Introduction.- At low temperatures and high carrier concentrations, the electrons and holes in semiconductors condense to form a high density metallic liquid. This electron-hole liquid (EHL) is a collective state of electrons and holes, which is unique in many respects: It is the most metallic of metals, and the most quantum of fluids. The experimental results have indicated that the EHL has the characteristic properties of a liquid. Its volume is conserved, it has an equilibrium density, etc. The EHL drops have a spherical shape with drop radius of \(10^{-3}-10^{-4}\) cm, which is much larger than the exciton (bound electron-hole pairs) radius \(a_x=10^{-5}-10^{-6}\) cm, which represents the characteristic interaction distance of the particles in the drop. Therefore, in calculating the bulk properties of the EHL, one can neglect the surface effects. Recently, a many-body variational approach for two-component systems has been applied to study the ground-state properties of such a fascinating system. Some of the results obtained for the EHL are in very good agreement with recent experiments.

The study of the EHL in two dimensions is also a very interesting problem for various reasons. Firstly, there are current experimental efforts to obtain such systems. Secondly, all earlier works were based on standard perturbative approaches. A major problem inherent in those methods, however, is the unphysical behavior of the short-range part of the correlation functions. Study of the inversion layer indicated that the problem of negative correlation functions at small distances becomes acute in two-dimensions, which affects the ground-state properties significantly.

In the following, we shall investigate the two cases: a) the layered electron-hole liquid with variable interlayer separations, and b) systems with many-valley structure.

Layered electron-hole liquid.- We consider a model where electrons and holes of finite two-dimensional density \(\rho\), move in two different planes separated by a distance \(c\). The tunneling of the particles between the planes is forbidden. This type of charge distribution is possible in
two adjacent parts of a semiconductor by an electrical discharge. The unperturbed eigenstates in this model are

\[ \Phi_{k,g,m}(r,z) = A^{-\frac{1}{2}} a_0 e^{i\mathbf{k}\cdot\mathbf{r}} \chi(z-m_i c) \]  

where \( A \) is the normalization area, \( \mathbf{k} \) is a two-dimensional vector, \( a_0 \) is the spin eigenfunction, \( m_i = 0 \) and 1 for electrons and holes respectively. We consider only the limit in which the one-dimensional eigenstate \( \chi(z) \) is arbitrarily highly localized, \( \chi^*(z)\chi(z) = \delta(z) \). The nondynamical correlations for the electrons or holes are

\[ g_\epsilon(r) = [1-\frac{1}{2}(2J_1(k_f r)/k_f r)^2] \]

where \( k_f \) is the radius of a two-dimensional 'Fermi disk' and \( J_1(x) \) is the Bessel function. The Fourier transform of (2) gives us the ideal gas structure function as,

\[ S_f(k) = \left\{ \begin{array}{ll} \frac{2}{\pi} \sin^{-1}(k/2k_f) + \frac{k}{\pi k_f} [1-(k/2k_f)^2]^{\frac{1}{2}}, & k < 2k_f \\ 1, & k > 2k_f \end{array} \right. \]

For a finite separation between the layers (\( c \neq 0 \)), the electron-hole interaction is, \( v_{eh} = e^2/|c+r_{-1} - r_j| \), \( c = (0,0,c) \) and \( r_j \) is a two-dimensional vector. Introducing the two-dimensional Coulomb units, \( a_x = \pi^2/2ue^2 \), \( E_x = 2ue^4/\pi^2 \) where \( u = m_e/(1+\sigma) \) is the reduced mass, \( \sigma = m_e/m_h \), the dimensionless variables \( x = r/r, q = kr \) and \( c_s = c/a_x, r = a_x r_s \), the mean interaction energy in the variational approach \(^3\text{,}^4\) is

\[ \epsilon_{\text{int}} = \frac{1}{r_s} \int_0^\infty [g_{ee}(x) + g_{hh}(x) - 2x(c_s^2/r_s^2 + x^2)^{-\frac{1}{2}} g_{eh}(x)] dx \]

where \( g_{ab}(x) \) are the partial pair-correlation functions. Noticing that, \( v_{eh}(q) = \frac{2\pi}{qr_s} \exp(-|qc_s/r_s|) \), \( \epsilon_{\text{int}} \) is expressed in terms of the partial static-structure-functions as

\[ \epsilon_{\text{int}} = \frac{2c_s}{r_s^2} + \frac{1}{2r_s} \int_0^\infty [S_{ee}(q) + S_{hh}(q) - 2S_{eh}(q) - 2] dq \]

\[ + \frac{c_s}{2r_s^2} \int_0^\infty [2 - qc_s/r_s + \ldots] S_{eh}(q) dq. \]

The first term is, as expected, the energy contribution from the electrostatic interaction between the planes of opposite charges. The ground-state energy in the Hartree-Fock approximation is given by

\[ \epsilon_{HF} = \frac{1}{r_s} - \frac{8/2}{3\pi} \frac{1}{r_s}. \]

Note the difference in the coefficient of the exchange energy, compared to Ref.7, because of our different choice of the 2D-units.

The basic equation for the pair-correlation functions \( g_{ab}(x) \),
\( \alpha, \beta = e, h \) in our approach is \(^3\) ;

\[
[-\nabla^2 + v_{\alpha\beta}(x) + W^b_{\alpha\beta}(x) + W^f_{\alpha\beta}(x)]g^{\frac{1}{2}}_{\alpha\beta}(x) = 0
\]  

(7)

where,

\[
v_{\alpha\beta}(x) = [\eta_{\alpha\beta} r_s / 2x - (1 - \delta_{\alpha\beta}) r_s / \sqrt{\pi} + c^2 / r_s^2]
\]

\[
\eta_{ee} = (1+\sigma), \quad \eta_{hh} = (1+1/\sigma), \quad \eta_{eh} = 0.
\]

The 'induced potentials' \( W_{\alpha\beta}(x) \) are given in the earlier works \(^3\), where the method of solving the above equations are also given.

**Systems with many-valley structure.**— So far, we have considered only the case of a single conduction band and a single valence band. In most cases, however, we have to take into account the effect of the valley degeneracy on the ground-state results to be discussed below. Therefore, in the following, we will consider a case where the conduction band has two minima and the valence band has a single maximum like in GaSe. The Hartree-Fock energy will now depend on the mass ratio as

\[
\varepsilon_{HF} = \frac{0.5 + \sigma}{1 + \sigma} \frac{1}{r_s^2} - \frac{9.66}{3\pi} \frac{1}{r_s}.
\]  

(8)

Earlier results in this multi-valley case \(^7\) have indicated that the ground-state energy depends more strongly on \( \sigma \) compared to the single valley case and the effect is more pronounced in two-dimensions. This is mainly due to the difference in the density of states between the two- and three-dimensional systems. The correlational energy is affected in our scheme, primarily through \( W^f_{\alpha\beta}(x) \), since the fermi momentum now is \( k_f^2 = 2\pi p / n_v \), where \( n_v = 2 \) for the electrons and 1 for the holes.

**Results and Discussions.**— In Figs.1 and 2, we have plotted the electr-
on-electron, hole-hole, and the electron-hole correlation functions as a function of $k_f r$, for various values of the dimensionless interlayer separation $c_s$, at $r_s = 3$. For $\sigma = 1$ (Fig.1), $g_{ee}(r)$ and $g_{hh}(r)$ are identical and they vary only slightly for different values of $c_s$. The effect of $c_s$ is much stronger in $g_{eh}(r)$, which tends to show less structure as $c_s$ is increased. For $m_h/m_e = 10$ (Fig.2), there is more or less a similar pattern in

Fig.2: Same as in Fig.1, but for $\sigma = 0.1$.

the distribution functions, except that the holes show stronger correlations among themselves. This behavior is reminiscent of the hole-hole distribution functions in three dimensions. The noticeable difference in the e-h correlation functions, as compared to those in three dimensions is that the enhancement is much reduced in two dimensions. In calculating the enhanced density, it should however, be noted that the ratio of the enhanced density to the exciton density is,

$$\frac{\rho_{eh}}{\rho_x} = \begin{cases} \frac{3g_{eh}(0)}{4r_s^3} & \text{; 3D} \\ \frac{g_{eh}(0)}{8r_s^2} & \text{; 2D} \end{cases}$$

(9)

In Fig.3, we have plotted the partial-static-structure functions, and $D(k) = S_{ee}(k)S_{hh}(k) - S_{eh}(k)^2$, as a function of $k/k_f$, for $\sigma = 1$ and $\sigma = 0.1$. All the functions rapidly approach to zero for small $k$. These functions were used to obtain the collective modes in the electron-hole liquid. They are obtained by generalizing the Bijl-Feynman equation for the two-component systems.
\[ \epsilon_{1,2}/\epsilon_f = q'/2D - \left( \frac{(\sigma S_{ee}+S_{hh})^2}{(\sigma S_{ee}+S_{hh})^2-4\sigma D} \right)^{\frac{1}{2}} \]  

(10)

where \( q' = k/k_f \) and \( \epsilon_f \) is the electron fermi energy. In Fig. 4, we have plotted the two branches of the excitation energies \( \epsilon_1 \) and \( \epsilon_2 \) in units of electron fermi energy as a function of \( k/k_f \), for \( \sigma = 1 \) and \( \sigma = 0.1 \). The dashed lines correspond to the threshold energy for the onset of Landau damping due to excitations of particle-hole pairs of the type 1

\[ \omega_e/\epsilon_f = q'(q'+2) \] and that of type 2 \[ \omega_h/\epsilon_f = q'(q'+2)\sigma \]. The plasmon mode rises sharply from zero with increasing \( k \) and has the characteristic plasma frequency \(^{13} \omega_p = [2\pi e^2/\rho k/\mu]^{\frac{1}{2}} \). The 'acoustic plasmon' mode (or ionic sound mode) \(^{14} \) exists only in the case where the holes are much heavier than the electrons \(^{12,15} \). This mode for \( \sigma = 0.1 \) is drawn as inset in Fig. 4.

Finally, in Fig. 5, we have plotted the ground-state energy minimum
as a function of the interlayer separation $c_S$, for different values of the electron-hole mass ratio. In the single valley case, the effect of the heavy holes is clearly insignificant. However, in the multi-valley case, we obtain a significant lowering of the energy. Comparing with the exciton energy in the ground state, we notice however, that the exciton state is clearly preferred. Nevertheless, a suitable choice of the electron-hole effective mass, the valley degeneracy and a finite separation between the layers, might lead to the energy being lower than the exciton energy. Our result is apparently in contrast with the results of Ref. 8 (curved marked AS), who obtained a lower energy for the EHL compared to the exciton energy $E_{\text{exc}}$ for all values of $c_s$. However, the unphysical behavior of the correlation functions at small distances and inadequate range in $k$, in their interpolation schemes, as pointed out in Ref. 10, introduce large amount of uncertainty in those energy values. Qualitatively, results similar to Fig. 5 were obtained in Refs. 7 and 10, for $c_s = 0$. More experimental efforts are undoubtedly needed to achieve a better understanding of this fascinating quantum liquid in two-dimensional systems.

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15. Tapash Chakraborty, to be published.