Half-Polarized Quantum Hall States

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We report a theoretical analysis of the half-polarized quantum Hall states observed in a recent experiment. Our numerical results indicate that the ground state energy of the quantum Hall $\nu = \frac{2}{3}$ and $\nu = \frac{2}{5}$ states versus spin polarization has a downward cusp at half the maximal spin polarization. We map the two-component fermion system onto a system of excitons and describe the ground state as a liquid state of excitons with nonzero values of exciton angular momentum.

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In recent years it has become increasingly clear that spin degree of freedom plays an important role in the fractional quantum Hall effect (FQHE) where many novel and interesting spin-related phenomena have been observed both theoretically and experimentally [1]. One of the important problems in this field is the influence of Zeeman splitting on the properties of FQHE systems, in particular, on the ground state spin polarizations. It is now well established that for some filling factors ($\nu = \frac{1}{m}$) the ground state is fully spin polarized for all values of Zeeman splitting, while for other filling factors (for example, $\nu = \frac{2}{3}, \frac{2}{5}$) the ground state is fully polarized only for large values of Zeeman splitting but unpolarized or partially polarized for small (or zero) values of Zeeman energy [1,2]. One interesting problem then is to find the state for intermediate values of Zeeman energy. That problem was highlighted in a recent experimental work [3], where the magnetic field driven spin transitions at various FQHE states were reported and, in particular, at $\nu = \frac{2}{3}$ and $\nu = \frac{2}{5}$, weak features (plateaulike singularities) were observed at half the maximal spin polarization of the system. Observed stability of the half-polarized states means that the ground state energy of the system as a function of spin polarization should have nonmonotonic behavior at half polarization. Our earlier work [4] did not provide much information about the nature of states at half polarization. In this paper, we have explored possible ground states at half polarization for filling factors $\nu = \frac{2}{5}$ and $\nu = \frac{2}{3}$. We find that the ground state energy versus spin polarization has a downward cusp at half polarization that might describe stability of the observed state.

The FQHE system at filling factor $\nu = \frac{2}{5}$ can be described as a composite fermion (CF) system with total filling factor $\nu = 2$ [5]. At high values of the Zeeman energy the composite fermions will occupy $n = 0$ ↑-spin and $n = 1$ ↑-spin Landau levels of composite fermions. As a result we have a fully spin-polarized state. At low Zeeman energies they will occupy $n = 0$ ↑-spin and $n = 0$ ↓-spin Landau levels, which will result in an unpolarized state. At intermediate values of Zeeman energy the CF will fully occupy the $n = 0$ ↑-spin Landau level and partially occupy $n = 0$ ↓-spin and $n = 1$ ↑-spin levels with filling factors ν_1 and ν_2, respectively, with $\nu_1 + \nu_2 = 1$. The half-polarized state corresponds to $\nu_1 = \nu_2 = \frac{1}{2}$. As the fully occupied state can be considered as a nondynamical background, the composite fermions occupying the partially filled levels can be thought of as a system consisting of two types of fermions with $\nu = 1$. The Hamiltonian of the two-component system has the form

$$\mathcal{H} = \frac{1}{2} \sum_{\alpha\beta} \iint d\vec{r}_1\, d\vec{r}_2\, V_{\alpha\beta}(|\vec{r}_1 - \vec{r}_2|) \times \psi_\alpha^+(\vec{r}_1)\psi_\beta^+(\vec{r}_2)\psi_\beta(\vec{r}_2)\psi_\alpha(\vec{r}_1)\,, \tag{1}$$

where $\alpha, \beta = 1$ and 2, V_{11} is the interaction potential between fermions of type 1, V_{22} is the interaction potential between fermions of type 2, and V_{12} is the potential between fermions of types 1 and 2. The specific feature of this system is that the Hamiltonian is completely "nonsymmetric," i.e., all interaction potentials V_{11}, V_{22}, and V_{12} are different.

Alternatively, we can also describe the $\nu = \frac{2}{5}$ state as the daughter state of the $\nu = \frac{1}{3}$ system [1]. Then the spin-polarized state of the $\nu = \frac{2}{5}$ system is due to condensation of spin-polarized quasiparticles with filling factor $\nu = \frac{1}{2}$ and the unpolarized state as the condensation of spin-reversed quasiparticles [6]. For intermediate polarization we have the system of spin-polarized and spin-reversed quasiparticles with $\nu = \frac{1}{2}$. Because they are Bose particles we can map this system into the system of fermions with $\nu = 1$. Here again, as for the composite fermion picture, we have two types of fermions with Hamiltonian (1). We also have a similar picture for the $\nu = \frac{2}{3}$ state, which can be described as the daughter state of the $\nu = 1$ system with condensation of spin-polarized and spin-reversed holes of $\nu = 1$.

For strong enough repulsion between the fermions at the same point we expect that the ground state of the system at $\nu_1 = \nu_2 = \frac{1}{2}$ and other values of ν_1 to be the Halperin-(1,1,1) state [1,7]

$$\psi = \prod_{i<j=1}^{N_1}(z_i - z_j)\prod_{k<m=1}^{N_2}(\tilde{z}_k - \tilde{z}_m)\prod_{i=1}^{N_1}\prod_{k=1}^{N_2}(z_i - \tilde{z}_k) \times \prod_{i=1}^{N_1} e^{-|z_i|^2/4\ell_0^2}\prod_{k=1}^{N_2} e^{-|\tilde{z}_k|^2/4\ell_0^2},$$

where N_1 is the number of fermions of type 1, N_2 is the number of fermions of type 2; $N_1 + N_2 = N$. Here z_j are the coordinates (complex) of fermions of type 1, and $\tilde{z}_j$ are those of fermions of type 2. If we consider the system of two types of fermions as a two-level system and introduce a pseudospin τ for the states at different levels, then the Halperin-(1,1,1) liquid state has $\tau = N/2$ and $N/2 \geq \tau_z \geq -N/2$. If this state is the correct ground state, then the transition from a polarized state to an unpolarized state of the system is just the rotation of the pseudospin vector from $\tau_z = -N/2$ to $\tau_z = N/2$ with a fixed value of the total pseudospin $\tau = N/2$. But then the ground state energy of the system is monotonic with polarization of the system (quadratic function) without any singularity at half polarization. This means that the Halperin state is not the true ground state of the half-polarized state.

In Ref. [8] it was proposed that the half-polarized ground state is a charge-density wave (CDW) of CF. To check this claim we have compared the energies of the proposed CDW state and the Halperin-(1,1,1) state [9]. The CDW of Ref. [8] in our notations is formed by type-1 and type-2 fermions on a square lattice. We calculate the cohesive energy of this state from [10]

$$E_{\mathrm{CDW}} = -\frac{1}{2}\sum_{\vec{Q}}[V_{11}^{\mathrm{HF}}(\vec{Q})\Delta_1(\vec{Q})\Delta_1(-\vec{Q}) + V_{22}^{\mathrm{HF}}(\vec{Q})\Delta_2(\vec{Q})\Delta_2(-\vec{Q}) + 2V_{12}^{\mathrm{H}}(\vec{Q})\Delta_1(\vec{Q})\Delta_2(-\vec{Q})], \quad (2)$$

where $V_{11}^{\mathrm{HF}}(\vec{Q})$ and $V_{22}^{\mathrm{HF}}(\vec{Q})$ are the Hartree-Fock potentials for fermions of types 1 and 2, respectively; $V_{12}^{\mathrm{H}}(\vec{Q})$ is the Hartree interaction potential between fermions of types 1 and 2. The order parameter $\Delta_\alpha(\vec{Q})$ of the CDW corresponding to wave vector $\vec{Q}$ for fermions of type α is taken to be nonzero only for reciprocal vectors: $\vec{Q} = (\pm Q_0, 0), (0, \pm Q_0)$ and $(\pm Q_0, \pm Q_0)$, $(\pm Q_0, \mp Q_0)$, where $Q_0^2\ell_0^2 = \pi$ (ℓ_0 is the magnetic length for composite fermions). The energy of the Halperin-(1,1,1) state is calculated from

$$E_{(1,1,1)} = -\frac{1}{4\pi}\int d^2r\, V_{\mathrm{eff}}(r)[g(r) - 1],$$

where $g(r) = 1 - \exp(-r^2/2\ell_0^2)$ is the correlation function of a fully occupied Landau level and

$$V_{\mathrm{eff}}(r) = \frac{1}{4}[V_{11}(r) + V_{22}(r) + 2V_{12}(r)]$$

is the effective interaction between composite fermions.

We have considered Coulomb interaction between composite fermions of types 1 and 2. The interaction asymmetry here results from their different form factors because fermions belong to different Landau levels. The energy of the Halperin liquid state at half polarization is $-0.196e^2/\varepsilon\ell_0$ and is *lower* than that of the proposed CDW state, $-0.123e^2/\varepsilon\ell_0$. This means that the proposed CDW state is not the lowest energy state and therefore not the ground state at half polarization.

To find the true ground state of the system that has lower energy than the Halperin state and that can also explain the half-polarized singularity, we considered the system described by the Hamiltonian (1) numerically in a spherical geometry [11,12]. All computations were done for a 12 fermion system on a sphere with sphere parameter $q = 5.5$, where the radius of the sphere $R = \sqrt{q} = 2.34$ in units of magnetic length ℓ_0. Because the Hamiltonian (1) does not change the number of fermions of a given type the eigenstates of the system can be classified by the number of fermions of type 1 ($12 \geq N_1 \geq 0$) and by angular momentum L (due to spherical geometry). In what follows, we have investigated three systems: (i) a "symmetric" system ($V_{11} = V_{22} = V_{12}$), where the interaction potentials are Coulombic except at the origin where it is taken to be less repulsive; (ii) the nonsymmetric system where the interactions between fermions were taken as the interactions between quasiparticles of the $\nu = \frac{1}{3}$ state—the "quasiparticle" system (these interaction potentials were found from finite size computations following the method of Ref. [13]); and (iii) the nonsymmetric system where the interactions between fermions were taken as the interactions between quasiholes of the $\nu = 1$ state—the "quasihole" system. For all these systems we found a similar behavior: The ground state energy as a function of filling factor ν_1 (N_1) has a *downward cusp* at $\nu_1 = \frac{1}{2}$ ($N_1 = 6$), as seen in Fig. 1. This point corresponds to a half-polarized state of the original system and the cusp indicates stability of the observed [3] half-polarized state.

It should be mentioned that for quasiparticle and quasihole systems we do not include in the Hamiltonian (1) the different creation energies for polarized and spin-reversed quasiparticles and quasiholes, which acts as an effective internal Zeeman splitting and makes the unpolarized state ($\nu_1 = 1$) the ground state at zero value of real Zeeman energy. These terms are monotonic functions of polarization and do not change the singular behavior at half polarization.

In order to analyze the symmetric system more carefully, we consider our two-component fermion system with $\nu = 1$ as a system of excitons, i.e., the electron-hole pairs. We choose type-1 fermions as electrons and the absence of type-2 fermions as holes. In this case the filling factors for electrons and holes are the same and are equal to ν_1, which is also the exciton filling factor.

It is well known that for a symmetric (Coulomb) multiexciton system the ground state is a Bose-condensed state

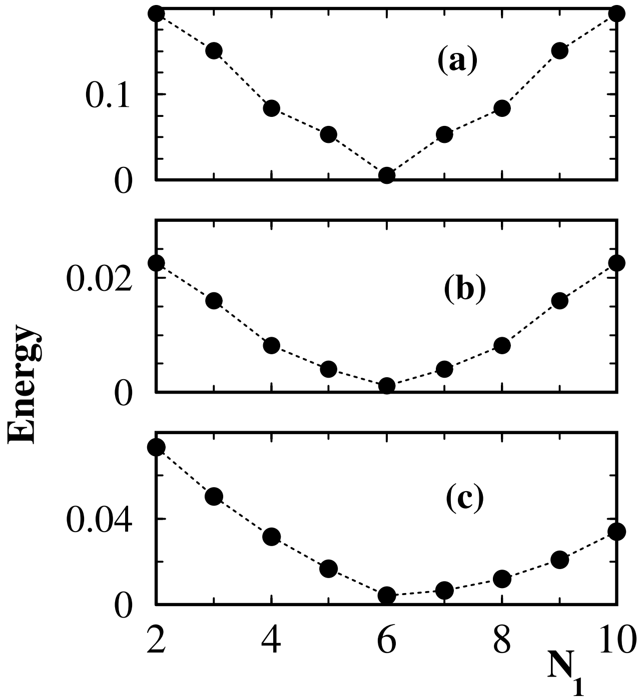

FIG. 1. Ground state energy as a function of the filling factor of fermions of type 1, $\nu_1 = N_1/12$, for (a) "symmetric," (b) "quasiparticle," and (c) "quasihole" systems. The energy is in units of $e^2/\epsilon\ell_0$.

of excitons with zero momentum [14–16]. In original fermion language these states are the Halperin states. For Coulomb interaction, these Bose-condensed states are the ground states for all values of filling factor ν_1 ($1 > \nu_1 > 0$). Let us now decrease the repulsion between original fermions at the origin. We do it numerically by decreasing the value of the pseudopotential with zero angular momentum, V_0. A decrease of V_0 by about 40% results in ground states of the multiexciton system that are not the Bose-condensed states of excitons with $L = 0$ (Fig. 1). These transitions are also accompanied by transition of the ground state of the one-exciton system to the state with nonzero angular momentum, $L = 1$.

Interestingly, the transition from a Bose-condensed state of zero-momentum excitons to a new state was also observed in a double layer system [17] with $\nu = 1$ when the separation between the layers is increased beyond some critical value [18,19]. At a critical layer separation of this system, the dispersion relation of the collective mode becomes negative at momentum $q \sim 1.3/\ell_0$ and it was proposed that this transition is the transition to a charge-density wave state [20]. In our case the dispersion relation of the collective mode of the Bose-condensed state also becomes negative but for much smaller momentum $q \sim L/R \sim 0.4/\ell_0$, which results from the fact that the ground state of the one-exciton system has angular momentum $L = 1$.

As stated above, for the Coulomb interaction the ground state of the one-exciton system has zero momentum and the multiexciton ground state is the Bose-condensed liquid state of excitons with zero momentum [16]. In analogy with the Coulomb system, we describe our multiexciton ground state as a liquid state of excitons with nonzero angular momentum $L = 1$, a *nonsymmetric exciton liquid.* Our multiexciton system does not contain $L = 0$ excitons because they are noninteracting, and removing one such exciton would not change the energy of the multiexciton system. However, in Fig. 1, the energy of our five-exciton system is higher then the energy of the six-exciton system. That means the six-exciton system can have only excitons with nonzero angular momentum.

In Fig. 2, the energy spectrum of the multiexciton symmetric system is shown for five- and six-exciton systems. We can see that the ground state of the six-exciton system has zero angular momentum, while the ground state of the five-exciton system has angular momentum $L = 1$. For the Halperin liquid each exciton effectively occupies only one state (electron and hole are in the same place) and the filling factor of the exciton is equal to the filling factor of electrons, ν_1. In the nonsymmetric exciton liquid, each exciton occupies effectively two states. As a result, the effective filling factor of excitons is $2\nu_1$. Then for $\nu_1 = \frac{1}{2}$ the filling factor of excitons is 1 which means that we have completely occupied the Landau level. Results of Fig. 2 do support this contention because removing one exciton with angular momentum $L = 1$ from the six-exciton system left the exciton hole with the same angular momentum, which means that the system behaves as though the

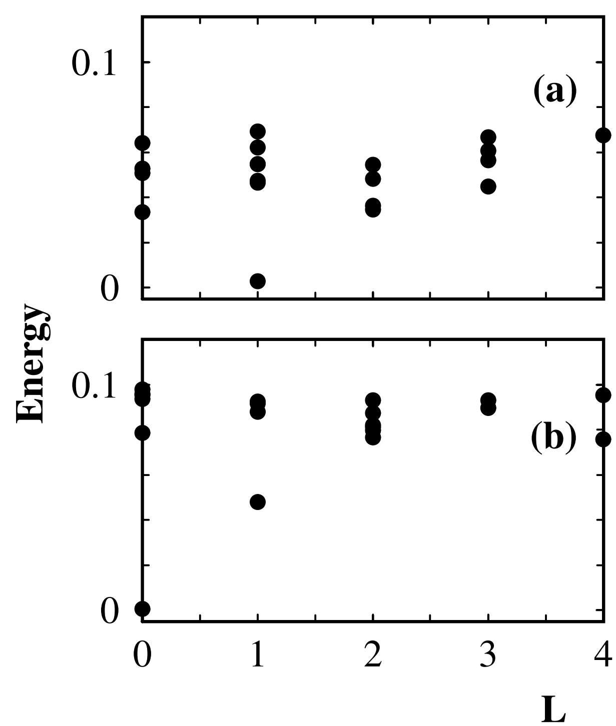

FIG. 2. Energy spectrum of the symmetric system for (a) $N_1 = 5$ and (b) $N_1 = 6$. Energy is in units of $e^2/\epsilon\ell_0$.

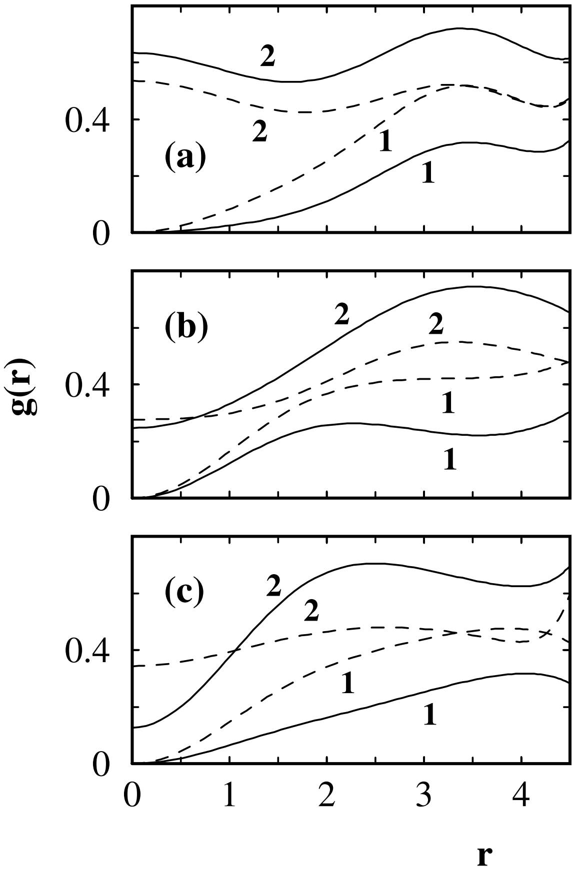

FIG. 3. Ground state pair correlation functions $g_{11}(r)$ (curves labeled "1") and $g_{12}(r)$ (curves labeled "2") for (a) symmetric, (b) quasiparticle, and (c) quasihole systems. The solid lines are for the $N_1 = 4$ systems, and dashed lines are for the $N_1 = 6$ systems. Correlation functions are shown in units of maximum electron density, and r is in units of magnetic length.

levels are completely filled. The energy spectrum of the six-exciton system, which corresponds to a half-polarized state of the original system, also has a gap. However, from the finite size results we cannot say with certainty if this gap will survive in the thermodynamic limit.

In Fig. 3, the pair correlation functions $g_{11}(r)$ and $g_{12}(r)$ are shown for $N_1 = 4$ and 6, for symmetric (a), quasiparticle (b), and quasihole (c) systems. The general feature of all these correlation functions is the nonzero value of $g_{12}(0)$. For the Bose-condensed state of excitons with zero momentum (the Halperin state) we would expect $g_{12}(0) = 0$, i.e., the holes in the multiexciton picture are sitting exactly at the position of the electrons. The nonzero values of $g_{12}(0)$ can therefore be directly associated with the cusp at $N_1 = 6$.

In closing, we have investigated the possible ground states at half the maximal spin polarization for $\nu = \frac{2}{5}, \frac{2}{3}$ FQHE states. Our results indicate that for the systems studied here there is a downward cusp at half polarization that reflects the observed structure in a recent experiment [3]. We interpret this result as due to condensation of a nonsymmetric exciton liquid.

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