Thermodynamics and Spin Polarizations of the Fractional Quantum Hall States

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The filling factor and temperature dependence of the electron spin polarization $\langle S_z(T)\rangle$ of a two-dimensional electron system have been studied for various values of Zeeman coupling. At $T = 0$, there are sharp spin transitions for all the filling fractions considered here except $\nu = \frac{1}{3}$. At low T, the appearance of a peak in $\langle S_z(T)\rangle$ at $\nu = \frac{2}{3}$ and $\nu = \frac{2}{5}$ is interpreted as a manifestation of the reentrant behavior observed earlier. At $\nu = \frac{3}{5}$, one sees a gradual trasition from one spin state to another at low T. At large T, $\langle S_z(T)\rangle$ generally decays as T^{-1}. These can be explored in the optically pumped nuclear magnetic resonance Knight shift measurements. [S0031-9007(96)00190-1]

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A two-dimensional electron system in a strong magnetic field exhibits a remarkable many-body effect, viz. the fractional quantum Hall effect (FQHE), which has been under intense investigations for well over a decade [1–4]. The effect is entirely due to electron correlations and as a result of that electrons condense into an incompressible liquid state with several unique properties [2–4]. At high magnetic fields and for large enough g factor, all electrons are expected to have their spins aligned with the magnetic field and one can safely ignore the spin degrees of freedom in the theory of the FQHE. But the g factor is small for electrons in GaAs while $1/m^*$ is large, which leads to a very small Zeeman energy relative to the cyclotron energy [3]. Therefore, complete polarization of the electron spins cannot be a good approximation for all filling fractions. Indeed, it has been known for a while from theoretical studies that the ground state spin polarization at various Landau level filling factors varies significantly from being fully spin polarized (such as at $\nu = \frac{1}{3}, \frac{5}{3}$, etc., where $\nu = 2\pi\ell_0^2 n$, n is the number of electrons per unit area, and $\ell_0^2 \equiv \hbar c/eB$ is the magnetic length) to partially spin-polarized states (as in $\nu = \frac{2}{3}, \frac{2}{5}$, etc.) and the electron-electron interaction is largely responsible for that [3–5]. The possibility of lowest-energy spin-reversed excitations in some of the filling factors where the ground state is either spin reversed or even fully spin polarized was also predicted theoretically [5]. Subsequent transport measurements, particularly in tilted magnetic fields [6], provided very convincing evidence in favor of those spin-reversed states. In fact, dramatic changes were observed in longitudinal resistivity at different filling factors when the tilt angle was increased and they were explained as due to various spin assignments of the ground state (and spin-reversed excitations) at those filling fractions [6]. For example, a sharp change in the dependence of the activation energy on tilt angle was observed at $\nu = \frac{8}{5}$ (electron-hole conjugate of $\frac{2}{5}$). This was described as a transition from a spin-polarized ground state at small angles to a polarized state at large angles. The linear behavior of the activation energy at two different ground states was identified with the Zeeman energy, and they appeared because of the spin-reversed quasiparticles and quasiholes [7].

A very ingenious approach (which is also more direct) to study the spin polarizations of two-dimensional electron systems in the QHE regime is the recently reported optically pumped nuclear magnetic resonance (OPNMR) measurements of the Knight shift K_s and spin-lattice relaxation rate T_1^{-1} of ^{71}Ga nuclei in electron-doped multiple quantum wells [8,9]. It has been already established earlier in experiments that the nuclear spin-lattice relaxation near a two-dimensional electron gas can provide information about the electronic density of states [10]. However, the strong temperature and filling factor dependence of the nuclear spin-lattice relaxation observed in recent experiments using OPNMR at $\nu \approx 1$ and $\nu \approx \frac{2}{3}$ cannot be explained in terms of the independent electron model, but by interaction induced spin-flip excitations [9]. In addition, the Knight-shift measurements turned out to be the first direct probe of the electron spin polarization of a two-dimensional electron system in a magnetic field [8]. Here one measures the shift between the lower frequency resonance, attributed to ^{71}Ga nuclei in the quantum wells, and the higher frequency resonance due to ^{71}Ga nuclei in the barrier. The shift is supposed to have occurred due to the magnetic hyperfine coupling between the ^{71}Ga nuclei and electrons in the wells. The hyperfine coupling constant was found to be isotropic in that experiment and therefore the observed NMR frequency shift is a direct measure of the electron spin polarization [8]. As yet, detailed studies have been limited to the integer quantum Hall regime ($\nu \geq 1$). In the case of $\nu = 1$, there is also indirect evidence of the existence of the spin excitations with topological characteristics (Skyrmions) [8,11]. Such excitations are important in the limit of vanishing Zeeman energy. When the Zeeman

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energy dominates, one would, of course, expect the lowest-energy excitations to be single spin-flip excitations predicted theoretically at $\nu = 1$ [12]. Similar studies in the fractional quantum Hall effect regime are expected to explore directly the spin-reversed ground state and spin-reversed excitations discussed above. In fact, NMR experiments of Barrett *et al.* already show indications that at $\nu = \frac{2}{3}$ the ground state and low-lying excitations might involve reversed spins. Further studies at very low temperatures are needed to get more information in the FQHE regime.

As mentioned above, one important aspect of the NMR Knight-shift studies of the two-dimensional electron gas is that it provides information about $\langle S_z \rangle$ as a function of temperature:

$$\langle S_z(T) \rangle \equiv \frac{1}{Z} \langle 0|S_z|0 \rangle + \sum \frac{1}{Z} e^{-\varepsilon_j/kT} \langle j|S_z|j \rangle ,$$

where $|0\rangle$ is the ground state, $Z = \sum_j e^{-\beta\varepsilon_j}$ is the partition function, and the summation is over all excited states $|j\rangle$ with energy ε_j. Here we report our studies of $\langle S_z(T) \rangle$ for various filling fractions where the ground states are not always expected to be fully spin polarized. As stated above, our earlier studies of the spin degrees of freedom in QHE (largely based on the finite electron systems in a periodic rectangular geometry) [5] revealed that the electrons in the ground state are fully spin polarized at $\nu = \frac{1}{m}$, spin unpolarized at $\nu = 2/(2m \pm 1)$, with $m = 1, 3, 5, \ldots$, and partially spin polarized at $\frac{3}{5}$, $\frac{3}{7}$, etc. Moreover, it was pointed out earlier [12] that in the lowest Landau level and in the presence of spin degrees of freedom the filling factors ν and $1 - \nu$ are no longer electron-hole symmetric. This was subsequently observed in transport experiments [6]. The behavior of $\langle S_z(T) \rangle$ in some of these filling fractions will be explored in this work.

The ground state and the excited states required to study the temperature dependence of $\langle S_z \rangle$ are obtained in the well known exact diagonalization of the few-electron system Hamiltonian in a periodic rectangular geometry [4]. The obvious advantage of this scheme over the others is that here the energy eigenstates can be calculated very accurately for various filling fractions. However, when we include the spin degrees of freedom, the size of the Hamiltonian matrix easily exceeds the size where a direct diagonalization is manageable. In practice we have to resort to an iterative scheme like the one described in [4]. In fact, the more electrons there are in the system the denser the energy spectrum is and more energy values are necessary for the convergence of the sums in $\langle S_z(T) \rangle$ at finite temperatures. While iterative methods are very efficient when only a few of the lowest eigenvalues are to be extracted, it soon becomes a formidable exercise when the number of electrons increases and the convergence is to be achieved. In this work we had to limit the orders of the Hamiltonian matrices to $\lesssim 10^5$ which naturally implies severe restrictions on the number of electrons which could be included in the system.

It should be pointed out that we can calculate $\langle S_z \rangle$ only in the presence of a Zeeman coupling, which is, of course, present in the experimental systems. This can be understood in a straightforward manner as follows: In the case when the Hamiltonian of the system $\mathcal{H}$ does not include the Zeeman term, for each state $|i\rangle$ with $S_z|i\rangle = s_z|i\rangle$ and $\mathcal{H}|i\rangle = \mathcal{E}|i\rangle$ there is a state $|i'\rangle$ for which $\mathcal{H}|i'\rangle = \mathcal{E}|i'\rangle$ but $S_z|i'\rangle = -s_z|i'\rangle$. These terms cancel each other in the sum of $\langle S_z \rangle$. On the other hand, if one includes the Zeeman energy in the Hamiltonian, the polarization $\langle S_z \rangle$ will differ from zero because these terms then sum up to

$$s_z e^{-\beta\mathcal{E}} [e^{\beta g \mu_B s_z B} - e^{-\beta g \mu_B s_z B}] = 2 s_z e^{-\beta\mathcal{E}} \times \sinh(\beta g \mu_B s_z B) ,$$

where g is the Lande g factor for electrons in the medium and μ_B is the Bohr magneton. Generally the sum over all energy states will then yield a nonvanishing polarization. The system can, however, still be unpolarized at zero temperature if the ground state, even in the presence of the Zeeman coupling, is unpolarized.

Numerical results for $\langle S_z(T) \rangle / \max\langle S_z \rangle$ as a function of T (in units of the potential energy) at $\nu = \frac{1}{3}, \frac{2}{3}, \frac{2}{5}$, and $\frac{3}{5}$ are shown in Figs. 1–4, respectively. Here the conversion factor for T is, e.g., $e^2/\epsilon\ell_0 = 51.67 B^{1/2}$ for parameters appropriate to GaAs, where the energy is expressed in K and the magnetic field B is expressed in tesla. In all our calculations, we have fixed the magnetic field at 10 T, but studied a range of g values (0.1–0.5). For $\nu = \frac{1}{3}$, we considered a five-electron system in a periodic rectangular geometry (Fig. 1). Here the ground state of the system is

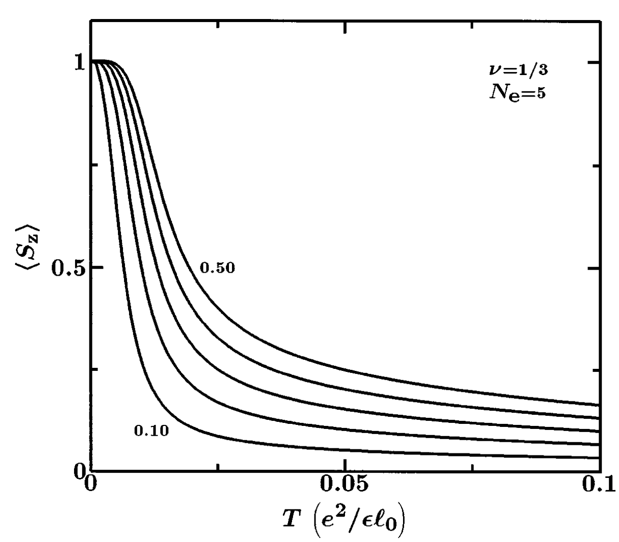

FIG. 1. Electron spin polarization $\langle S_z(T) \rangle$ vs the temperature T (in units of $e^2/\epsilon\ell_0$) at $\nu = \frac{1}{3}$ at a magnetic field of 10 T and various values of the g factor ($g = 0.1$–0.5). The number of electrons in the system is also indicated.

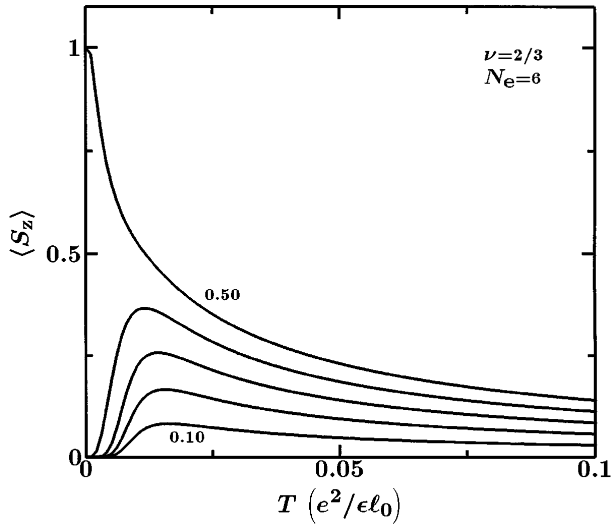

FIG. 2. Electron spin polarization vs T (in units of $e^2/\epsilon\ell_0$) at $\nu = \frac{2}{3}$ for various g values and the number of electrons as indicated in the figure.

known to be fully spin polarized [3–5], and, except for a very small Zeeman energy, the excited states are also supposed to be spin polarized. This is precisely what is seen at $T = 0$ for all the g values considered here. Further, it is interesting to note that as g is decreased (i.e., the Zeeman energy is decreased) $\langle S_z(T)\rangle$ drops off more rapidly with increasing temperature. This result is, in fact, consistent with the observation by Barrett *et al.* [8]. Their result for the Knight shift as a function of T, which supposedly represents the finite-size Skyrmions, decreases more rapidly than the results for model calculations at $\nu = 1$ with spin waves as the low-lying excitations. The latter case can be realized here, as explained above, when the Zeeman energy is strong enough to shrink the Skyrmions into single spin-flip excitations. For large T, our results indicate a $1/T$ decay of $\langle S_z(T)\rangle$. This can be understood as follows: When we note that there are states with both $+s_z$ and $-s_z$ whose energies differ by the Zeeman energy, then it is easy to see that in the limit $T \rightarrow \infty$ the leading term in the expansion of $\langle S_z(T)\rangle$ is

$$s_z^{(0)}[1 - \exp(-2\beta g \mu_B s_z^{(0)} B)]$$

when the ground state has the nonvanishing polarization $s_z^{(0)}$. If the ground state is unpolarized the leading term is

$$-2s_z^{(1)} e^{-\beta(\mathcal{E}_1 - \mathcal{E}_0)} \sinh(\beta g \mu_B s_z^{(1)} B)\,,$$

where $\mathcal{E}_0$ is the energy of the ground state and $\mathcal{E}_1 - g\mu_B s_z^{(1)} B$ the lowest energy with nonvanishing polarization $s_z^{(1)}$. At the high-temperature limit these terms above are both proportional to B/T. Thus at high temperatures the system behaves like a Curie paramagnet.

The results for $\nu = \frac{2}{3}$, calculated for a six-electron system, and $\nu = \frac{2}{5}$, calculated for a four-electron system, are presented in Figs. 2 and 3, respectively. At $T = 0$, one observes the spin-singlet state for these two fractions at low values of g. This was known from the earlier work [4–6]. In this spin state, as the temperature increases the curves peak at $T \sim 0.01$ and then at high temperatures they decrease as $1/T$. The appearance of the peak is presumably related to the "reentrant" behavior of the activation energy observed earlier for these two filling fractions in transport measurements. That behavior was associated with a phase transition from one spin ground state (unpolarized) to the other (polarized but with spin-reversed excited states) [6,7]. We speculate that, at the low-temperature side of the peak, the system has a spin-flip ground state as well as spin-flip excitations. At the

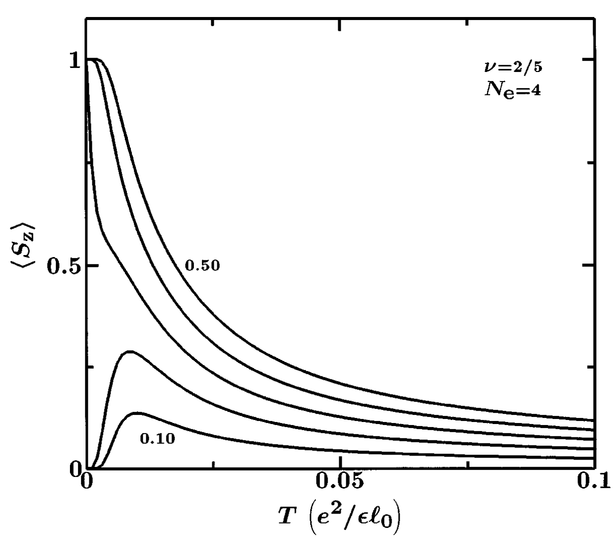

FIG. 3. Electron spin polarization vs T (in units of $e^2/\epsilon\ell_0$) at $\nu = \frac{2}{5}$ for various g values and the number of electrons as indicated in the figure.

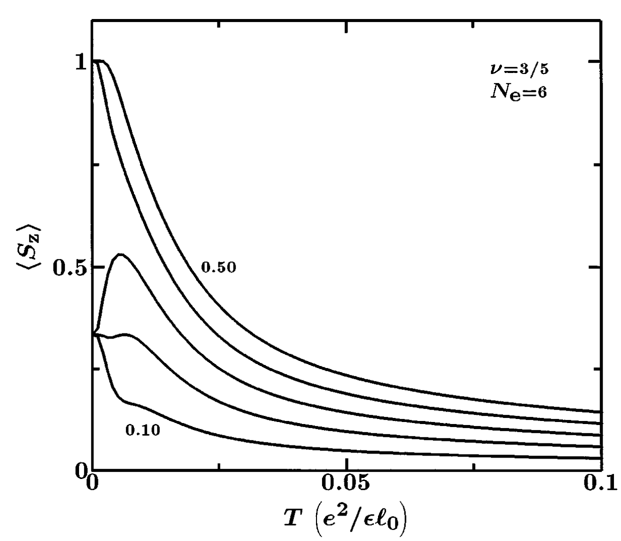

FIG. 4. Electron spin polarization vs T (in units of $e^2/\epsilon\ell_0$) at $\nu = \frac{3}{5}$ for different g values and the number of electrons as indicated in the figure.

high-temperature side of the peak, the system, on the other hand, has a spin-polarized ground state but the excitations are still spin reversed. The peak is much sharper for $\nu = \frac{2}{5}$ (Fig. 3) than it is for $\nu = \frac{2}{3}$ (Fig. 2), again consistent with what was observed in transport measurements where there was a sharp transition for $\nu = \frac{8}{5}$ [6], but a not so sharp transition at $\nu = \frac{2}{3}$. At low temperatures, there seems to be an abrupt transition from the spin-singlet to the fully spin-polarized state. The threshold g (or the Zeeman energy) where that spin transition takes place is much lower for $\nu = \frac{2}{5}$ than for $\nu = \frac{2}{3}$. This might explain why spin-reversed states are already observed in transport experiments at $\nu = \frac{2}{3}$ but not at $\nu = \frac{2}{5}$. At $g = 0.5$, the system is fully spin polarized for all values of T at these two filling fractions, and there one observes a sharp fall of spin polarization with increasing temperature as for $\nu = \frac{1}{3}$.

In the case of $\nu = \frac{3}{5}$ we find that the ground state is at $S = 1$ (and correspondingly $S_z = 0$ and $S_z = 1$ are degenerate in the absence of Zeeman coupling). This means that the system will be at least partially polarized no matter what value of g one takes (except for $g = 0$, when, of course, $\langle S_z \rangle = 0$). For low values of g and at low temperatures, one sees gradual formation of a peak in $\langle S_z(T) \rangle$ and a transition to the fully polarized state when g is further increased. This behavior is consistent with the behavior of $\langle S_z(T) \rangle$ at other fractions considered here and can be interpreted as transitions from the partially polarized to a spin-singlet state and eventually to a fully polarized state. The high-temperature behavior is, however, the same as in all other fractions considered in this work.

In closing, we have studied the spin polarization of the fractional quantum Hall states as a function of temperature for a range of filling fractions which are known to have different spin states at $T = 0$. The results at low temperatures exhibit interesting structures which are interpreted as consequences of different spin polarizations of the ground state and excited states. We expect that OPNMR measurement of K_s at these filling fractions might be useful to explore the new and interesting features discussed above.

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