Electron correlations in antidot arrays in a magnetic field

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A model for periodic array of scatterers in a two-dimensional electron gas (antidots) subjected to a strong perpendicular magnetic field is presented and its influence on the correlated states of the electrons is studied. We observe unique interaction- and disorder-driven spin transitions in the system. For a Gaussian form of antidot potential, the ground state at $\frac{1}{3}$-filled lowest Landau level gradually transforms from a fully spin-polarized state to a spin-partially polarized and eventually to a spin-unpolarized state as the potential strength and width are increased. The signature of those transitions is also evident in the lowest-energy spin and charge excitations.

Investigations of the electron states in quantum-confined systems like quantum dots,[1–5] quantum rings,[6] and various other mesoscopic systems in recent years have revealed a wealth of information on the role of the confinement potential, electron correlations, etc. in low-dimensional electron systems. The purpose of this paper is to present a model calculation of one other such interesting system, which may be considered the reverse of the quantum dots, viz., antidot arrays,[7,8] in the extreme quantum limit. The antidot arrays were first created by Weiss *et al.*[7] by imposing (lithographically) a periodic array of strong scatterers upon an otherwise defect-free two-dimensional electron gas (2DEG). This is normally achieved by punching holes at a regular interval in a high-mobility 2DEG. Transport measurements on these systems[7] have shown interesting magnetoresistance oscillations at low magnetic fields. The periodicity of these oscillations was found to correspond to the condition that the cyclotron orbit radius is an integer multiple of the modulation period. Many other peculiarities were also observed in Hall-effect[7] and far-infrared magnetospectroscopy[8] measurements. Theoretical attributes to these observed effects have been the classical nonchaotic[9] and chaotic[10] electron dynamics and Landau band quantization due to a weak periodic 1D potential.[11] Studies of antidot arrays have recently taken a very interesting turn with the observation of the fractional quantum Hall effect (FQHE) in antidot arrays.[12] This effect, first discovered in a 2DEG,[13] is entirely due to electron correlations.[14] It arises due to the formation of an energy gap separating the ground state, which is a uniform-density liquid state, and the quasiparticle excitations.[14,15] The energy gaps are present because of the incompressibility of the electron system at certain fractional Landau level fillings. At these densities, there are positive discontinuities in the chemical potential that indicate incompressibility of the ground state and are a measure of the energy gap.[14,15] The FQHE in a pure 2DEG has been studied quite exhaustively[15] since its discovery and in the light of the recent experiment mentioned above, it is important to find out how the FQHE states are influenced by the antidot arrays as compared to the case of a pure 2DEG.

It should be pointed out that the original motivation behind searching for the FQHE in antidot arrays[12] was primarily to look for a signature of the presence of the ''Chern-Simons gauge field'' particles.[16] In the mean-field approximation, these objects are expected to behave as noninteracting fermions in a magnetic field (except at half-filled Landau level where the magnetic field is exactly canceled by the so-called gauge field) and should therefore have a cyclotron radius (effective) at the prominent odd-denominator FQHE states. If that cyclotron radius matches with the modulation period one would expect oscillations in magnetoresistance. However, since in what follows we focus on the FQHE states at 1/3-filled lowest Landau level, the Laughlin state is identical to the state of gauge-transformed particles and we need to consider only the system of two-dimensional electrons in the presence of antidot arrays. We should point out that the observed fractions in antidot arrays[12] did not include 1/3.

In the case of a pure 2DEG one begins with two-dimensional electrons in a periodic rectangular geometry that is a well-established method for accurate evaluation of the FQHE states.[15,17] Accordingly, we consider a rectangular cell containing N_e number of electrons. We ignore for simplicity the Landau-level mixing, and impose periodic boundary conditions such that the cell contains an integer number N_s of flux quanta. We also consider the electrons to be in the lowest Landau level. In the present case of antidot arrays, the rectangular cell now has, in addition, a static antidot in the middle of the cell. The antidot potential, just like the Coulomb interaction, is periodically repeated (with period $\sqrt{2\pi N_s}\ell_0$, for the square cell considered here, where $\ell_0^2=\hbar c/eB$ is the magnetic length) when the periodic boundary condition is imposed in both directions of the two-dimensional plane. The two-body terms $u_{j_1j_2j_3j_4}$ in the Hamiltonian

$$\mathscr{H}=\sum_{j_1,j_2} t_{j_1j_2}a^\dagger_{j_1}a_{j_2}+\sum_{j_1,j_2,j_3,j_4} u_{j_1j_2j_3j_4}a^\dagger_{j_1}a^\dagger_{j_2}a_{j_3}a_{j_4}$$

to be diagonalized are the matrix elements of the Coulomb potential described earlier.[15,18] The periodically repeated antidots interact with an electron at $\mathbf{r}$ via the potential

$$V(\mathbf{r})=\sum_{k,l} V^{\mathrm{antidot}}(\mathbf{R}+ka\hat{\mathbf{x}}+lb\hat{\mathbf{y}}-\mathbf{r}),$$

where $\mathbf{R}=(X,Y)$ is the position of the antidot within the cell of size $a\times b$. Defining the Fourier transform of the antidot potential as

$$\tilde{V}^{\mathrm{antidot}}(\mathbf{q})=\frac{1}{ab}\int V^{\mathrm{antidot}}(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}d\mathbf{r}$$

and denoting by λ the aspect ratio a/b the one-body matrix elements in the Hamiltonian can be written in the lowest Landau level in the form

$$t_{j_1j_2}=\sum_{k,l} e^{i\sqrt{2\pi/\ell_0^2N_s\lambda}Xk}e^{i\sqrt{2\pi\lambda/\ell_0^2N_s}Y(j_1-j_2+N_sl)}\tilde{V}^{\mathrm{antidot}}\left(\sqrt{\frac{2\pi}{\ell_0^2N_s\lambda}}[k^2+\lambda^2(j_1-j_2+N_sl)^2]^{1/2}\right)$$
$$\times e^{i(\pi/N_s)(j_1+j_2-N_sl)k}e^{-(\pi/2N_s\lambda)[k^2+\lambda^2(j_1-j_2+N_sl)^2]}.$$

In what follows, we use a Gaussian form of the potential for the scatterers:

$$V^{\mathrm{antidot}}(\mathbf{r})=V_0e^{-(\mathbf{r}-\mathbf{R})^2/d^2},$$

where V_0 (same units as energy, $e^2/\epsilon\ell_0$, where ϵ is the background dielectric constant) is the potential strength, d (in units of magnetic length). In the limit $d\to 0$, one gets the δ-function potential, which was considered earlier by other authors[19] within the Hartree approach and is supposed to be a good approximation in the case of a steep potential of the scatterer. As we shall see below, our choice is better in the magnetic field regime where electron correlations are dominant. There are also other choices available in the literature such as the product of cosine functions, but our model, where we impose periodicity of the antidot potential explicitly, should effectively be the same as that choice. Our results indicate unique spin transitions at the 1/3-filled lowest Landau level, which is known to be fully spin polarized in the absence of antidot arrays.

Before we present the results of our present work, let us briefly recapitulate what we know about the 1/3 filling of the lowest Landau level, studied earlier in this model. The ground-state energy obtained in the finite-size systems compares extremely well with the many-body calculations.[15] The energy gap and the elementary excitations are also well described by the present model for the pure 2DEG in the FQHE regime and are in good agreement with the many-electron results. It has been established theoretically[15,17,20] as well as experimentally[21] that in the limit of low magnetic fields, several filling fractions tend to have spin-reversed states. However, the state at 1/3-filled lowest Landau level remains fully spin polarized, even in the limit of vanishing Zeeman energy.[15,17]

The results for the ground-state energy of the antidot system are shown in Fig. 1 for (a) $d=0.5$, (b) 1.0, and (c) 1.5. For $V_0=0$, we recover the earlier result of a pure 2DEG,[15,18] but as V_0 is increased, the ground-state energy increases monotonically. For a repulsive scattering center an increase in energy is, of course, expected. The important result here is that as V_0 increases the lowest-energy state no longer remains spin polarized, but gradually transforms into a spin-partially-polarized state ($S=1$) and then to a spin-unpolarized state $S=0$, where S is the total spin of the four-electron system considered here. These spin transitions can be explained as follows: In the absence of electron-electron interactions, but in the presence of antidot potentials, the degeneracies of the states of the noninteracting systems are lifted and the system is in the spin-unpolarized state. On the other hand, in the absence of antidot potentials but for the interacting systems, the ground state is fully spin polarized as discussed above. Therefore, for strong antidot potentials (large values of V_0), the ground state is still unpolarized, while for moderate to weak antidot potentials the electron-electron interaction has the tendency to polarize the ground state.

The spin transitions also depend on the width of the Gaussian potential (Fig. 1), which understandably works in a similar way to the V_0 (i.e., the effect is dominant when d is increased). We also find that in the region of V_0 where spin transitions take place, the spin of the state is not well defined because there the spin states are degenerate. Away from those regions, the spin of the ground state is well defined. Interestingly, similar studies with a δ-function potential revealed that the effect of antidot potentials is insignificant. This is in agreement with our findings that the spin transitions take place only for *large* values of d. A numerical study of the effect of a δ-function potential in a 2DEG was reported earlier by Rezayi and Haldane.[22] For a six-electron system in spherical geometry, they found that the Laughlin ground state is stable regardless of the potential strength.

The stability of the spin-reversed ground states in Fig. 1 depends crucially on the energy gaps, which are different for different spin states. The spin-reversed excitations have been observed earlier in the activation energy measurements.[21] We have calculated the quasiparticle-quasihole energy gap from the discontinuity of chemical potential at the ground-state filling factor.[17,23] The results for the lowest-energy excitations are presented in Fig. 2, where we have included the contribution due to the Zeeman energy. The results are for $d=1.0$ and for various values of the potential strength V_0. For $V_0=0$, the earlier results are recovered,[17] where, for low magnetic fields, the spin-reversed quasiparticle and spin-polarized quasihole pair have the lowest energy and beyond a crossover point ($\sim$12 T), the fully spin polarized

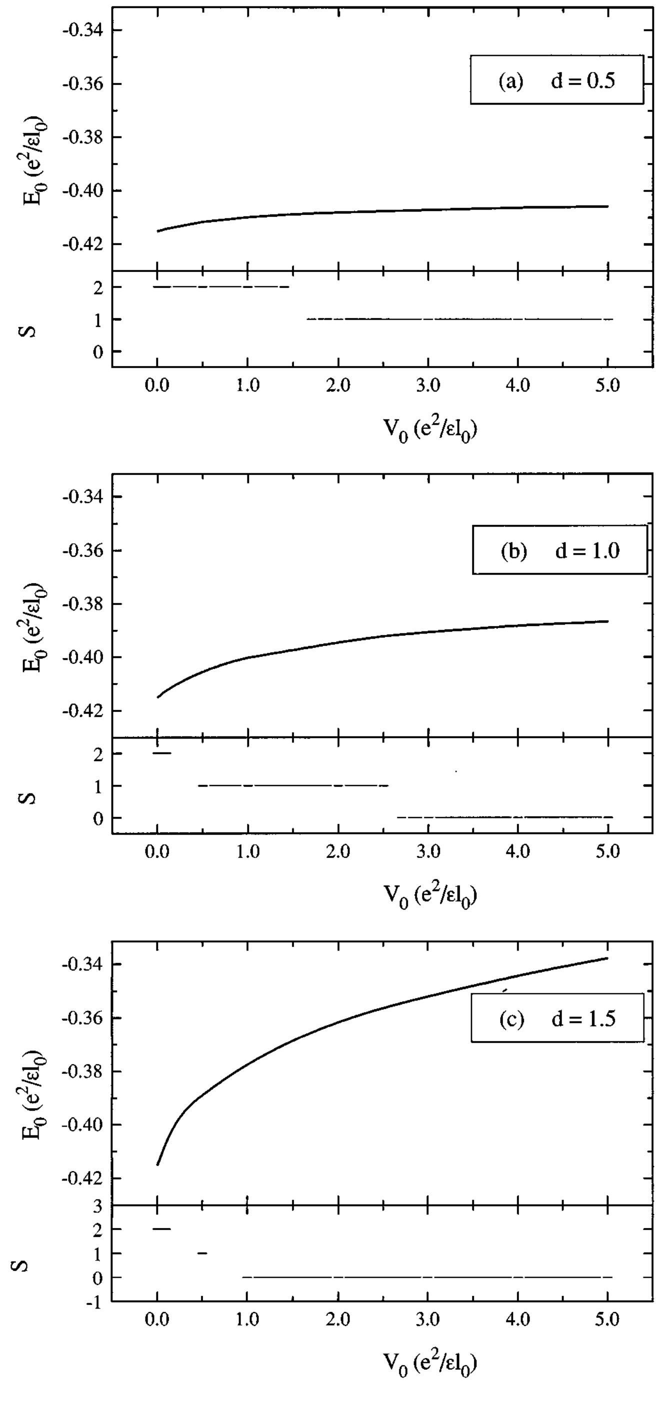

FIG. 1. Ground-state energy (per particle) and the total spin S of 1/3-filled lowest Landau level as a function of the antidot potential strength V_0 and width $d/\ell_0=0.5$ (a), 1.0 (b), and 1.5 (c).

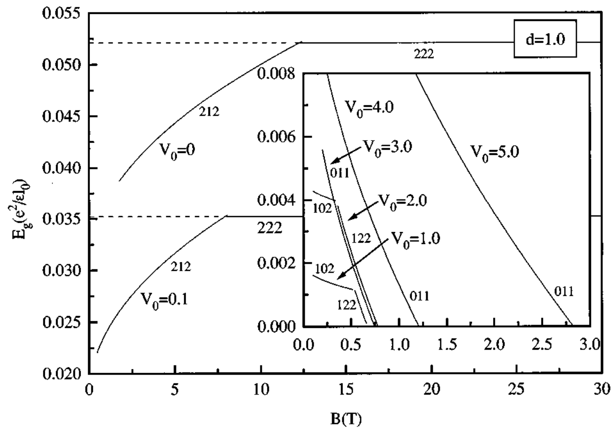

FIG. 2. The energy gap (units of $e^2/\epsilon\ell_0$) as a function of the magnetic field B (Tesla) for various values of the potential strength V_0 (units of energy). The total spin for the ground state, the quasiparticle, and the quasihole (S_G, S_p, S_h) at a given V_0 are also presented. For example, for $V_0=0$ and 0.1, the ground-state spin has $S=2$ (four-electron system), and the lowest-energy gap corresponds to spin-reversed quasiparticle– $(S=1)$ spin-polarized-quasihole $(S=2)$ pair up to a crossover point, beyond which the gap is due to fully spin-polarized-quasiparticle– $(S=2)$ quasihole $(S=2)$ pair. The spin-reversed excitations in the low-field regime are given in the inset.

quasiparticle-quasihole gap (Laughlin gap) has the lowest energy. For $V_0=0.1$, we have a similar situation with a crossover point of ~ 8 T, except that the energy gap is lowered. Such a lowering of the gap is expected in the presence of any impurity potentials. Interestingly, we find that the energy gap vanishes at $V_0=0.5$. This is the region of V_0 where the spin of the ground state changes from being fully spin polarized to a spin partially polarized state (Fig. 1). As V_0 is increased further, various spin-reversed excitations start to have the lowest energy and they all decrease with increasing magnetic field. For a spin-reversed ground state (at other filling fractions) this behavior has been established earlier theoretically.[23] The energy gaps for $V_0=2.0$ and 3.0 are very close and in this region of V_0 there is one other spin transition where the ground state changes from spin-partially-polarized to spin-unpolarized $(S=0)$ states. All of these spin-reversed excitation energies decrease with increasing magnetic field.

Let us now discuss some of the approximations involved in our model and their possible implications. First, it should be pointed out that in our model, only V_0, d, and the filling factor are the independent variables. The periodicity of the antidot potentials is inversely proportional to the magnetic field B. Although it does not affect our results, this condition need not be present in a better model for antidot arrays. As shown in Fig. 2, in the high-magnetic-field region, only small values of V_0 persist and their sole effect is to reduce the gap. Here the nature of the ground state and excitations is the same as that for $V_0=0$. One should note that although the system studied here is rather small, it has already been established earlier[15,17] that the ground-state energy obtained for the $V_0=0$ case is almost identical to the many-electron system results. Therefore, the results presented above are perhaps reliable representatives of a many-electron system, at least in the case of weak antidot potentials. Of course, in a more realistic situation we need to include several other corrections arising from the finite-thickness effects, Landau-level mixing, etc. The latter effect is known to be quite important at low fields. Earlier work on the effect of repulsive impurities on the FQHE states considered only the lowest Landau level even in the presence of very strong impurities.[22] We would like to mention that, since for a given number of electrons and the filling fraction, the area of the cell is directly related to the magnetic length, d cannot be made arbitrarily large compared to the magnetic length. In fact, for $d \gg \ell_0$, the energy rises very rapidly with increasing V_0 and the spin transitions are somewhat anomalous at large V_0. Our results remain quite stable for a large range of $d \sim 0 - 2\ell_0$ and V_0, which might provide an indication that the Landau-level mixing may not substantially change the result. We expect that the approximations discussed above affect primarily the crossover points in spin transitions. Further work is needed to improve upon these approximations in order to study the actual physical systems.

In summary, we have studied the effect of antidot arrays on the FQHE states at 1/3-filled lowest Landau level. In contrast to the pure 2DEG where the ground state at this filling fraction is most stable and is fully spin polarized, the ground state in our model of antidot arrays changes to spin-reversed states and various spin-reversed excitations are favored as a function of the magnetic field in the presence of antidot arrays. More theoretical work is needed to improve the model proposed here. Tilted-field experiments on antidot arrays at 5/3 filling factor,[21] for a suitable choice of antidot parameters, might also be useful to explore these spin transitions.

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