Temperature Dependence of Spin Polarizations at Higher Landau Levels

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We report our results on the temperature dependence of spin polarizations at $\nu = 1$ in the lowest as well as in the next higher Landau level. They compare well with recent experimental results. At $\nu = 3$, spin polarization has a smaller magnitude but is not much influenced by higher Landau levels. In sharp contrast, for filling factor $\nu = \frac{8}{3}$ we predict that, unlike in the case of $\nu = \frac{2}{3}$, the system remains fully spin polarized even at vanishingly small Zeeman energies.

It has been long established that spin degrees of freedom play a very important role in the quantum Hall effects [1,2], that are unique demonstrations of electron correlations in nature. At the Landau level filling factor $\nu = 1$ ($\nu = n_e/n_s$, where n_e is the electron number and $n_s = AeB/\hbar c = A/2\pi \ell_0^2$ is the Landau level degeneracy, A is the area of the system and ℓ_0 is magnetic length) the ground state is fully polarized with total spin $S = n_e/2$ [3]. A fully spin polarized state is also expected for $\nu = \frac{1}{3}$, while a spin unpolarized state is predicted for the filling factor $\nu = 2/m$, where m is an odd integer [2]. Recently, a new dimension to those studies was introduced by Barrett *et al.* [4] (see also [5]) in their work on spin excitations around $\nu = 1$ and also temperature dependence of spin polarizations at $\nu = 1$. Since then several experimental groups have explored spin polarization of various other filling factors. In these experiments, direct information about electron spin polarization at various filling factors can be obtained via nuclear magnetic resonance (NMR) spectroscopy. Information about spin polarization of the two-dimensional electron gas in an externally applied high magnetic field is derived here from measurements of Knight shift of ^{71}Ga NMR signal due to conduction electrons in GaAs quantum wells in the quantum Hall effect regime. For a fully polarized ground state, as is the case for $\nu = 1$ and $\nu = \frac{1}{3}$, experimental results indicate that spin polarization saturates to its maximum value at very low temperatures and drops rapidly as the temperature is raised (Fig. 1, and also reported earlier in Refs. [4,5]). At large T, spin polarization is expected to decay as T^{-1} [6–8] and was found experimentally to behave that way [4,5].

Recently, Song *et al.* [9] reported NMR spectroscopy in a somewhat similar setup as that of Barrett *et al.* [4] in order to explore $\nu = 1$ and $\nu = 3$. Interestingly, temperature dependence of spin polarization at $\nu = 3$ revealed a different behavior as compared to that at $\nu = 1$. More specifically, the results of Song *et al.* indicated that even at the lowest temperature studied, electron spin polarization at $\nu = 3$ does not show any indication of saturation,

and with increasing temperature it sharply drops down to zero (Fig. 2). In this paper, we investigate spin polarization versus temperature at $\nu = 1$ in the lowest Landau level as well as in the next Landau level. We also compare our results with experimental results of Ref. [9]. At low temperatures, the behavior of spin polarization at $\nu = 3$ is similar to that at $\nu = 1$, but of much smaller magnitude. These results agree reasonably well with available experimental data at $\nu = 3$. However, discrepancies between our theoretical results and the experimental data remain at higher temperatures. We also present theoretical results for $\nu = \frac{2}{3}$ in the next higher Landau level. At $\nu = \frac{2}{3}$, convincing evidence exists about the spin polarization in the lowest Landau level [2,10,12]. But there are no experimental data available as yet for spin polarizations in the next higher Landau level, i.e., at $\nu = \frac{8}{3}$. We find (somewhat unexpectedly) that for $\nu = \frac{8}{3}$, even at a vanishingly small Zeeman energy, electrons in the higher Landau level remain fully spin polarized.
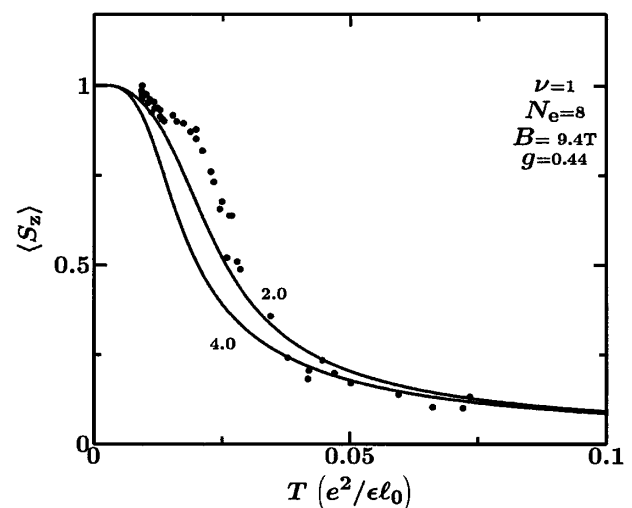

FIG. 1. Temperature dependence of spin polarization at $\nu = 1$ for two different values of finite-thickness parameter β. Experimental data points are from Ref. [9].
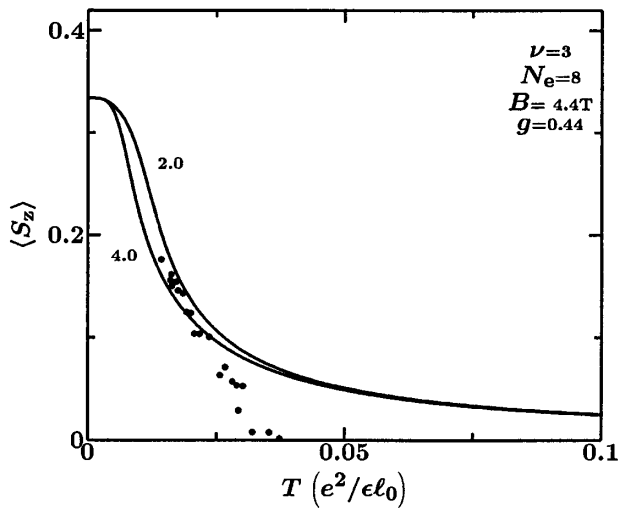
FIG. 2. Temperature dependence of spin polarization at $\nu = 3$ for different values of β. Experimental results are from Ref. [9].

We have calculated the temperature dependence of spin polarization for different filling factors from [6–8],

$$\langle S_z(T) \rangle \equiv \frac{1}{Z} \sum e^{-\varepsilon_j/kT} \langle j|S_z|j \rangle,$$

where $Z = \sum_j e^{-\varepsilon_j/kT}$ is the canonical partition function, and the summation is over all states including all possible polarizations. Here ε_j is the energy of the state $|j\rangle$ with Zeeman coupling included. They are evaluated for finite-size systems in a periodic rectangular geometry [2]. Our earlier theoretical results indicated that at small values of the Zeeman energy, temperature dependence of spin polarization is nonmonotonic for filling factors $\nu = 2/m$, $m > 1$ being an odd integer. In particular, for $\nu = \frac{2}{3}$ and $\nu = \frac{2}{5}$, we found that spin polarization initially *increases* with temperature, reaching a peak at $T \sim 0.01$ K when it falls as $1/T$ with increasing temperature. Appearance of the peak was associated with spin transitions at these filling factors and was found to be in good agreement with the experimental observation [10]. For $\nu = 1$ and $\nu = 1/3$, our results are also in excellent agreement [7] with the earlier available experimental results [4,5].

In our present work, energies are evaluated via exact diagonalization of a few electron system in a periodic rectangular geometry [2]. Since even at the lowest experimental magnetic field the Landau level separation $\hbar\omega_c$ is still an order of magnitude greater than typical energies due to the Coulomb interaction, electrons in the lowest Landau level can be treated as inert. In the calculations that follow we can therefore consider the lowest Landau level to be a uniform background causing merely a constant shift to interaction energies. The higher Landau levels then enter the system Hamiltonian via a modified interaction potential [11]. More specifically, for a finite number of active electrons N_e in a rectangular cell and choosing the Landau-gauge vector potential, the Hamiltonian in the $n = 0, 1$ Landau levels is (ignoring the kinetic energy and

single-particle terms in the potential energy which are constants [2])

$$\mathcal{H} = \sum_{j_1, j_2, j_3, j_4} \mathcal{A}_{nj_1, nj_2, nj_3, nj_4} a^{\dagger}_{nj_1} a^{\dagger}_{nj_2} a_{nj_3} a_{nj_4},$$

$$\mathcal{A}_{nj_1, nj_2, nj_3, nj_4} = \delta'_{j_1 + j_2, j_3 + j_4} \mathcal{F}_n(j_1 - j_4, j_2 - j_3),$$

$$\mathcal{F}_n(j_a, j_b) = \frac{1}{2ab} \sum_{\mathbf{q}} \sum_{k_1} \sum_{k_2}{}' \delta_{q_x, 2\pi k_1/a} \delta_{q_y, 2\pi k_2/b} \delta'_{j_a k_2}$$

$$\times \frac{2\pi e^2}{\epsilon q} \left[\frac{8 + 9(q/b') + 3(q/b')^2}{8(1 + q/b')^3} \right]$$

$$\times \mathcal{B}_n(q) \exp\left(\frac{1}{2} q^2 \ell_0^2 - 2\pi i k_1 j_b/n_s \right),$$

$$\mathcal{B}_n(q) = \begin{cases} 1 & \text{for } n = 0, \\ (1 - \frac{1}{2} q^2 \ell_0^2)^2 & \text{for } n = 1. \end{cases}$$

$$n_e = \begin{cases} N_e & \text{for } n = 0, \\ \frac{\nu}{\nu - 2} N_e & \text{for } n = 1. \end{cases}$$

Here a and b are the two sides of the rectangular cell that contains the electrons. The Fang-Howard variational parameter b' is associated with the finite-thickness correction [2], ϵ is the background dielectric constant, and the results are presented in terms of the dimensionless thickness parameter $\beta = (b'\ell_0)^{-1}$. The Kronecker δ with prime means that the equation is defined modn_s, and the summation over q excludes $q_x = q_y = 0$. This numerical method has been widely used in the quantum Hall effect literature [2] and is known to be very accurate in determining the ground state and low-lying excitations in the system.

Our results for $\langle S_z(T) \rangle / \max\langle S_z(T) \rangle$ vs T for an eight-electron system in a periodic rectangular geometry at $\nu = 1$ are presented in Fig. 1 where we also present the experimental data of Ref. [9] for comparison. Here the temperature is expressed in units of $e^2/\epsilon\ell_0$ and the conversion factor to K is $e^2/\epsilon\ell_0[\text{K}] = 51.67(B[\text{T}])^{1/2}$ appropriate for the system studied in experiments. In our calculations, we fix the parameters as in the experimental systems: the Landé g-factor is 0.44 and the magnetic field is $B = 9.4$ T. The curves that are close to the experimental data (and presented here) are for $\beta = 2$–4. As we discussed above, at low temperatures there is a rapid drop in spin polarization and for high temperatures spin polarizations decay as $1/T$. Our results are in good agreement with those experimental features. They were also in good qualitative agreement with the earlier experimental results at this filling factor [7]. While not entirely new, these results are presented with the intention of comparing them with the temperature dependence of spin polarization at $\nu = 3$. The results in the latter case are shown in Fig. 2 (again for an eight-electron system in a periodic rectangular geometry). In drawing this figure, we have taken the following facts from the experimental results of Ref. [9] into consideration: (a) the maximum $\langle S_z \rangle$ is in fact 1/3 and not 1 as in $\nu = 1$; (b) the experimental scale at $\nu = 3$ of Ref. [9] is the same as that at $\nu = 1$; and (c) spin polarization at $\nu = 3$

is drawn in Fig. 2 in the same scale as for $\nu = 1$. All the parameters except the magnetic field are kept the same as in the case of $\nu = 1$. Just as in the experimental situation, we fix the magnetic field for $\nu = 3$ at a much lower magnetic field of $B = 4.4$ T. The filled Landau levels, however, are still found to be inert at this low field and do not influence our chosen Hamiltonian. As seen in Fig. 1, numerical values of spin polarization are much smaller here than those for $\nu = 1$. Our theoretical results for $\beta = 2-4$ agree reasonably well with the experimental results of Ref. [9] except in the high temperature regime where the experimental data drop down to zero. Theoretical results, in contrast, have the usual $1/T$ tail. We should point out, however, that due to discreteness of the energy spectrum for a finite number of electrons the terms with S_z and $-S_z$ in the polarization cancel each other at high temperatures like $1/T$, and we will always end up with $1/T$ decay of $\langle S_z(T) \rangle$ vs T [6]. Therefore, we cannot predict with certainty how a macroscopic system would behave at high T. However, given the fluctuations in data points for $\nu = 1$ and $\nu = 3$ and the fact that the last few data points for $\nu = 3$ are extremely small, it is not clear if one expects saturation of points with $1/T$ behavior or the spin polarization actually vanishes. Clearly, experimental data at high temperatures do not show any sign of saturation and in order to settle the question of actual vanishing of $\langle S_z(T) \rangle$ it would be helpful to have more data in the high temperature regime. Saturation is also not visible in the low-temperature region of the experimental data. In order to clarify many of these outstanding issues, it is rather important to have a more experimental probe of temperature dependence at this filling factor.

Influence of higher Landau levels is found to be quite significant for filling factor $\nu = \frac{2}{3}$. As we have demonstrated earlier [6], at low Zeeman energies the system at this filling factor is spin unpolarized, and with increasing Zeeman energies the system undergoes a phase transition to a fully spin polarized state. A similar result is also expected for $\nu = \frac{2}{5}$. These theoretical predictions are now well established through a variety of experiments [10,12,14–17]. Our results for $\langle S_z(T) \rangle$ vs T at $\nu = \frac{8}{3}$ are shown in Fig. 3, where we present results for a six-electron system and a magnetic field value of 4.4 T. In Fig. 3, we present our results for $\beta = 2, 4$, but the spin polarization is rather insensitive to the finite-thickness correction. We also consider two different values of the Landé g-factor: 0.44 (solid curves) and 0.05 (dashed curves). Interestingly, the results indicate that the total spin S of the active electrons, unlike in the lowest Landau level, is at its maximum value $S = N_e/2$ even without Zeeman coupling. Hence even an infinitesimal Zeeman coupling will orient the spins in the active system resulting in the polarization being $1/4$. That is at odds with the conventional composite fermion model which predicts fractions of the form $2 + 2/m$, m odd, to be unpolarized [18]. This somewhat surprising behavior can be thought to be due to more repulsive effective inter-
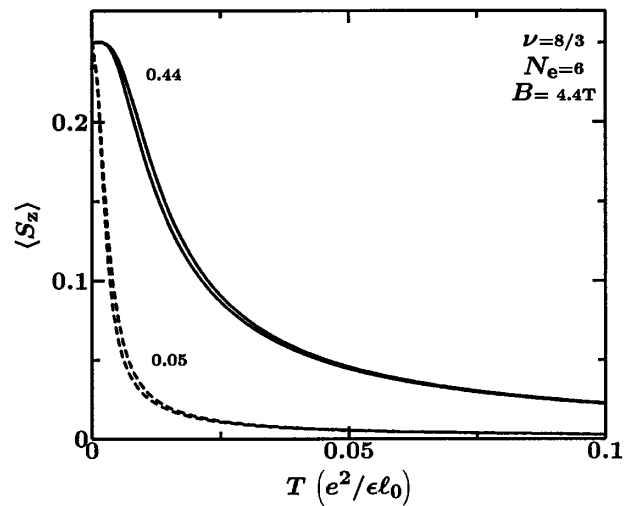

FIG. 3. Temperature dependence of spin polarization at $\nu = \frac{8}{3}$ for $\beta = 2, 4$ and two different values of the Landé g-factor ($g = 0.44$ and $g = 0.05$). The results are almost independent of β.

actions forcing the electrons, according to Hund's rule, to occupy the maximum spin state more effectively as compared with electrons on the lowest Landau level. In order to demonstrate this behavior we have considered the case of a very small Zeeman energy (dashed curves), but the results still indicate full spin polarization of the active system. At this low Zeeman energy, spin polarization drops rather rapidly from its maximum value as the temperature is increased. In this context we should mention that the idea of an extremely small Zeeman energy is not that far fetched: in recent experiments, a significant reduction in Zeeman energy has been achieved by application of a large hydrostatic pressure on the heterostructure [16,19,20]. It is even possible to have situations close to zero Zeeman energy [21]. With the help of all the different techniques available in the literature to study spin polarization, it should be possible to explore $\langle S_z(T) \rangle$ for $\nu = \frac{8}{3}$.

In closing, we have investigated spin polarization as a function of temperature for $\nu = 1$ and $\nu = \frac{2}{3}$ in the higher Landau level. Our results indicate that for $\nu = 3$ our theoretical results are not much influenced by the higher Landau level (except being much lower in magnitude). Available experimental results are incomplete at low and high temperature regions where no saturation of data points has been observed. Our results at $\nu = \frac{8}{3}$ reveal that the system is always fully spin polarized even at very small Zeeman energies. That is in contrast to the behavior at $\nu = \frac{2}{3}$ which at low Zeeman energies has a spin unpolarized state [2] that is well supported by various experimental investigations. More experimental data points at $\nu = 3$ in the low and high-temperature regime would be very helpful. An experimental probe of $\nu = \frac{8}{3}$ with NMR and optical spectroscopy should be able to explore the spin states predicted in the present work.

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