quasiparticle-quasihole energy gap E_g, derived from the positive discontinuity of the chemical potential at the filling factor ν [2].

As shown in Fig. 2 for four electrons and a filling factor of $\frac{1}{3}$ and in Fig. 3 for even smaller values of $\nu = \frac{1}{5}$ and $\frac{1}{7}$ [11], large values of α cause the enhancement of E_g. This enhancement is larger for small magnetic fields

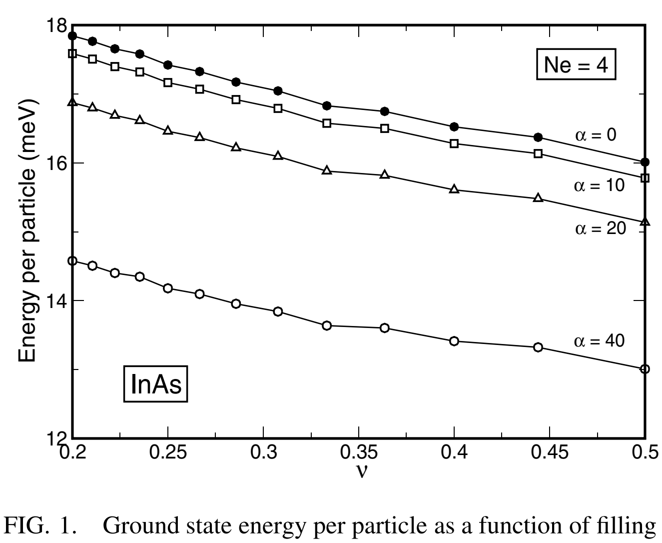

FIG. 1. Ground state energy per particle as a function of filling factor ν for four different values of the SO coupling constant $\alpha = 0, 10, 20, 40$ calculated for four electrons in the lowest two Landau levels in InAs ($B = 10$ Tesla).

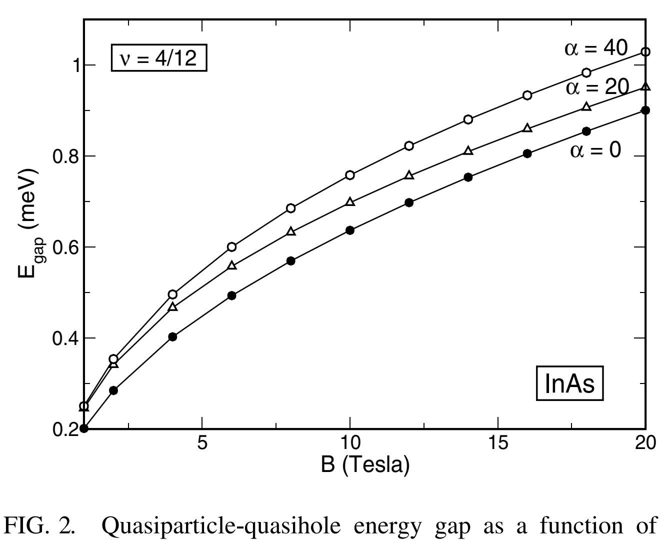

FIG. 2. Quasiparticle-quasihole energy gap as a function of magnetic field B for a filling factor $\nu = 1/3$ and three different values of the SO coupling constant $\alpha = 0, 20, 40$ calculated for four electrons in the lowest two Landau levels in InAs.

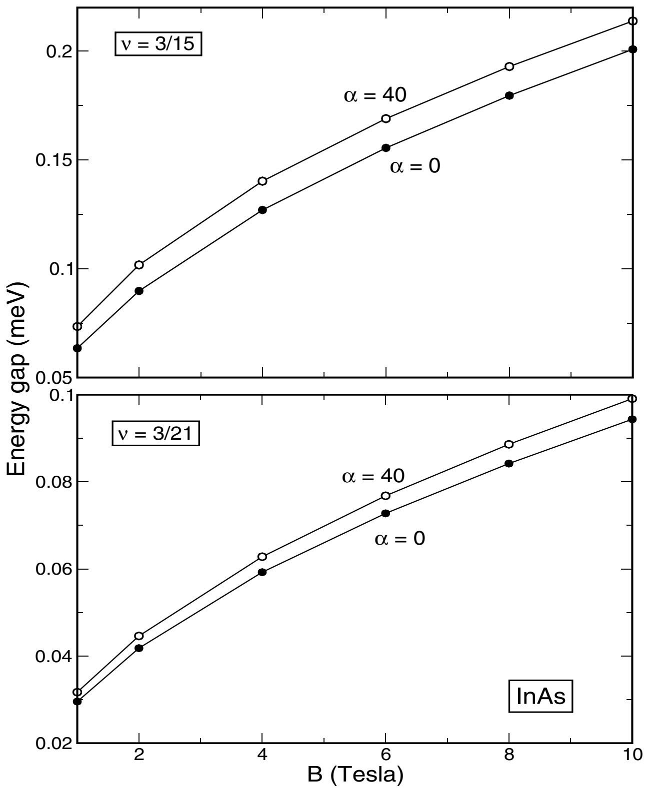

FIG. 3. Quasiparticle-quasihole energy gap as a function of magnetic field B for a filling factor $\nu = 1/5$ and $1/7$ and two different values of the SO coupling constant $\alpha = 0$, 40 calculated for three electrons in the lowest two Landau levels in InAs.

($B \sim 1$ Tesla, i.e., fields for which the Bychkov-Rashba term is still comparable to the Zeeman term), and can be of the order of 25% for $\nu = \frac{4}{12}$ and midrange values of the coupling constant ($\alpha = 20$). This is seen in Fig. 2, where E_g increases from 0.20 meV for $\alpha = 0$ to $\sim$0.25 for $\alpha = 20$. Smaller increases occur at smaller filling factors, where E_g shows a 17% and 7% increase for $\nu = \frac{3}{15}$ and $\frac{3}{21}$, respectively. Reasons for this behavior derive from a complex interplay between the different one-body [Eq. (10)] and two-body [Eq. (12)] terms. For $\alpha \neq 0$, one clear effect of the SO interaction is that the kinetic energy is no longer constant for a given value of the magnetic field [Eq. (7)] but depends on the strength of the SO coupling. This, coupled with the fact that the interaction terms are profoundly modified by the SO interaction, results in a change in $\partial E_0/\partial \nu$. This is reflected in an increase of the energy gap.

In summary, we have investigated the influence of the SO coupling (Bychkov-Rashba) on the incompressible state proposed by Laughlin at $\nu = \frac{1}{3}$, using the exact diagonalization scheme for finite-size systems in a periodic rectangular geometry. We found that, as the SO coupling strength is increased, there is an increase of the quasiparticle-quasihole gap. This is particularly advantageous for filling factors $\nu < 1/5, 1/7$ where a larger gap would signify a more stable liquid state that will push the liquid-solid transition further down in the density.

The work of T. C. has been supported by the Canada Research Chair Program and the Canadian Foundation for Innovation Grant. The work of M. C. has been supported by NSERC.

*Electronic address: tapash@physics.umanitoba.ca

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