Spin polarizations at and about the lowest filled Landau level

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Abstract. – The spin polarization *vs.* temperature at or near a fully filled lowest Landau level is explored for finite-size systems in a periodic rectangular geometry. Our results at $\nu = 1$ which also include the finite-thickness correction are in good agreement with the experimental results. We also find that the interacting electron system results are in complete agreement with the results of the sigma model, *i.e.* skyrmions on a torus have a topological charge of $Q \geq 2$ and the $Q = 1$ solution is like a single spin-flip excitation. Our results therefore provide direct evidence for the skyrmionic nature of the excitations at this filling factor.

At the Landau level filling factor $\nu = 1$ ($\nu = N_e/N_s$, where N_e is the electron number and $N_s = AeB/hc = A/2\pi\ell_0^2$ is the Landau level degeneracy and ℓ_0 is the magnetic length) the ground state is fully spin polarized with total spin $S = N_e/2$ [1]. In recent Knight-shift spin polarization measurements [2], a precipitous fall in the spin polarization was observed when either one moves slightly away from $\nu = 1$ or the temperature is increased at $\nu = 1$. This effect has also been observed in subsequent experiments with tilted magnetic field as well as optical absorption studies [3]. Theoretically, such a result is explained as due to the fact that the low-energy charged excitations are spin textures (skyrmions) [4], [5] instead of the single spin-flip excitations [1] (the latter excitations are, of course, possible only for large values of the Zeeman energy, or large values of the g-factor). In fact, finite-size system calculations in a spherical geometry indicated that, when one adds or removes a flux quantum at $\nu = 1$ the total spin changes to $S = 0$ [4].

Theoretical studies at $\nu = 1$ indicated that [4] for large values of g the excitations are of single-particle type, *i.e.*, they carry charge $\pm e$ and spin $S_z = \frac{1}{2}$ and they have the size of magnetic length ℓ_0. As g is decreased, the excitations still carry charge $\pm e$, but they cover an extended region and have a nontrivial spin order: at the boundary of the system the local spin takes the value of the ground state and reversed at the center of the skyrmion. Along any radius, the spin gradually twists between these two limits. The size of the skyrmion is determined by the competition between the interaction energy and the Zeeman energy. The former favors a

large size in order to have uniform charge density, while the latter, with increasing strength, tends to reduce the size.

Earlier theoretical studies of spin polarization *vs.* temperature at $\nu = 1$ involved a continuum quantum field theory of a ferromagnet as a model and its properties at finite temperatures [6]. The other work was based on a many-body perturbation theory [7]. A qualitative agreement between the calculated temperature dependence of the spin polarization from these theories and the observed results was achieved.

We have employed finite-size systems in a periodic rectangular geometry to study the spin polarization $\langle S_z \rangle$ as a function of temperature at and about $\nu = 1$. Here

$$\langle S_z(T) \rangle \equiv \frac{1}{Z} \langle 0 | S_z | 0 \rangle + \sum \frac{1}{Z} \, e^{-\varepsilon_j / kT} \langle j | S_z | j \rangle \, ,$$

where $|0\rangle$ is the ground state, $Z = \sum_j \, e^{-\beta \varepsilon_j}$ is the partition function and the summation is over all excited states $|j\rangle$ with energy ε_j. The ground state and the excited states are calculated from the exact diagonalization of a few-electron system Hamiltonian in a periodic rectangular geometry [8].

The results for $\langle S_z(T) \rangle$ *vs.* T (in units of $e^2/\epsilon \ell_0$, where ϵ is the background dielectric constant) in an eight-electron system at $\nu = 1$ is plotted in fig. 1 a), where the magnetic field is held fixed ($B = 10\,\mathrm{T}$) but the g-factor is varied (0.1–0.5). In recent experiments [9] it was shown that applying hydrostatic pressure on the electron system, one can vary the g-factor at a given magnetic field. The experimental results by Barrett *et al.* [2] are also plotted in fig. 1 for comparison. The observed data show a much sharper drop with increasing temperature than the theoretical results obtained here. As mentioned above, the size of spin-excitations is dictated by the competition between the Zeeman energy (which is controlled here by the g-factor) and the interaction energy. Interestingly, the interaction potential can also be modified by including the finite-thickness correction to the electron-electron interaction, in the calculation [10], [11]. This is shown in fig. 1 b) where we present the results for a finite-thickness correction parameter $\beta = 0.5$ as an example [10], [11]. The agreement with the experimental results now improves noticeably. The changes in our results are most pronounced for large g, which is a direct evidence of the competition between the Coulomb and Zeeman energies mentioned above. Our results are also in good agreement with recent magnetoabsorption spectroscopy results [12] (fig. 1).

The skyrmion description of quasiparticle (hole) excitations near $\nu = 1$ [4], [13] assumes that the low-energy, long-wavelength effective Hamiltonian of the system is given by

$$\mathcal{H} = \int \mathrm{d}\mathbf{x} \, \gamma \, \partial_i \mathbf{n}(x) \cdot \partial_i \mathbf{n}(x) + \frac{1}{2} \int \mathrm{d}\mathbf{x} \int \mathrm{d}\mathbf{y} \, q(x) \, U(x,y) \, q(y) \, . \tag{1}$$

Here $\hat{\mathbf{n}}(\mathbf{x})$ is a unit vector field representing the local spin polarization, $U(x,y)$ is the inter-electron interaction potential and γ is a constant which is related to the spin stiffness. In our following arguments we shall consider the $g = 0$ case. The deviation of the electron charge density from its $\nu = 1$ value is given in terms of the spin density

$$q(x) = \frac{1}{4\pi} \epsilon_{ij} \, \mathbf{n}(x) \cdot (\partial_i \mathbf{n}(x) \times \partial_j \mathbf{n}(x)) \, , \tag{2}$$

where the right-hand side is the topological charge density. It is to be noted that $Q = \int_x q(x)$ is always an integer which is equal to the number of times $\hat{\mathbf{n}}(\mathbf{x})$ wraps around the sphere as $\mathbf{x}$ varies over all space.

In eq. (1), both the terms are small if $\mathbf{n}(x)$ is slowly varying, *i.e.* if its derivatives are small. We therefore expect that the *smoother* the configuration, the smaller its energy would be. The
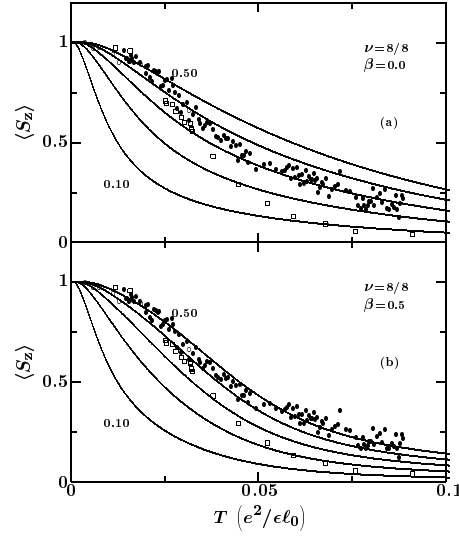
Fig. 1. – Electron spin polarization $\langle S_z(T) \rangle$ as a function of temperature T at $\nu = 1$ for an 8-electron system without (a) and with (b) finite-thickness correction included. Experimental results of [2] ($\square$) and [12] ($\circ, \bullet$) are also given for comparison.

smoothest spin configuration that one can imagine on any closed surface (*e.g.*, a sphere or a torus) is the *hedgehog* configuration. That is the spin configuration where the spin is always normal to the surface. As we go over all space of a sphere the spin density vector **n** covers the sphere exactly once. The charge of this state is, therefore, $Q = 1$ and the total spin is zero. This effect has been observed numerically in the interacting electron systems [4].

Let us consider the electron system on a plane with periodic boundary conditions, neglect the interaction term initially and consider the energy functional of the sigma model (the first term of eq. (1)). All the multisoliton solutions of the sigma model are exactly known [14]. In terms of the variables $w = \cot(\frac{\theta}{2})e^{i\phi}$ (which correspond to the stereographic projection of the sphere onto the complex plane) the solutions are analytic and antianalytic functions. They saturate the Bogomol'nyi bound for the energy which is given by

$$E \geq 8\pi\gamma|Q|. \tag{3}$$

For periodic boundary conditions the solutions are doubly periodic analytic functions, namely elliptic functions. A basic property of elliptic functions is that the sum of its residues at its poles in the fundamental domain is zero [15]. The charge Q of the configuration is therefore at least two. The winding number is calculated as [14] $Q = \sum_i n_i$, where i runs over all the poles and n_i is the order of the i-th pole. Now, if the sum of the residues is to vanish, then the elliptic function must have at least two simple poles or a second-order pole. Therefore, we must have $Q \geq 2$ for the solutions in this geometry. Pictorially, this would mean that the hedgehog on a torus has $Q = 0$ —the sphere is covered twice as we go over a torus, but it is covered in the opposite sense such that the total winding number is zero. In other words, the hedgehog on a torus can be viewed as a *skyrmion-antiskyrmion pair*. But if we reflect all the spins in the inner half of the torus about the xz (or yz) plane, then the sign of the topological charge density flips in the inner half and we get $Q = 2$. The total spin of both the above configurations is zero by symmetry.

In what follows, we show that the $Q = 1$ configurations that saturate the bound in eq. (3)
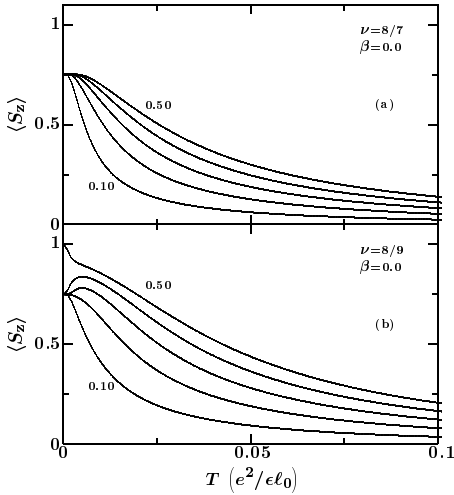
Fig. 2

Fig. 3

Fig. 2. – Electron spin polarization *vs.* T at $\nu = 1$ for an 8-electron system with one flux quantum added or removed.
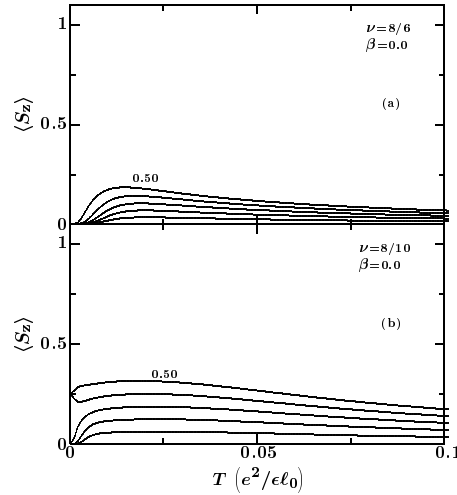
Fig. 3. – Electron spin polarization *vs.* T at $\nu = 1$ for an 8-electron system with two flux quanta added or removed.

are singular and pointlike. Consider the configuration $w(z, \bar{z}) = A\,\text{sn}(z)\,e^{i\Omega(z,\bar{z})}$, where $\text{sn}(z)$ is one of the Jacobian elliptic functions with periods $4K, 2iK'$ [15], and A is a constant. If we choose $2K = L_x$ and $2K' = L_y$, where L_x and L_y are the lengths of the sides of the rectangle, then $\text{sn}(z)$ is periodic in the y direction and antiperiodic in the x direction. If $\Omega(z + L_x, \bar{z} + L_x) = \Omega(z, \bar{z}) + \pi$ and it is periodic in the y direction, then $w(z, \bar{z})$ is periodic. It is easily verified that if the derivatives of Ω are periodic then $w(z, \bar{z})$ has $Q = 1$. The energy of w is

$$E[w] = 8\pi\gamma + 4\gamma \int \text{d}\mathbf{x}\ \frac{(A|\text{sn}(z)|)^2}{(1 + (A|\text{sn}(z)|)^2)^2}\ (\partial_i \Omega)^2\ .$$

In the limit $A \to 0$, the first factor of the integrand is sharply peaked about the pole of $\text{sn}(z)$. Therefore, if the derivatives of Ω vanish near that point then the bound (3) is saturated in the above limit. The $Q = 1$ configuration that minimizes the energy is therefore singular and pointlike. Inclusion of the Coulomb interaction is likely to make it regular but it will still be localized in a small region near the pole. We, therefore, conclude that the $Q = 1$ excitation in the rectangular periodic geometry is more like a single spin-flip excitation and not a spin texture. We should point out here that the above arguments apply only to the $g = 0$ case where the solutions for w are analytic (and antianalytic) and therefore sensitive to boundary conditions.

We now argue that in the presence of Coulomb repulsion the $Q = 2$ solutions will have *zero spin*. We take as an ansatz, $w(z) = A\,\text{sn}(z)$, where $L_x = 4K$ and $L_y = 2K'$. The Jacobian elliptic functions have well-seperated poles and zeros. Since the poles and zeros correspond to north and south poles, respectively, these correspond to slowly varying spin configurations. Further, if we choose $A = \sqrt{k}$, where k is the modulus [15], then using the property $\text{sn}(z + iK') = [k\,\text{sn}(z)]^{-1}$, we have $w(z + iL_y/2) = [w(z)]^{-1}$, and as a result,
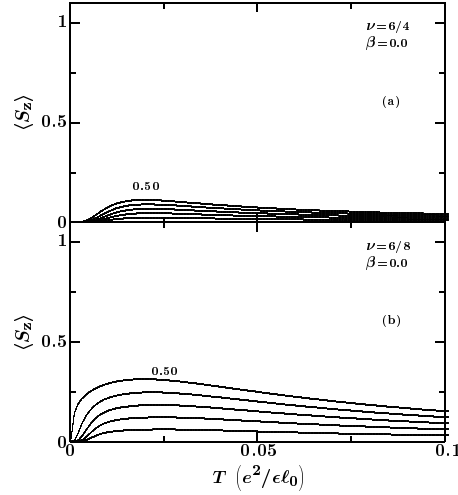
Fig. 4. – Same as in fig. 3 but for a 6-electron system.

$q\left(z + iL_y/2\right) = q(z)$. This choice of A would give the lowest value of the Coulomb energy since the charge is spread out symmetrically. Since $\cos\theta = (w\bar{w} - 1)/(w\bar{w} + 1)$, the transformation property of $w(z)$ mentioned above can be used to show that the z component of the total spin vanishes. The x and y components of the total spin can be shown to be zero using the property that $\mathrm{sn}(z + 2K) = -\mathrm{sn}(z)$. The periodic rectangular geometry therefore provides a unique test for the geometrical and topological aspects of the sigma model scenario. It predicts that the spin of the $Q = 1$ excitations should not deviate very much from the ground state spin whereas the spin of the $Q = 2$ excitation should drop to zero.

Our exact diagonalization studies provide strong support for these predictions. In fig. 2, we present the results for $\langle S_z(T)\rangle$ at $\nu = 1$ with one flux quantum added ($\nu = 8/9$) or removed ($\nu = 8/7$). Obviously, the spin polarization does not drop to zero, in contrast to what one expects in a spherical geometry, but has the nature of a single spin-flip excitation as anticipated above. The rapid drop in spin polarization takes place only when we add or remove *two* flux quanta from the system, *i.e.* at $\nu = 8/10$ or at $\nu = 8/6$. Indeed, for both fractions $\langle S_z(T \sim 0)\rangle = 0$ when g is small (fig. 3). It should be pointed out, however, that both $\nu = \frac{4}{5}$ and $\nu = \frac{4}{3}$ are genuine fractional quantum Hall states which were observed experimentally [11], [16]. Additionally, for the latter fraction, the fact that it has $S = 0$ was already established theoretically as well as experimentally at $T = 0$ [11]. It is quite difficult to envision these many-body states as two-skyrmion excitations of $\nu = 1$. Although desirable, it is a formidable task to diagonalize even larger systems. Therefore, we opted for a six-electron system instead. Here, for $\nu = 6/8$ or $\nu = 6/4$ we do not expect, *a priori*, that the ground state has $S = 0$. However, as shown in fig. 4, both these states have $S = 0$ at $T = 0$ and they also show the other expected behavior of a spin-singlet state [8], [17]. Observation of such a state at these fractions therefore lends support to our prediction above that the toroidal geometry supports at least two skyrmions as lowest-energy spin excitations at $\nu = 1$.

In conclusion, at $\nu = 1$ our finite-size system results are in good agreement with the experimental results. Finite-thickness correction of the interaction is found to improve the agreement, thereby providing strong support for the skyrmionic picture of excitations near $\nu = 1$. We have also demonstrated that the results of the interacting electron system are in complete agreement with the sigma model predictions.

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