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Fractional oscillations of electronic states in a quantum ring

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PACS. 73.20Dx – Electron states in low-dimensional structures (including quantum wells, superlattices, layer structures, and intercalation compounds).
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Abstract. – We have studied the energy states and the optical-absorption spectra for interacting electrons in a quantum ring with the spin degrees of freedom included. We observe features unique to the spin and interactions *viz* decrease of the period and amplitude of oscillations of the ground-state energy (fractional Aharonov-Bohm effect). This period decrease is explained in terms of the lifting of spin singlet-triplet degeneracy due to the interactions and Hund's rule. The optical-absorption spectra also show several unique features which are a combined effect of spin, impurity and interactions.

Despite considerable progress in recent years, much remains to be understood on the effects of electron correlations in various mesoscopic systems like quantum dots [1]-[4], antidots [5], and quantum rings [6]-[10]. Although a systematic effort to understand the role of impurities and interactions in a quantum ring has been reported recently [8]-[10], a clear picture of the combined effect of interactions, spin, and impurities on the electronic states of a quantum ring has not yet emerged. In this letter, we report novel structures in the energy spectra and optical-absorption spectra of interacting electrons with spin in a quantum ring. In our quantum ring model, the electrons in the presence of a symmetry-breaking scattering center of Gaussian form,

$$V^{\mathrm{i}}(\mathbf{r}) = V_0^{\mathrm{i}}\, e^{-(\mathbf{r}-\mathbf{R})^2/d_{\mathrm{i}}^2}\,,$$

are confined by a potential of the form [9]

$$U(\mathbf{r}) = V^{\mathrm{i}}(\mathbf{r}) + \begin{cases} \dfrac{1}{2} m^* \omega_0^2 (r-r_0)^2, \; r > r_0\,, \\ U_0 \exp[-r^2/d^2], \quad 0 \le r \le r_0\,, \end{cases}$$

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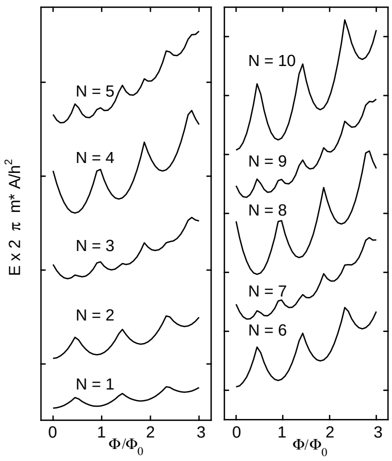

Fig. 1. – Ground-state energy *vs.* Φ/Φ_0 for up to ten noninteracting electrons. The energies are scaled to demonstrate how the periodicity is sensitive to the number of electrons in the ring.

where r_0 is the radius of the ring, and U_0 is the strength of the repulsive potential constraining the electrons to a narrow ring. Here the length is measured in units of r_0 and the energy in units of $h^2/2m^*\pi A$, where $A = \pi r_0^2$ is the area of the ring. Let us introduce the dimensionless quantities: $\alpha = \omega_0 m^* A/h$, $x = r/r_0$, where α is inversely proportional to the diameter of the ring [8] and $\mathcal{U} = (2m^*\pi A/h^2)U_0$. The confinement potential then takes the form

$$U(\mathbf{x}) = V^{\mathrm{i}}(\mathbf{x}) + \begin{cases} 4\alpha^2(x-1)^2, & x > 1\,, \\ 4\mathcal{U}\exp[-x^2/d^2], & 0 \le x \le 1\,. \end{cases}$$

For $\alpha = 20$, $\mathcal{U} = \alpha^2$ and $d = 0.5$, the system is close to an ideal one-dimensional ring. The Coulomb interaction is then written in these units as

$$\frac{e^2}{\epsilon r} = 9.45 m^* R_0 \frac{1}{\epsilon x}\,,$$

where $R_0 = 20$ is the radius of the ring in nanometers. We have performed exact diagonalization studies to extract the electronic states of a few-electron system in a quantum ring and the optical-absorption spectrum is calculated in the electric-dipole approximation [10]. For the energy spectra, we considered only $V_0^{\mathrm{i}} = 0$, *i.e.* an impurity-free system.

It has been demonstrated earlier [11], [12] that when one considers particles of spin $\frac{1}{2}$ (noninteracting) in a ring, the major consequence is period and amplitude halving of the current in a single ring. The effects of spin are strongly particle number dependent (*modulo* 4). The observed Φ_0-period current is only possible for an even number of electrons or for a single electron. In a ring free of any impurities, Fermi statistics would dictate that the persistent current has $\frac{1}{2}\Phi_0$ periodicity rather than the Φ_0 periodicity of spinless electrons. These are verified below in our present model.

One important finding of our present work is the decrease of period and amplitude of the oscillations of the ground-state energy. This so-called "fractional Aharonov-Bohm effect" with period $1/N_{\mathrm{e}}$ (for N_{e} particles in the ring), which arises solely due to electron correlations, was indicated earlier from Hubbard and Luttinger models [13], [14] suitable for a discrete ring. In those studies, electron correlations are believed to produce charge and spin separation in a ring. The charge and spin excitations then make a different number of loops in the ring and the period is thereby affected [13]. The other interpretations [14] involve the spin-flip

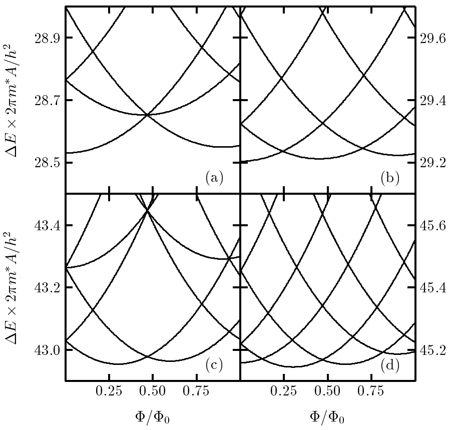

Fig. 2.

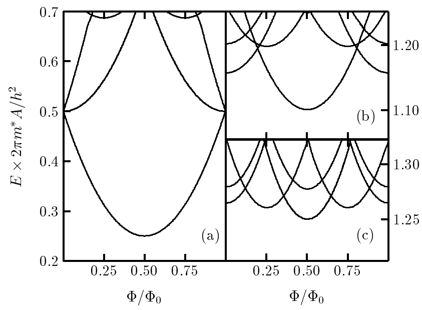

Fig. 3.

Fig. 2. – The few low-lying energy states for a ring containing *a*) two noninteracting electrons, *b*) two interacting electrons, *c*) three noninteracting electrons, and *d*) three interacting electrons.

Fig. 3. – Few lowest-lying energies of a four-electron ring. The interaction electrons are of the delta-function type with *a*) $V_0 = 0$ (noninteracting), *b*) $V_0 = 0.5$ and *c*) $V_0 = 1.0$.

processes where the number of spin-up electrons jump by unity for each period and the spin magnetization of the system changes over its maximum possible range as the flux increases by one flux unit. Here we present an alternative (and more rigorous) explanation for the period decrease, which is based on the energy states that we calculate via the exact diagonalization of the many-electron Hamiltonian.

In fig. 1, we present the noninteracting ground-state energies for up to ten electrons in our model. The energies are scaled in the figure in order to show how the periodicity of the ground-state energy is sensitive to the number of electrons in the ring. With odd number of electrons ($N > 1$), the period and the amplitude of the oscillations are, as expected, halved. As discussed above, such a result was anticipated earlier by simple extension of the well-known results for noninteracting spinless fermions [15] to the case of noninteracting spin-$\frac{1}{2}$ particles.

The role of interactions on the electrons in a quantum ring is subtle and, as yet, unclear [16]. In our approach, we have been able to study the effects of interaction with or without impurities in a systematic manner [8]-[10]. As we shall see below, Coulomb interaction has a profound effect on the energy spectra in the presence of spin. In fig. 2, we present the few low-lying energy states for a two *a*) noninteracting and *b*) interacting electron system. In the noninteracting system, as the flux is increased, the angular-momentum quantum number (L) of the ground state is increased by two, *i.e.* the ground state changes like $0, 2, 4, \ldots$. The period of the magnetization is one flux quantum. The ground state in the noninteracting case is always a spin-singlet. The first excited state is, however, spin degenerate. When the Coulomb interaction is turned on, this singlet-triplet degeneracy is lifted. This is due to the exchange term in the Coulomb matrix element. As a consequence, the triplet state comes down in energy with respect to the others, and, therefore, the period of the ground-state oscillations is halved. Also the amplitude of the oscillations is halved. In the presence of the electron-electron interactions the angular momentum of the ground state changes like $0, 1, 2, 3, 4, \ldots$. The singlet state now goes up in energy and is not shown in the figure.

In fact, the same kind of effect is seen with three electrons ($S_z = 1/2$) in fig. 2 *c*) for noninteracting and *d*) for the interacting cases. Without interactions the ground state changes its L quantum number as a function of magnetic flux like $1, 2, 4, 5, 7, 8, \ldots$. When the Coulomb interaction is turned on all the spin degeneracies are lifted again. Now the ground state oscillates with a suprisingly short period (one third of the flux quantum). Again the L quantum number changes like $0, 1, 2, 3, \ldots$.

In order to further investigate these features for a four-electron system, we first consider a strictly one-dimensional ring and assume interparticle interactions of the form

$$v(\mathbf{r}_1, \mathbf{r}_2) = V_0\, \delta(\theta_1 - \theta_2)\,.$$

Although this choice of the interaction is somewhat unrealistic, we hasten to add that it is the closest to a potential which resembles the interactions in discrete Hubbard models studied by many authors. In fig. 3 *a*), we present the few low-lying energy states for the noninteracting case. The lowest state is, of course, a spin singlet. The next state is spin degenerate. By turning on the interaction $V_0 = 0.5$ (in units of $h^2/2\pi m^* A$) (fig. 3 *b*)), the degeneracy is lifted. The ground state is now a spin-triplet state and the energy oscillates with a period $\Phi_0/2$. A further increase of the interaction strength $V_0 = 1.0$ (fig. 3 *c*)) results in oscillation of the ground-state energy with a period of $\Phi_0/4$.

The results for the Coulomb interaction are presented in fig. 4 for the noninteracting (*a*) $S_z = 1$, *c*) $S_z = 0$) and interacting (*b*) $S_z = 1$, *d*) $S_z = 0$) four-electron systems. For interacting electrons, when $S_z = 1$ the results again demonstrate Φ_0/N_e periodicity, consistent with the results for the other systems discussed above. The other spin configuration, $S_z = 0$, shows $\Phi_0/2$-periodicity. In this case, the system is at lower energies if the Zeeman energy is ignored. However, if the Zeeman energy is taken into account the $S_z = 1$ configuration becomes lower in energy and the $\Phi_0/4$ periodicity is recovered.

The general result, therefore, is that the Coulomb interaction (or in fact, any type of repulsive interaction) favors spin-triplet ground states. Without interaction the ground states are spin singlets and as a function of Φ/Φ_0 are parabolas with the minimum at about the integer values (in the case of an ideal ring, exactly at integer values). When we turn on a repulsive interaction the singlet states rise in energy more than the triplet state. If the repulsion is strong enough one can see period decrease.

One possible way to explain these results is by Hund's rule [17], *i.e.* the lower the symmetry of the spatial part of the wave function, the lower the interaction energy. In quantum rings the noninteracting ground states are all in spin-singlet states and, therefore, their spatial wave functions have to be symmetric with respect to particle exchanges. According to Hund's rule the interaction energies in those states are large. The noninteracting excited states, which can have their minima between the minima of the noninteracting ground states, have more freedom to arrange their spin states. In the absence of any interaction, all possible spin states originating from the same spatial configuration are degenerate. When the interaction is turned on, the states with lower symmetry in the spatial part rise less than the ones with higher symmetry. In particular, the noninteracting ground states gain much more interaction energy than some of the excited states. Depending on the strength of the interaction (in the Coulomb case the crucial quantity is the density) it may happen that some of the excited noninteracting states have lower energies than the noninteracting ground states. As a result, we have fractional period in the oscillations of the ground-state energy.

Why then in the presently available experiments no $1/N_\mathrm{e}$ behavior of the period is seen? The crucial quantity is the electron density. The experimental density, for $N_\mathrm{e} \sim 10^4$ electrons in a ring of radius $r_0 \sim 1.3$ μm [7], would correspond to a ring of radius $\sim$ 0.2–0.3 nm in the two-electron system. If the density is that high there are no correlations in the system and the free-particle picture is valid. Furthermore, when the mutual interactions can be neglected it is very likely that all single-particle states are doubly occupied with spin-up and -down electrons

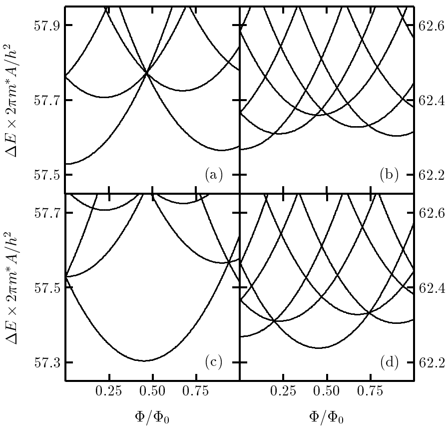

Fig. 4.

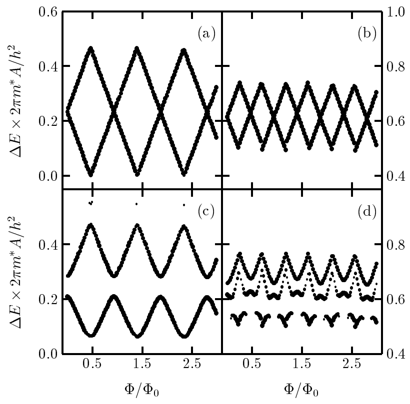

Fig. 5.

Fig. 4. – The few low-lying energy states for a ring containing four *a*) noninteracting electrons ($S_z = 1$), *b*) interacting electrons ($S_z = 1$), *c*) noninteracting electrons ($S_z = 0$), and *d*) interacting electrons ($S_z = 0$). The interelectron interactions are of Coulomb form.

Fig. 5. – Absorption spectra for a quantum ring with *a*) two noninteracting electrons, *b*) two interacting electrons, *c*) two noninteracting electrons with moderate impurity ($V_0 = 1.0, d = 0.2$ at $r = r_0$) and *d*) two interacting electrons with the same impurity strength. The areas of the filled circles are proportional to the calculated absorption intensities.

coming from the surrounding material acting as an electron reservoir. Therefore, in most cases the total number of electrons is even and the resulting period is Φ_0.

When the Coulomb interaction is strong enough, the spin-triplet state (in the two-electron system) lies, due to the exchange term, energetically so low that it becomes a new ground state of the system in the vicinity of the intersection points of the noninteracting spin-singlet ground states. When the Coulomb interaction is further increased, the energy separation of these spin-triplet and spin-singlet states tends to saturate. Due to this saturation, the fractional period of the ground state is preserved even in the limit of strong interactions. Therefore, the size of the ring is an essential quantity to observe the fractional oscillations in the ground-state energy. Recent experiments on the few-electron quantum dot ground state [4], [18] could, in principle, be extended to observe the novel phenomena in the ground state discussed here.

Finally, we show the results of our optical-absorption studies of quantum rings with noninteracting and interacting electrons and an impurity of moderate strength ($V_0^{\mathrm{i}} = 1.0, d_{\mathrm{i}} = 0.2$, $R = r_0$) [10]. The results clearly show the strong effect of Coulomb interaction on the electrons with spin. For the noninteracting impurity-free system, the two-electron results ($S_z = 0$) are shown in fig. 5 *a*). The results for the same system with interactions included are shown in fig. 5 *b*). With a moderate impurity in the system the results are presented in fig. 5 *c*) (noninteracting) and *d*) (interacting). It is quite obvious that the Coulomb interaction introduces a lot of structures into the optical-absorption spectrum of the noninteracting system. As can be seen from fig. 5, the observation of the fractional oscillations of the electronic states is possible via the optical-absorption measurements. In fact, absorption spectra reflect the behavior of the energy levels and impurity and interactions present a rather complicated structure (fig. 5 *d*)) of the energy levels. Complexity notwithstanding, impurities do not destroy the fractional periodicity of electronic states.

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