PHYSICAL REVIEW B VOLUME 55, NUMBER 4 15 JANUARY 1997-II

Specific heat of quantum Hall systems

Tapash Chakraborty
Institute of Mathematical Sciences, Taramani, Madras 600 113, India

P. Pietiläinen
Theoretical Physics, University of Oulu, Linnanmaa, FIN-90570 Oulu, Finland

(Received 22 October 1996)

The specific heat of a two-dimensional electron gas in the quantum-Hall-effect regime (fractional and integral) is studied. The fractional-filling factors chosen here are known to have various spin configurations other than a fully spin-polarized state and we find that the specific heat versus temperature curves provide important information about those spin states. In the case of the lowest-filled Landau level the peak shifts to higher temperatures and the results are generally g-factor independent. [S0163-1829(97)51404-4]

It has long been established that at the Landau-level filling factor $\nu=1$ ($\nu=N_e/N_s$ where N_e is the electron number and $N_s=AeB/hc=A/2\pi l_0^2$ is the Landau level degeneracy and ℓ_0 is the magnetic length) the ground state is fully spin polarized with total spin $S=N_e/2$.[1] Recent Knight-shift spin polarization measurements,[2] reveal a precipitous fall in the spin polarization when one either moves slightly away from $\nu=1$ or the temperature is increased at $\nu=1$. This effect has also been observed in subsequent experiments with tilted magnetic field as well as optical absorption studies.[3] Theoretically, such a result is explained as due to the fact that the low-energy charged excitations are spin textures (Skyrmions) (Ref. 4) for very small values of the g factor. These excitations carry a charge $\pm e$ (as in the single-particle case), but they cover an extended region and have a nontrivial spin order: at the boundary of the system the local spin takes the value of the ground state and reverses at the center of the skyrmion. Along any radius, the spin gradually twists between these two limits. Numerical diagonalization scheme when applied to a few-electron system in a periodic rectangular geometry supports the above picture of spin excitations.[5] In the fractional-quantum-Hall-effect regime[6,7] one notices more structures in the spin-polarization results as a function of the temperature[8] due primarily to spin configurations different from a fully spin-polarized state (except of course, at $\nu=\frac{1}{3}$, which is fully spin polarized[9]).

The formation of a crystalline state of the multiskyrmion system has also been considered recently[10] and the observation of a sharp peak in low-temperature heat capacity near $\nu=1$(Ref. 11) has been attributed to melting of the skyrmion lattice. Measurement of equilibrium properties like the specific heat of a two-dimensional electron gas provides information about the form of the density of states.[12] At very low temperatures and for filling factor close to lowest-filled Landau level ($\nu=0.81$), experiments have revealed a very sharp peak in the specific heat which is suggestive of a phase transition involving skyrmionic excitations. In this paper, we present results for the specific heat of the quantum Hall systems in the integer and fractional regime. Our method is comprised of calculating the eigenstates exactly for a few-electron system in a periodic rectangular geometry.[5,7,8] The system is supposed to be in the lowest Landau level, with densities corresponding to various filling factors whose spin configurations are already known from previous work.[5,7–9]

Numerical diagonalization of the Hamiltonian (in the lowest Landau level, and for an impurity-free two-dimensional electron gas[7]) provides us with all the energy levels in the system. The evaluation of the specific heat is then quite straightforward. The canonical partition function can be obtained from

$$Z=\mathrm{Tr}e^{-\beta\mathcal{H}}$$

and

$$E=\langle\mathcal{H}\rangle=-\frac{\partial \ln Z}{\partial\beta}=T^2\frac{\partial \ln Z}{\partial T}.$$

The heat capacity c_v is given by

$$c_v=-T\left(\frac{\partial^2 F}{\partial T^2}\right)_{V,N_e}=\frac{1}{T^2}\langle(\mathcal{H}-\langle\mathcal{H}\rangle)^2\rangle,$$

where F is the free energy

$$F=-T\ln Z=-T\ln \mathrm{Tr}e^{-\beta\mathcal{H}}.$$

At $T\approx 0$, we get

$$\langle(\mathcal{H}-\langle\mathcal{H}\rangle)^2\rangle\approx\Delta^2 e^{-\Delta/T},$$

where Δ is the energy difference between the ground and first excited states. Correspondingly, the specific heat behaves like

$$C\propto\frac{1}{T^2}e^{-\Delta/T}.$$

The exact coefficients in the above expression depend strongly on the system and, in particular, on the density of states. In finite systems the energy spectrum is discrete and there is always an energy gap $\Delta\neq 0$ independent on the filling fraction. In the real systems, when the filling fraction does not correspond to a quantum Hall state, the gap van-

0163-1829/97/55(4)/1954(4)/$10.00

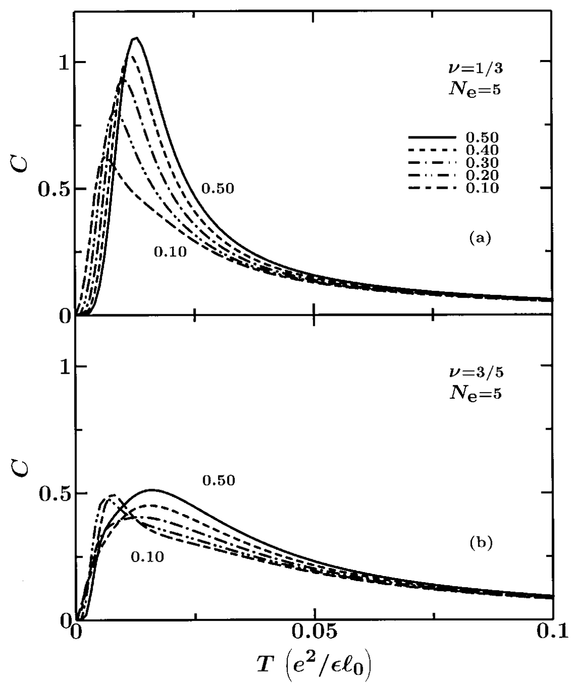

FIG. 1. The specific heat per electron versus temperature (a) at $\nu=\frac{1}{3}$ and (b) at $\nu=\frac{3}{5}$ for various values of the g factor. The magnetic field is held fixed at $B=10$ T.

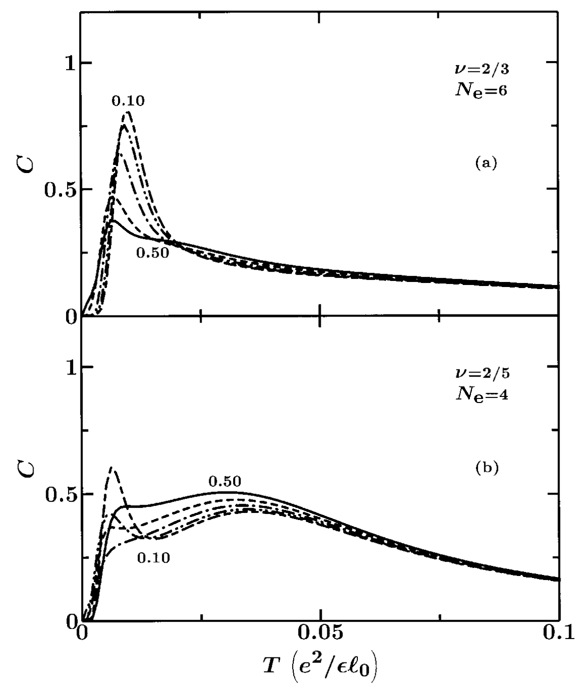

FIG. 2. The specific heat per electron versus temperature for (a) $\nu=\frac{2}{3}$ and (b) $\nu=\frac{2}{5}$. The number of electrons in the system is also given here.

ishes and the specific heat will not necessarily any more obey the exponential law given above. Even at the quantum Hall states, the gap in the real system will be reduced due to the impurities and defects generating states in the gap of the pure system discussed in this paper. The finite-size effects at low temperatures are thus quite dominant in the results for specific heat C presented below. It should be mentioned that at very low temperatures it would be a formidable task to distinguish anything which goes like $e^{-\Delta/T}$ from the large specific heat of the underlying lattice ($\propto T^3$).

In Figs. 1–5, we present the results of specific heat as a function of temperature (in units of the potential energy, $e^2/\epsilon l_0$, where ϵ is background dielectric constant) for $\nu=1,\frac{1}{3},\frac{2}{3},\frac{2}{5}$, and $\frac{3}{5}$. It is known from earlier studies that the electrons in the ground state are fully spin polarized at $\nu=1/m$, spin unpolarized at $\nu=2/(2m\pm1)$, with $m=1,3,5,\ldots$, and partially spin polarized at $\frac{3}{5},\frac{3}{7}$, etc.[7] The numerical results are obtained here for a fixed value of the magnetic field ($B=10$ T) while the g factor is varied. It should be mentioned at this point that a very interesting development in the experimental study of spin configurations of quantum Hall states has been the realization that the g factor can be reduced by applying hydrostatic pressure.[13] In particular, in the fractional quantum Hall regime, it has been experimentally demonstrated[13] that the effect of the hydrostatic pressure on the system is analogous to that of a tilted magnetic field.[14] Therefore, variation of the g factor for a fixed value of the magnetic field studied by us[5,8] has now become a realistic possibility.

As mentioned above, the system is spin polarized at $\frac{1}{3}$ for all values of g.[7] At this filling factor the results for specific heat versus the temperature reveal sharp Schottky-like peaks at low temperatures because of the energy gap in the low-energy excitation spectrum.[15] The gap arises due to the incompressibility of the system at this filling fraction.[7] Clearly, the heights of the peaks in the C plots [Fig. 1(a)] are roughly proportional to g, i.e., the larger the g is, the higher is the peak. The actual peak height may depend slightly on the size of the system (see below). At $\nu=\frac{3}{5}$ [Fig. 1(b)], the system

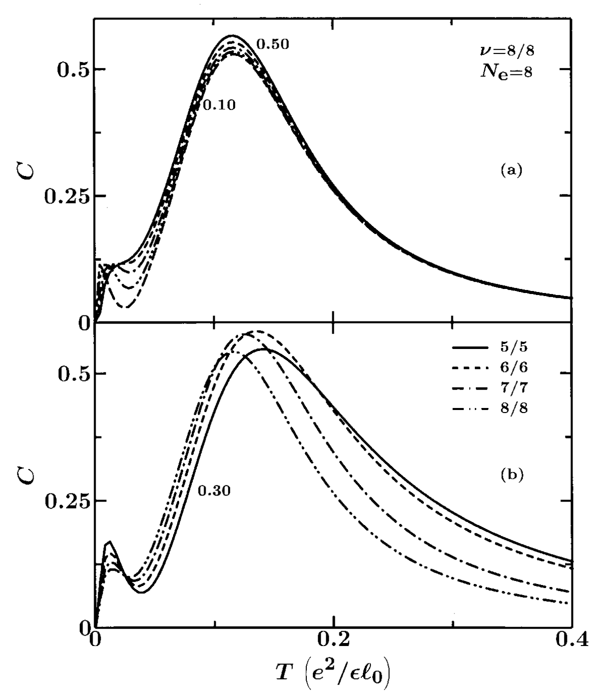

FIG. 3. The specific heat versus temperature at (a) $\nu=1$ for various values of the g factor and (b) for different number of electrons and a fixed $g=0.3$.

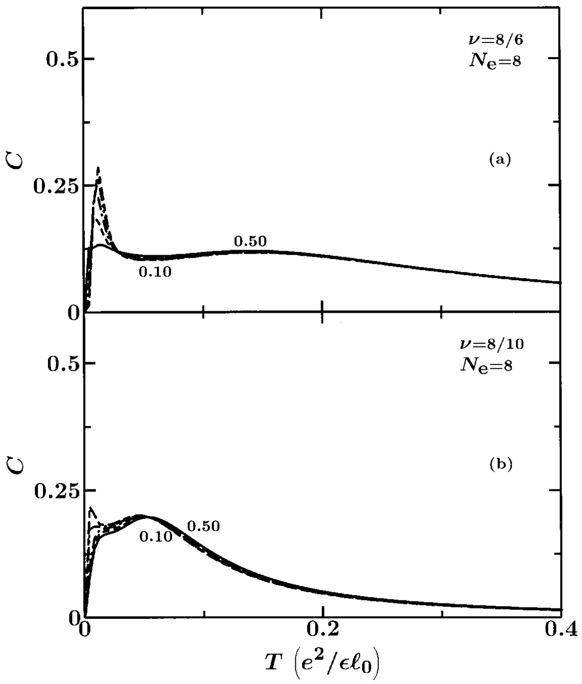

FIG. 4. The specific heat versus temperature $\nu=1$ with two flux quanta (a) removed or (b) added.

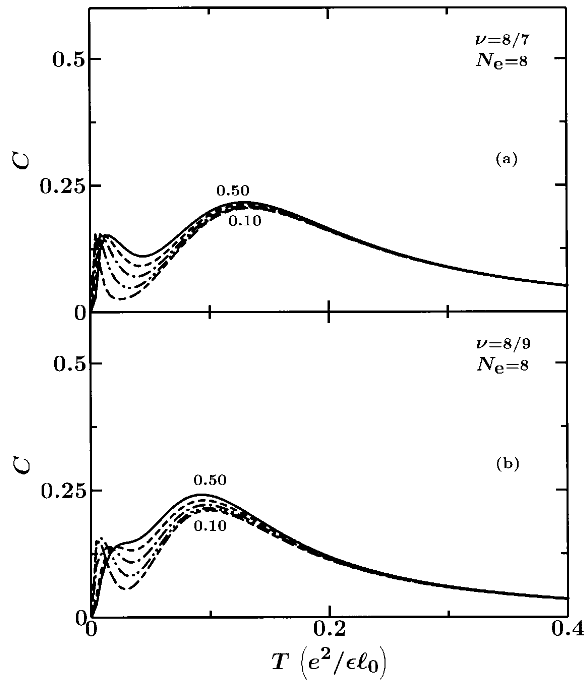

FIG. 5. The specific heat versus temperature $\nu=1$ with one flux quanta (a) removed or (b) added.

remains spin polarized down to $g \sim 0.3$.[8] The peaks in corresponding C curves behave just like in the $\nu=\frac{1}{3}$ system. However, when $g \leqslant 0.2$ the system becomes spin unpolarized. It is to be noted that near the spin transition point (in this case when $g \sim 0.2$) the energy gap Δ is rather small. The position of the peak is roughly proprtional to Δ as one can see by seeking for the maximum of $1/T^2 e^{-\Delta/T}$. This implies that the peak shifts to lower temperatures and starts again to go to higher temperatures when g is further reduced. These arguments are, of course, not very rigorous because there are more than two states in our system.[8] However, one definitive conclusion here is that the behavior of C changes when the spin polarization of the system changes.

The same pattern is also valid for the $\nu=\frac{2}{3}$ case [Fig. 2(a)]. Here the system is almost degenerate already at $g=0.5$ and hence the peak of C is at very low T. However, one can still see some remnants of the peak corresponding to the spin-polarized state near $T=0.025$. Apparently, the height of the peak is roughly proportional to the energy gap. Now, if the system remains spin polarized down to $g=0$ the energy gap is more or less due to the Zeeman energy and decreases when g decreases. Hence we have a situation like the $\nu=\frac{1}{3}$ case. If the system is spin-polarized at large values of g but goes to a spin-unpolarized (or spin-partially-polarized) state when g is reduced the energy gap must first decrease to zero (there is no gap at the point where spin-polarized and spin-unpolarized states have equal energy) and then start to increase when the value of g is further reduced. Provided that the conjecture about the proportionality of the gap and peak height is correct the peak must first go down and then start to rise again. This is what seems to happen at $\nu=\frac{2}{3}$. The $\nu=\frac{2}{5}$ [Fig. 2(b)] system seems to be somewhere between $\nu=\frac{1}{3}$ and $\nu=\frac{2}{3}$. For $T>0.03$, the behavior of the specific-heat curves is the same for all these filling factors.

The results for the lowest-filled Landau level ($\nu=1$) is shown in Fig. 3(a). Here the noticeable feature is that there is a sharp peak at a higher temperature, while at low temperatures, there are some weak structures. The other striking feature of the result is that it is largely g-factor independent. We have also checked the dependence of C at $\nu=1$ on the system size. Calculations for $N_e=5-8$ reveal that as the electron number increases the low-T structures become less prominent, but the high-temperature peak gets somewhat sharper and falls somewhat more rapidly at higher temperatures [Fig. 3(b)]. Overall, there is no dramatic change of the results as the system size is increased.

Our earlier studies[5] indicated that, in a toroidal geometry the lowest-energy spin excitations near $\nu=1$ are the two-skyrmion excitations. Single-Skyrmion excitations do not exist in this geometry. Instead, when one adds or removes a flux quanta at $\nu=1$, there is just a single spin-flip excitation. However, it was also found that when one adds or removes two flux quanta at $\nu=1$, the spin polarization drops from maximum polarization of $N_e/2$ to zero at very low temperatures, thereby indicating the existence of skyrmionic excitations. At nonzero temperatures, these filling factors have features of spin-singlet states.[8] The specific-heat results at $\nu=8/6$ and $\nu=8/10$ (Fig. 4) are however, basically featureless (except near $T=0$ where there is a sharp peak, albeit small) with no indication of any sharp peak that is present at $\nu=1$ for $T \sim 0.1e^2/\epsilon l_0$. The specific heat results for $\nu=\frac{8}{6}$ and $\nu=\frac{8}{10}$ show similarities with those of $\nu=\frac{2}{3}$ and $\nu=\frac{2}{5}$, respectively. The only noticeable feature for $\nu=\frac{8}{7}$ and $\nu=\frac{8}{9}$ (Fig. 5) is the hump at $T \approx 0.1$ which develops into a peak as one arrives at $\nu=1$. These two fractions should correspond to single-flip excitations rather than a spin texture[5] as discussed above. The results are again totally independent of the g factor. One possible explanation of the g-factor independence could be that the values of specific heat at these filling factors are very small and therefore there cannot be any large variation when some parameters are varied.

In conclusion, the specific heat versus temperature results at various filling factors indicate that there is a sharp peak at low temperatures whose height depends on the energy gap as well as the g factor. At $\nu=1$, the peak shifts to higher temperatures while some structures still linger on at low temperatures. At and around $\nu=1$ the results are practically g-factor independent.

[1] F. C. Zhang and T. Chakraborty, Phys. Rev. B **34**, 7076 (1986).

[2] S. E. Barrett, G. Dabbagh, L. N. Pfeiffer, K. W. West, and R. Tycko, Phys. Rev. Lett. **74**, 5112 (1995); S. E. Barrett, R. Tycko, L. N. Pfeiffer, and K. W. West, *ibid.* **72**, 1368 (1994); R. Tycko, S. E. Barrett, G. Dabbagh, L. N. Pfeiffer, and K. W. West, Science **268** 1460 (1995).

[3] A. Schmeller *et al.*, Phys. Rev. Lett. **75**, 4290 (1995); E. H. Aifer *et al.*, *ibid.* **76**, 680 (1996); M. J. Manfra *et al.* (unpublished).

[4] S. L. Sondhi *et al.*, Phys. Rev. B **47**, 16 419 (1993); see also H. A. Fertig *et al.*, *ibid.* **50**, 11 018 (1994); K. Moon *et al.*, *ibid.* **51**, 5138 (1995).

[5] T. Chakraborty, P. Pietiläinen, and R. Shankar (unpublished).

[6] D. C. Tsui, H. L. Störmer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982); H. L. Störmer, Physica B **177**, 401 (1992); J. P. Eisenstein and H. L. Störmer, Science **248**, 1510 (1990).

[7] T. Chakraborty and P. Pietiläinen, *The Quantum Hall Effects: Integral and Fractional*, 2nd ed. (Springer, New York, 1995), second edition; T. Chakraborty, in *Handbook on Semiconductors*, edited by P. T. Landsberg (North-Holland, Amsterdam, 1992), Vol. 1, Chap. 17.

[8] T. Chakraborty and P. Pietiläinen, Phys. Rev. Lett. **76**, 4018 (1996).

[9] B. I. Halperin, Helv. Phys. Acta **56**, 76 (1983); T. Chakraborty and F. C. Zhang, Phys. Rev. B **29**, 7032 (1984); F. C. Zhang and T. Chakraborty, *ibid.* **30**, 7320 (1984); T. Chakraborty, P. Pietiläinen, and F. C. Zhang, Phys. Rev. Lett. **57**, 130 (1986).

[10] A. G. Green, I. I. Kogan, and A. M. Tsvelik (unpublished); L. Brey *et al.*, Phys. Rev. Lett. **75**, 2562 (1995).

[11] V. Bayot *et al.*, Phys. Rev. Lett. **76**, 4584 (1996).

[12] E. Gornik *et al.*, Phys. Rev. Lett. **54**, 1820 (1985).

[13] N. G. Morawicz *et al.*, Semicond. Sci. Technol. **8**, 333 (1993); S. Holmes *et al.*, *ibid.* **9**, 1549 (1994); R. J. Nicholas (private communication).

[14] J. P. Eisenstein *et al.*, Phys. Rev. Lett. **62**, 1540 (1989); R. G. Clark *et al.*, *ibid.* **62**, 1536 (1989); J. E. Furneaux, D. A. Syphers, and A. G. Swanson, *ibid.* **63**, 1098 (1989); A. G. Davis *et al.*, Phys. Rev. B **44**, 13 128 (1991); L. W. Engel *et al.*, *ibid.* **45**, 3418 (1992); T. Sajoto *et al.*, *ibid.* **41**, 8449 (1990).

[15] D. Yoshioka, J. Phys. Soc. Jpn. **56**, 1301 (1987); note that here C in Eq. (2) is actually C/β.