CORRELATIONS IN THE LAYERED ELECTRON-HOLE LIQUID

Tapash Chakraborty and C.E.Campbell
School of Physics and Astronomy
University of Minnesota,Minneapolis,MN 55455,USA.

Introduction.- At low temperatures and high carrier concentrations, the electrons and holes in semiconductors condense to form a high density metallic liquid.This electron-hole liquid(EHL)[1,2]is a collective state of electrons and holes,which is unique in many respects:It is the most metallic of metals,and the most quantum of fluids.The experimental results have indicated that the EHL has the characteristic properties of a liquid.Its volume is conserved,it has an equilibrium density etc. The EHL drops have a spherical shape with drop radius of 10^{-3}-10^{-4}cm, which is much larger than the exciton (bound electron-hole pairs) radius $a_x=10^{-5}$- 10^{-6}cm,which represents the characteristic interaction distance of the particles in the drop.Therefore,in calculating the bulk properties of the EHL,one can neglect the surface effects.Recently,a many-body variational approach[3,4]for two-component systems has been applied to study the ground-state properties of such a fascinating system.Some of the results obtained for the EHL are in very good agreement[5] with recent experiments.

The study of the EHL in two dimensions is also a very interesting problem for various reasons.Firstly,there are current experimental efforts[6] to obtain such systems.Secondly,all earlier works[7,8,10]were based on standard perturbative approaches[9].A major problem inherent in those methods,however,is the unphysical behavior of the short-range part of the correlation functions.Study of the inversion layer[9] indicated that the problem of negative correlation functions at small distances becomes accute in two-dimensions,which affects the ground-state properties significantly.

In the following,we shall investigate the two cases:a)the layered electron-hole liquid with variable interlayer separations,and b) systems with many-valley structure.

Layered electron-hole liquid.- We consider a model[11] where electrons and holes of finite two-dimensional density ρ,move in two different planes separated by a distance c.The tunneling of the particles between the planes in forbidden.This type of charge distribution is possible in two adjacent parts of a semiconductor by an electrical discharge.The unperturbed eigenstates in this model are

$$\Phi_{\vec{k},\sigma,m}(\vec{r},z) = A^{-\frac{1}{2}} a_\sigma e^{i\vec{k}.\vec{r}} \chi(z-m_i c) \tag{1}$$

where A is the normalization area,$\vec{k}$ is a two-dimensional vector,a_σ is the spin eigenfunction,m_i=0 and 1 for electrons and holes respectively. We consider only the limit in which the one-dimensional eigenstate $\chi(z)$ is arbitarily highly localized,$\chi^*(z)\chi(z) = \delta(z)$.The nondynamical correlations for the electrons or holes are

$$g_f(r) = [1-\tfrac{1}{2}(2J_1(k_f r)/k_f r)^2] \tag{2}$$

where k_f is the radius of a two-dimensional 'Fermi disk' and $J_1(x)$ is the Bessel function.The Fourier transform of (2) gives us the ideal gas structure function as,

$$S_f(k) = \begin{cases} \frac{2}{\pi} \sin^{-1}(k/2k_f) + \frac{k}{\pi k_f}[1-(k/2k_f)^2]^{\frac{1}{2}}, & k<2k_f \\ 1 & , k>2k_f \end{cases} \tag{3}$$

For a finite separation between the layers ($c\neq 0$),the electron-hole interaction is, $v_{eh}=e^2/|\vec{c}+\vec{r}_i-\vec{r}_j|$, $\vec{c} = (0,0,c)$ and $\vec{r}$ is a two dimensional vector.Introducing the two-dimensional Coulomb units,$a_x=\hbar^2/2\mu e^2$, $E_x=2\mu e^4/\hbar^2$ where $\mu=m_e/(1+\sigma)$ is the reduced mass, $\sigma=m_e/m_h$,the dimensionless variables $x=r/\bar{r}$,$q=k\bar{r}$ and $c_s=c/a_x$, $\bar{r}=a_x r_s$, the mean interaction energy in the variational approach[3,4] is

$$\varepsilon_{int} = \frac{1}{r_s} \int_0^\infty [g_{ee}(x)+g_{hh}(x)-2x\{c_s^2/r_s^2+x^2\}^{-\frac{1}{2}} g_{eh}(x)]dx \tag{4}$$

where $g_{\alpha\beta}(x)$ are the partial pair-correlation functions.Noticing that, $v_{eh}(q)=\frac{2\pi}{qr_s}\exp(-|qc_s/r_s|)$, ε_{int} is expressed in terms of the partial static-structure-functions as

$$\varepsilon_{int} = \frac{2c_s}{r_s^2} + \frac{1}{2r_s}\int_0^\infty [S_{ee}(q)+S_{hh}(q)-2S_{eh}(q)-2]dq$$
$$+ \frac{c_s}{2r_s^2}\int_0^\infty [2 - qc_s/r_s + \ldots]S_{eh}(q)q\,dq. \tag{5}$$

The first term is,as expected,the energy contribution from the electrostatic interaction between the planes of opposite charges.The groundstate energy in the Hartree-Fock approximation is given by

$$\varepsilon_{HF} = \frac{1}{r_s^2} - \frac{8\sqrt{2}}{3\pi}\frac{1}{r_s}. \tag{6}$$

Note the difference in the coefficient of the exchange energy,compared to Ref.7,because of our different choice of the 2D-units.

The basic equation for the pair-correlation functions $g_{\alpha\beta}(x)$,

α,β =e,h in our approach is[3,4];

$$[-\nabla^2 + v_{\alpha\beta}(x) + W^b_{\alpha\beta}(x) + W^f_{\alpha\beta}(x)]g^{\frac{1}{2}}_{\alpha\beta}(x)=0 \qquad (7)$$

where, $$v_{\alpha\beta}(x)=[\eta_{\alpha\beta}r_s/2x-(1-\delta_{\alpha\beta})r_s/\sqrt{x^2+c_s^2/r_s^2}]$$

$$\eta_{ee}=(1+\sigma),\ \eta_{hh}=(1+1/\sigma),\ \eta_{eh}=0.$$

The 'induced potentials' $W_{\alpha\beta}(x)$ are given in the earlier works[3,4],where the method of solving the above equations are also given.

Systems with many-valley structure.- So far,we have considered only the case of a single conduction band and a single valence band.In most cases,however,we have to take into account,the effect of the valley degeneracy on the ground-state results to be discussed below.Therefore,in the following,we will consider a case,where the conduction band has two minima and the valence band has a single maximum like in GaSe.The Hartree-Fock energy will now depend on the mass ratio as

$$\varepsilon_{HF}=\left(\frac{0.5+\sigma}{1+\sigma}\right)\frac{1}{r_s^2} - \frac{9.66}{3\pi}\frac{1}{r_s}. \qquad (8)$$

Earlier results in this multi-valley case[7] have indicated that the ground-state energy depends more strongly on σ,compared to the single valley case and the effect is more pronounced in two-dimensions.This is mainly due to the difference in the density of states between the two- and three-dimensional systems.The correlational energy is affected,in our scheme,primarily through $W^f_{\alpha\beta}(x)$,since the fermi momentum now is $k_f^2=2\pi\rho/n_v$,where n_v=2 for the electrons and 1 for the holes.

Results and Discussions.- In Figs.1 and 2,we have plotted the electr-

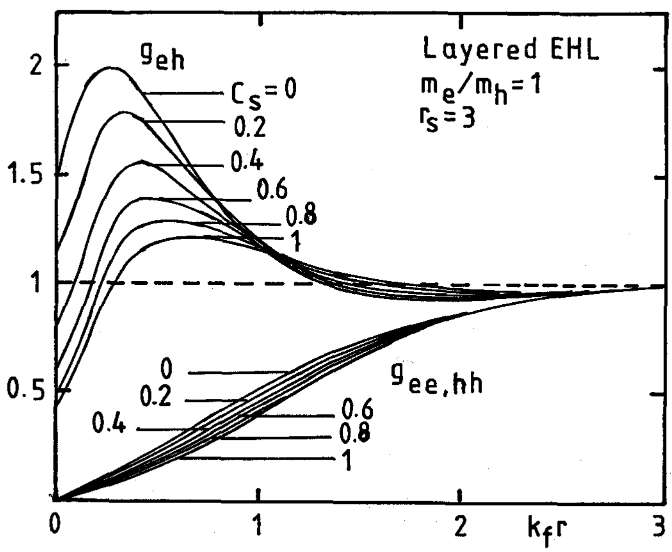

Fig.1: Pair-correlation functions $g_{ee}(r)$, $g_{hh}(r)$ and $g_{eh}(r)$ vs $k_f r$ for σ=1.

on-electron, hole-hole, and the electron-hole correlation functions as a function of $k_f r$, for various values of the dimensionless interlayer separation c_s, at r_s=3. For σ=1 (Fig.1), $g_{ee}(r)$ and $g_{hh}(r)$ are identical and they vary only slightly for different values of c_s. The effect of c_s is much stronger in $g_{eh}(r)$, which tends to show less structure as c_s is increased. For m_h/m_e=10 (Fig.2), there is more or less a similar pattern in

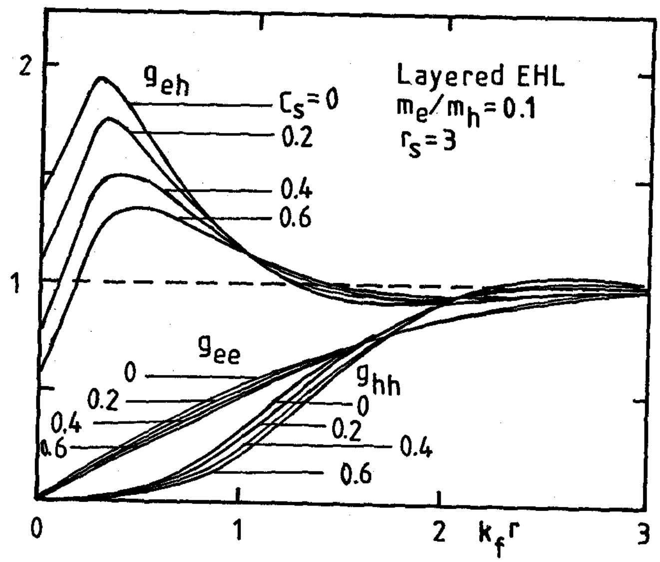

Fig.2: Same as in Fig.1, but for σ=0.1.

the distribution functions, except that the holes show stronger correlations among themselves. This behavior is reminiscent of the hole-hole distribution functions in three dimensions[3]. The noticeable difference in the e-h correlation functions, as compared to those in three dimensions is that the enhancement is much reduced in two dimensions. In calculating the enhanced density[5], it should however, be noted that the ratio of the enhanced density to the exciton density is,

$$\rho_{eh}/\rho_x = \begin{cases} 3g_{eh}(0)/4r_s^3 & ;3D \\ g_{eh}(0)/8r_s^2 & ;2D \end{cases} \tag{9}$$

In Fig.3, we have plotted the partial-static-structure functions, and $D(k)=S_{ee}(k)S_{hh}(k)-S^2_{eh}(k)$, as a function of k/k_f, for σ=1 and σ=0.1. All the functions rapidly approach to zero for small k. These functions were used to obtain the collective modes in the electron-hole liquid. They are obtained by generalizing the Bijl-Feynman equation for the two-component systems[12]

$$\varepsilon_{1,2}/\varepsilon_f = \frac{q'^2}{2D}[(\sigma S_{ee}+S_{hh})\pm\{(\sigma S_{ee}+S_{hh})^2-4\sigma D\}^{\frac{1}{2}}] \quad (10)$$

where $q'=k/k_f$ and ε_f is the electron fermi energy. In Fig.4, we have plotted the two branches of the excitation energies ε_1 and ε_2 in units of electron fermi energy as a function of k/k_f, for $\sigma=1$ and $\sigma=0.1$. The dashed lines correspond to the threshold energy for the onset of Landau damping due to excitations of particle-hole pairs of the type 1

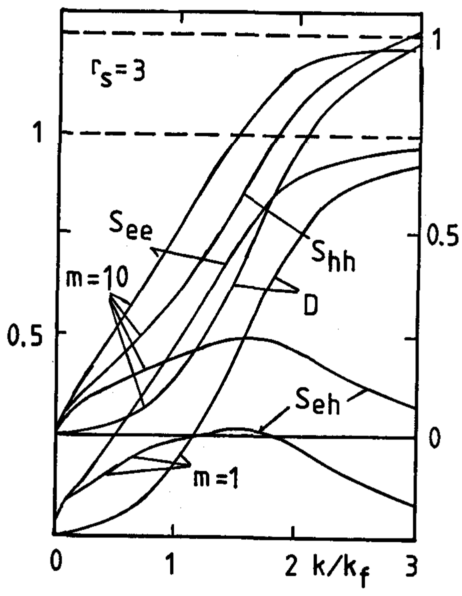

Fig.3: Static-structure functions $S_{ee}(k)$, $S_{hh}(k)$, $S_{eh}(k)$ and $D(k)$ as a function of k/k_f for $\sigma=1,0.1$.

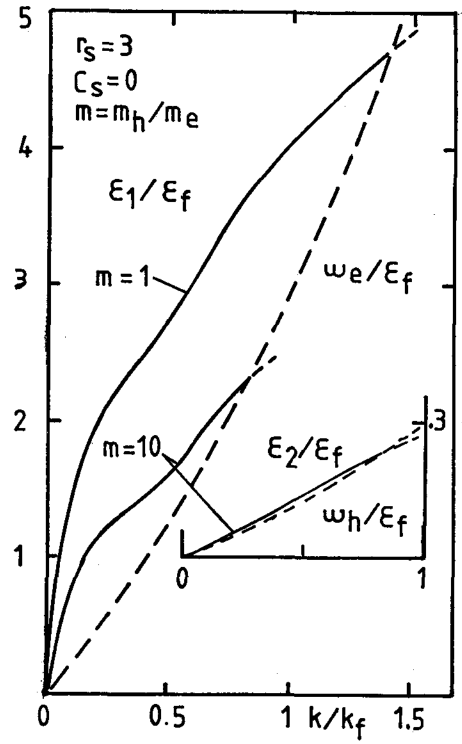

Fig.4: Dispersion relations of the two-dimensional plasmons in the EHL for m = 1,10, and the 'acoustic plasmon' mode for m=10, as inset.

$[\omega_e/\varepsilon_f=q'(q'+2)]$ and that of type 2 $[\omega_h/\varepsilon_f=q'(q'+2)\sigma]$. The plasmon mode rises sharply from zero with increasing k and has the characteristic plasma frequency[13] $\omega_p\approx[2\pi e^2\rho k/\mu]^{\frac{1}{2}}$. The 'acoustic plasmon' mode (or ionic sound mode)[14] exists only in the case where the holes are much heavier than the electrons[12,15]. This mode for $\sigma=0.1$ is drawn as inset in Fig.4.

Finally, in Fig.5, we have plotted the ground-state energy minimum as a function of the interlayer separation c_s,for different values of the electron-hole mass ratio.In the single valley case,the effect of the heavy holes is clearly insignificant.However,in the multi-valley case,

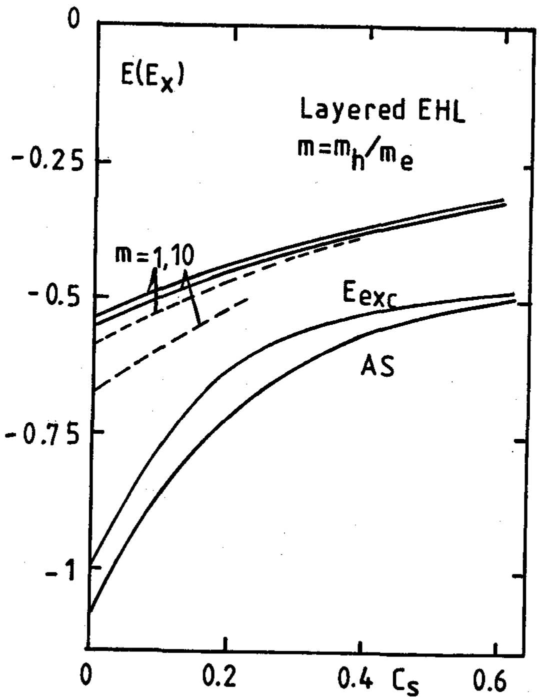

Fig.5: The ground-state energy minimum as a function of the dimensionless interlayer separation c_s.The solid lines are the single-valley case and the short-dashed lines are the many-valley results. The curve E_{exc} represents the exciton energy in the ground state and the curve 'AS' is the result of Ref.8.

we obtain a significant lowering of the energy.Comparing with the exciton energy in the ground state[8],we notice however,that the exciton state is clearly preferred.Nevertheless,a suitable choice of the electron-hole effective mass,the valley degeneracy and a finite seperation between the layers,might lead to the energy being lower than the exciton energy.Our result is apparently in contrast with the results of Ref.8 (curved marked AS),who obtained a lower energy for the EHL compared to the exciton energy E_{exc} for all values of c_s.However,the unphysical behavior of the correlation functions at small distances[8]and inadequate range in k,in their interpolation schemes,as pointed out in Ref.10, introduce large amount of uncertainty in those energy values.Qualitatively,results similar to Fig.5 were obtained in Refs.7 and 10,for c_s=0. More experimental efforts are undoubtedly needed to acieve a better understanding of this fascinating quantum liquid in two-dimensional systems.

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