TAKING STOCK OF THE QUANTUM HALL EFFECTS: THIRTY YEARS ON

BY TAPASH CHAKRABORTY AND KLAUS VON KLITZING

The quantum Hall effects (QHEs) are now about 30 years old. The date of birth of the original effect was duly recorded as February 5, 1980 at around 2 a.m. during an experiment at the High Magnetic Field Laboratory in Grenoble, France [1], while its fractional counterpart was discovered on October 7, 1981 at the Francis Bitter Magnet Laboratory, Massachusetts, USA [2]. The objectives of the Grenoble experiment were to answer some of the fundamental questions in the electronic transport of silicon field effect transistors, such as, how can one improve the mobility of these devices, or what are the dominant scattering processes in the dynamics of electrons at the nanometer scale at the interface between silicon and silicon dioxide. Specially designed devices (Hall devices), such as the one shown in Fig. 1, which allow direct measurement of the resistivity tensor were considered. Low temperatures (typically 4.2 K) were used so that the scattering processes involving electron-phonon interactions were suppressed. Application of a magnetic field was an already established method to gather information about the microscopic details of a semiconductor [3].

It was known since 1966 that electrons accumulated at the surface of a silicon crystal by a positive voltage at the gate (i.e., a metal plate parallel to the surface) form a two-dimensional electron gas (2DEG) [4]. The energy for electron motion perpendicular to the surface is quantized (dimensional quantization), while in the presence of a perpendicular magnetic field, the motion of electrons in the plane is also quantized (Landau quantization). In the ideal case, the energy spectrum of a 2DEG in strong magnetic fields consists of discrete Landau energy levels (normally broadened due to impurity scattering) with an equal energy spacing. The QHE is observed if the Fermi energy lies in the energy gap and if the temperature is so low that excitations across the gap are not possible.
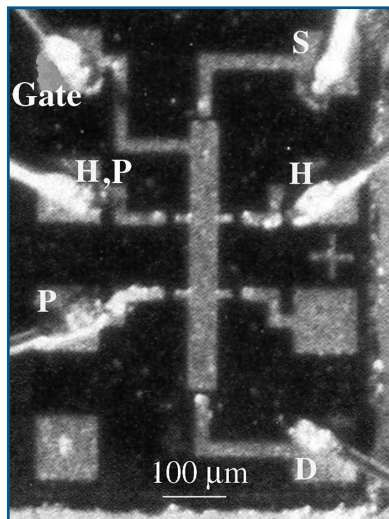

Fig. 1 A typical silicon MOSFET device used for measurements of the Hall effect.

The experimental results that led to the discovery of the QHE are shown in Fig. 2. The blue curve is the electrical resistance of the silicon field effect transistor as a function of the gate voltage. Since the electron concentration increases linearly with increasing gate voltage, the electrical resistance decreases monotonically. Further, the Hall voltage (if a constant magnetic field of, say, 19.8 Tesla is applied) decreases with increasing gate voltage, since it is inversely proportional to the electron concentration. The black curve shows the Hall resistance with a clear plateau at a gate voltage where the longitudinal resistance vanishes. The uniqueness of these findings was that the Hall plateau can be expressed with high precision as $R_H = \rho_{xy} = h/ne^2$ (h is the Planck constant, e is the elementary charge, and n is the number of fully occupied Landau levels).

The epoch making discovery was the 'exact quantization' of Hall resistance to a fundamental value of $h/e^2 = 25812.807...$ Ohm that is incredibly robust. This value is independent of the material, geometry and microscopic details of the semiconductor [5,6]. Measurements of the Hall conductance have been found to be integral multiples of e^2/h to nearly one part in a billion. This has facilitated
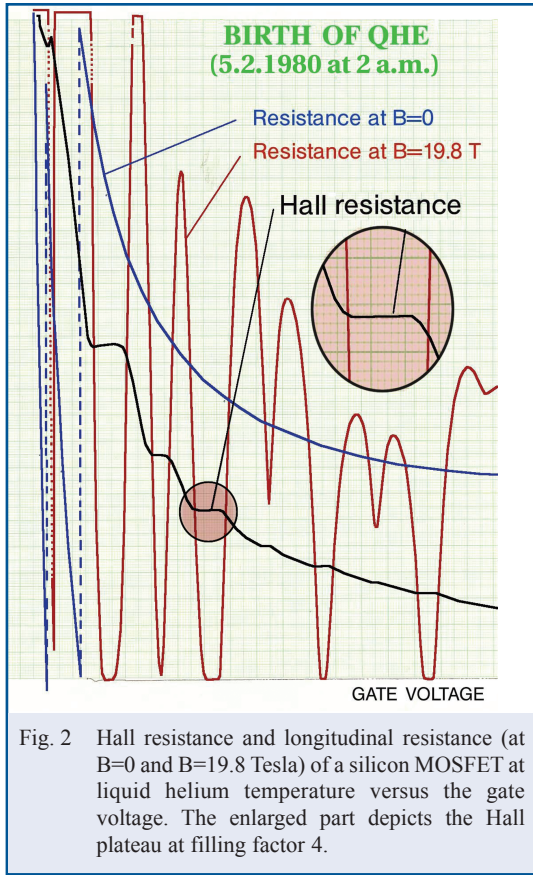
SUMMARY

The quantum Hall effects, discovered about thirty years ago have remained one of the most spectacular discoveries in condensed matter physics in the past century. Those discoveries triggered huge expansion in the field of low-dimensinal electronic systems, the area grew at an unprecedented rate and continues to expand. Novel and challenging observations, be it theoretical or experimental, have been reported since then on a regular basis. Additionally, the effects have inspired physicists to find analogous situations in far-flung fields as disparate as string theory or black hole physics.

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BIRTH OF QHE
(5.2.1980 at 2 a.m.)

Resistance at B=0
Resistance at B=19.8 T

Hall resistance

GATE VOLTAGE

Fig. 2   Hall resistance and longitudinal resistance (at B=0 and B=19.8 Tesla) of a silicon MOSFET at liquid helium temperature versus the gate voltage. The enlarged part depicts the Hall plateau at filling factor 4.

TABLE 1
Summary of high precision data for the quantized Hall resistance until 1988 which led to the fixed value of 25818.807 Ohm recommended as a reference standard for all resistance calibrations after January 1, 1990.

(Hall-) Resistance	R_H
PRL 45, 494 (1980)	25 812.68 (8) Ω
BIPM (France)	25 812.809 (3) Ω
PTB (Germany)	25 812.802 (3) Ω
ETL (Japan)	25 812.804 (8) Ω
VSL (The Netherlands)	25 812.802 (5) Ω
NRC (Canada)	25 812.814 (6) Ω
EAM (Switzerland)	25 812.809 (4) Ω
NBS (USA)	25 812.810 (2) Ω
NPL (UK)	25 812.811 (2) Ω
1.1. 1990	**25 812.80700 Ω**

and the resistivity vanishes at sufficiently low temperatures. Due to the presence of impurities in the sample, there are localized states that can be filled but they do not contribute to the conductivity. The remarkable precision of Hall quantization which is oblivious to the material characteristics, impurities, and different geometries, was attributed to the subtle manifestation of the principle of gauge invariance [11].

The QHEs are characterized by the filling factor ν (ν = total number of electrons/number of flux quanta passing through the sample=$n_s \Phi_0/B$, where n_s is the carrier density, $\Phi_0 = h/e$ is the flux quantum, and B is the magnetic field). The integer QHE (IQHE) corresponds to ν being a simple integer. In 1982, Tsui, Störmer and Gossard discovered [12] that in devices with much less disorder, the QHE appears with ν having rational fractional values (Fig. 3). This fractional QHE (FQHE) arises purely due to electron-electron interactions. The original observation of a FQHE at ν = ⅓, ⅔ was superbly described by Laughlin [13,14] who introduced a many-body wavefunction that was based on an inspired guess. It was confirmed subsequently by various numerical studies [14]. The novelty of the Laughlin state was that, it described an incompressible state of the electron liquid whose low-energy excitations are fractionally-charged quasi-particles and quasiholes [13,14], not unlike quarks [15]. They also obey fractional statistics [16,17], which means that the interchange of two such objects multiplies the wavefunction by a phase which may take any value ("anyons") instead of just +1 (bosons) or -1 (fermions). The QHE depends crucially on the existence of a gap in the excitation spectrum. In the case of the IQHE, the gaps are the single-particle type kinetic energy gap between Landau levels, the spin gap, and the valley gap in Si. The gap in the FQHE on the other hand, arises purely from electron-electron interactions. Electron spin was also found to play an important role in the FQHE ground state and excitations (*spin-reversed quasiparticles*). [14,18].

When better quality samples started revealing more and more filling fractions [19], it soon became clear that Laughlin's theory

the definition of a new practical standard for electrical resistance based on the resistance quantum given by the *von Klitzing constant* R_K = 25 812.807449 ± 0.000086 Ohm. Since 1990, a fixed value of R_{K-90} = 25 812.807 Ohm has been adopted internationally as a standard for resistance calibration [7,8] (Table 1). Recent discussions about a new definition of the units of measurements based on fundamental constants led to the recommendation [9] to fix not only the value of the velocity of light but also the values of the Boltzmann constant, the Avogadro constant, the Planck constant and the elementary charge, which automatically means that also the von Klitzing constant will be a fixed number within such a new system of international units (SI system). Within the present SI system the QHE provides an extremely precise independent determination of the fine structure constant which is "one of the fundamental constants of nature characterizing a whole range of physics including elementary particle, atomic, mesoscopic and macroscopic systems" [10].

In a simple minded picture to explain the observed step-like behaviour of resistivity, one could begin with the non-interacting electron system in a perpendicular magnetic field. The Landau levels (LLs) are known to be highly degenerate, with degeneracy defined as the number of states per unit area, eB/h. As each of the degenerate states is filled, fewer states remain unoccupied and the resistivity decreases. Once the LL is completely filled, there remains a gap to the next energy LL
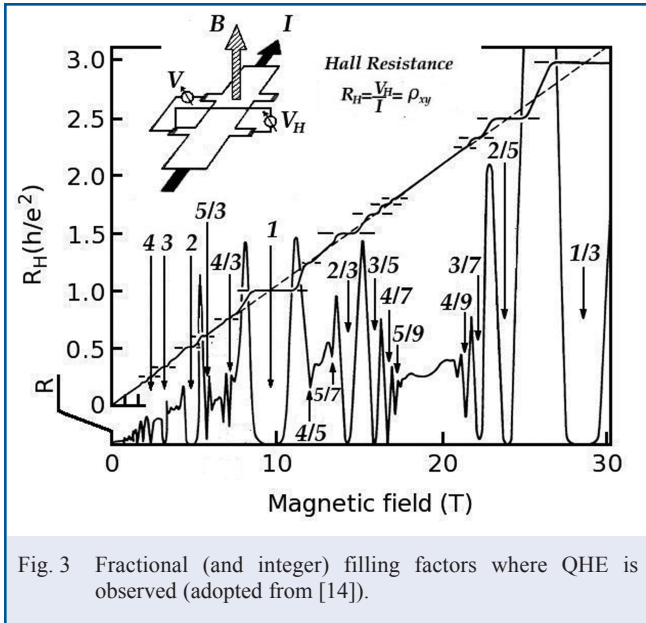
Fig. 3   Fractional (and integer) filling factors where QHE is observed (adopted from [14]).

was inadequate to describe those higher-order filling factors. In the composite fermion picture [20], trial wave functions for ground states and excitations were introduced that correctly predict the most prominent observed FQH states. In contrast to the observation of the FQHE at odd-denominator filling factors, for most even-denominator fractions, and in particular at $\nu = \frac{1}{2}$, no effect has been observed. According to the fermion-Chern-Simon picture [21], the system at $\nu = \frac{1}{2}$ is compressible and is not expected to display any QHE. Exactly at $\nu = \frac{1}{2}$ and within the mean-field approximation, the actual electron system becomes equivalent to a gas of fermions in zero magnetic field. There are experimental indications in support of this idea [22]. One surprising discovery in the FQHE was the strongly correlated electronic state at the half-filled second orbital ($\nu = \frac{5}{2}$) Landau level [23]. This observation surely did not tally with the existing theories for all other FQH states. It has been proposed theoretically that this filling fraction corresponds to degenerate ground states and fractionally-charged non-Abelian quasiparticles [24]. Inter-change of two quasiparticles of this type would shift the system between orthogonal ground states. This state is proposed to have properties appropriate for fault tolerant quantum computation [25]. The non-Abelian nature of the 5/2 state is yet to be directly confirmed by experiments. However, in recent experiments [26], the quasiparticle charge was determined to be $e_{5/2}^* = e/4$, in agreement with the proposed paired FQH state at $\nu = 5/2$ [24]. This property is different from all other observed FQH states.

The theory of Laughlin which introduced several novel concepts in correlated quantum fluids, inspired analogous effects in other subfields of physics. The QHE was generalized to four dimensions [27] in order to study the "interplay between quantum correlations and dimensionality in strongly correlated systems". Two-dimensional electron systems were modeled by strings interacting with D-branes [28]. Here the fractionally-charged quasiparticles and composite fermions were described in the language of string theory. An interesting analogy between the QHE and black hole has been reported, and in particular, the edge properties of a QHE system have been used to model black hole physics from the point of view of an external observer [29]. Important developments of the QHE have also taken place from the field theoretical point of view [30].

There has been a lot of excitement recently about a new state of matter, the topological insulator [31], which has a bulk insulating gap, but gapless electronic states (topologically protected against scattering by time-reversal symmetry) on the sample boundary. In two-dimensions the topolgical insulator is a quantum spin Hall system, somewhat akin to the IQH state. Finally, the QHE has played a crucial role in a novel two-dimensional system discovered recently, graphene. The latter is a single-atom-thick layer of carbon atoms arranged in a hexagonal lattice with remarkable attributes [32,33]. Charge carriers in graphene behave as massless Dirac fermions, whose dynamics is governed by the Dirac equation. The quantization condition of Hall resistance in graphene is different from that in a conventional 2DEG by a half-integer shift [34], and has been reliably measured even at room temperature [35] which is attributed to large cyclotron gaps of Dirac fermions in graphene. The fractional QHE in graphene was studied theoretically [36] and was subsequently observed [37].

The QHEs are truly remarkable macroscopic quantum phenomena observed in two-dimensional electron systems. Discovery of IQHE was clearly just the beginning of a long sequence of discoveries in this field. Experiments on the QHE continue to reveal a countless number of often unexpected and challenging results. Theorists have been busy developing novel concepts in order to deal with these phenomena. What is happening now in the field of low-dimensional electron systems is nothing short of a revolution that shows no signs of running out of steam in the immediate future.

ACKNOWLEDGEMENTS

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