Spin precession in a fractional quantum Hall state with spin-orbit coupling

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In order to investigate the fundamental question of whether inter-electron interactions are important in planar electron-based spintronics devices, we have carried out a many-body study of the spin configuration in a quantum Hall state in the presence of Bychkov-Rashba-type spin-orbit interaction. We find that the spin orientation is position dependent and can be tuned via the applied electric field. We propose that this and other properties of such a system are ideally suited for exploitation in spin devices, and that the performance can be further optimized by a careful choice of the growth orientation. © *2005 American Institute of Physics*. [DOI: 10.1063/1.2045546]

Spintronics has become a fast developing field in which the electron spin degree of freedom is exploited to create novel electronic devices.[1] Of paramount importance in this pursuit is the ability to manipulate spins in a controlled and reliable way. The spin-orbit (SO) interaction provides a bridge between spin and charge properties which, by coupling the electron momentum with its spin, makes it possible to control the spin dynamics using electric fields. After the initial proposal of a spin device based on this concept,[2] the scientific literature has been saturated by theoretical studies on the effects of SO coupling in inversion-asymmetric two-dimensional (2D) systems. Most of these,[3,4] however, rely on a simplified single-particle picture, where electron-electron interaction is neglected. Although many different devices based upon this paradigm have been proposed theoretically, the experimental realization of much of them has remained elusive. This raises a question about the wisdom of ignoring the influence of many-body effects on the spin configuration.

A crucial factor that has also contributed to such a delay in the realization of many theoretical schemes is the short spin lifetime in conventional two-dimensional semiconductors. A 2D electron gas (2DEG) in the quantum Hall (QH) state with odd integer ($\nu=1$) or fractional ($\nu=1/3$) filling factor, however, is completely spin polarized[5] and there is no spin scattering in such a system. The mobility is very high and therefore there is also very little scattering from impurities. This removes the problem of spin decoherence found in conventional 2DEG systems due to spin decay mechanisms such as the Dyakonov–Perel relaxation.[6] For the same reason (i.e., the spin polarization), these QH states might well be efficient spin injectors.

The special features of QH systems with integer ($\nu=1$) filling factors have already found application in different spin devices: the use of edge states in the QH regime has been proposed as a spin polarizer for spin readout in single-spin memory devices,[7] to achieve pure-state initialization of the qubit,[8] to enhance Coulomb Blockade measurements,[9] and more generally as an efficient spin injector into semiconductors.[10] Furthermore, weak spin relaxation in QH

states was exploited recently in the design of spin devices by Pala *et al.*[11] All these applications exploit the absence (or weakness) of spin relaxation mechanisms in QH systems and the high spin polarization of QH states. Despite increasing interest in QH systems for spintronics applications and the important role played in this context by the SO interaction in a 2DEG, no many-body study on the spin configuration in QH states in the presence of SO coupling exists to date.

In this letter we present a theoretical investigation into the effects of Bychkov–Rashba SO coupling in the presence of Coulomb interactions on the spin configuration in a fractional quantum Hall (FQH) state. Our approach is a generalization to a FQH system[5,12] of the spin precession concepts developed in a recent paper by Koga and co-workers.[4] According to that simple single-particle picture of Koga *et al.*, assuming the electron wave vector $\mathbf{k}\|\hat{x}$, and taking the spin basis along the z axis perpendicular to the 2DEG plane, the two (Bychkov–Rashba) spin split states with energies $E=\hbar^2 k^2/2m^* \pm \alpha k$, can be written as

$$\Psi_{\mathbf{k}\uparrow} = \frac{1}{2}\begin{pmatrix} 1-i \\ 1+i \end{pmatrix} e^{ikx} \tag{1}$$

$$\Psi_{\mathbf{k}\downarrow} = \frac{1}{2}\begin{pmatrix} 1+i \\ 1-i \end{pmatrix} e^{ikx}. \tag{2}$$

One can then build a linear combination of these two states at a given energy (in this case the wave vectors are written as $\mathbf{k}=(k\pm\Delta k,0,0)$, respectively, for spin-down and -up states)

$$\Psi_{\mathbf{k}} = \frac{1}{\sqrt{2}}(\Psi_{\mathbf{k}-\Delta\mathbf{k}\uparrow} + \Psi_{\mathbf{k}+\Delta\mathbf{k}\downarrow}) \tag{3}$$

$$= \frac{1}{2\sqrt{2}}\left\{ \begin{pmatrix} 1-i \\ 1+i \end{pmatrix} e^{i(k-\Delta k)x} + \begin{pmatrix} 1+i \\ 1-i \end{pmatrix} e^{i(k+\Delta k)x} \right\}$$

$$= e^{ikx}\frac{1}{\sqrt{2}}\begin{pmatrix} \cos(\Delta kx) - \sin(\Delta kx) \\ \cos(\Delta kx) + \sin(\Delta kx) \end{pmatrix} \tag{4}$$

whose spin orientation depends on the position along x and on the strength of the spin-orbit interaction ($\Delta k \sim \alpha$). Therefore, as $\Psi_{\mathbf{k}}$ propagates along x, the spin precesses.

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We have performed a many-body analog to this simple single-particle analytical treatment of the effects of Bychkov–Rashba coupling in a $\nu=1/3$ FQH state, where the many-body Schrödinger equation was solved by means of the exact diagonalization scheme for four electrons per supercell.[5] The many-body wave functions were expanded in terms of a complete basis obtained as superposition of solutions of the single-particle Hamiltonian

$$H = \frac{(\mathbf{p} - e\mathbf{A})^2}{2m^*} + \frac{\alpha}{\hbar}[\boldsymbol{\sigma} \times (\mathbf{p} - e\mathbf{A})]_z + \frac{1}{2}g\mu_B B\sigma_z$$

that includes the Bychkov–Rashba term[13] and the Zeeman term. Here $\mathbf{p}$ is the momentum operator, α is the SO coupling strength, $\boldsymbol{\sigma}=(\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices and B is along z. Solutions of this Hamiltonian are also spinors but have a more complex form than Eqs. (1) and (3), due to the presence of the external magnetic field (see Ref. 12 for details)

$$\psi_{s,j}^+(\mathbf{r}) = \frac{1}{\sqrt{\sqrt{\pi}\ell_0 L_y A_s}}\sum_n \exp\left[i(X_j + nL_x)\frac{y}{\ell_0^2}\right.$$
$$\left. - \frac{(X_j + nL_x - x)^2}{2\ell_0^2}\right]$$
$$\times \begin{pmatrix} -iD_s\beta_{s-1}H_{s-1}\left(\frac{(X_j + nL_x - x)}{\ell_0}\right) \\ \beta_s H_s\left(\frac{(X_j + nL_x - x)}{\ell_0}\right) \end{pmatrix}$$

$$\psi_{s,j}^-(\mathbf{r}) = \frac{1}{\sqrt{\sqrt{\pi}\ell_0 L_y A_s}}\sum_n \exp\left[i(X_j + nL_x)\frac{y}{\ell_0^2}\right.$$
$$\left. - \frac{(X_j + nL_x - x)^2}{2\ell_0^2}\right]$$
$$\times \begin{pmatrix} \beta_{s-1}H_{s-1}\left(\frac{(X_j + nL_x - x)}{\ell_0}\right) \\ -iD_s\beta_s H_s\left(\frac{(X_j + nL_x - x)}{\ell_0}\right) \end{pmatrix}$$

and the corresponding energies

$$E_s^\pm = s\hbar\omega_c \pm \sqrt{E_0^2 + 2s\alpha^2/\ell_0^2}.$$

Here

$$D_s = \frac{\sqrt{2s}\alpha/\ell_0}{E_0 + \sqrt{E_0^2 + 2s\alpha^2/\ell_0^2}}$$

$A_s = 1 + D_s^2$, and $E_0 = 1/2(\hbar\omega_c - g\mu_B B)$, $g=-14$, $H_n(x)$ is the Hermite polynomial of degree n, $\ell_0 = (\hbar/m^*\omega_c)^{1/2}$ is the radius of the cyclotron orbit with frequency $\omega_c = eB/m^*$ and center $X_j = k_y\ell_0^2$, $L_x L_y \propto \ell_0^2$ is the supercell area, and $\beta_n = 1/\sqrt{2^n n!}$.

We then constructed a state as linear combination of two many-body wave functions, eigenstates of Coulomb interaction, relative to two degenerate excited states with different total momentum J (in analogy to Eq. (3); in our case, however, the two spins are not pointing in opposite directions due to the spin polarized nature of the FQH state[5,14]), and calculated the expectation value of the spin components $S_x = \langle\sigma_x\rangle$, $S_y = \langle\sigma_y\rangle$ and $S_z = \langle\sigma_z\rangle$ along the three principal directions.
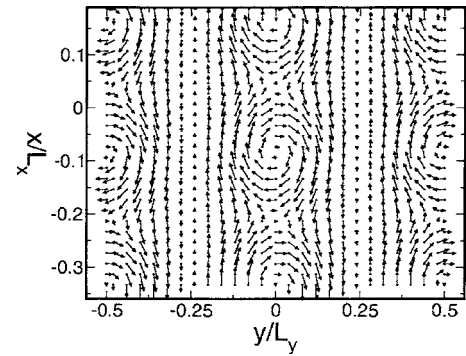

FIG. 1. Projection of $\langle S \rangle$ on the xy plane, calculated for $\alpha=40$. Only a portion of the supercell is shown, as the behavior is periodic in both directions (the motion is along the y direction).

Our results (Figs. 1–3) show a more complex picture than that described by Eq. (4) for the resulting superposition of spin eigenstates, that, however, retains the main feature of a position-dependent spin orientation (i.e., the precession). As shown in Fig. 1 the spin rotation has a period of $L_y/2$. Our calculated spin precession length is therefore $L_{sp} = 4.1\ell_0$, which for $B=1$ T is of the order of 100 nm. While L_{sp} depends only on the applied *magnetic* field, we find that the value of the different angles θ_i ($i=x,y,z$), which the electron spin forms with the principal axes, also depends on the applied *electric* field through the SO coupling strength α.

This property of the $\nu=1/3$ FQH state could be exploited in a spin device (e.g., in a spin transistor). The use of such a state in a spin device would, in fact, have two main
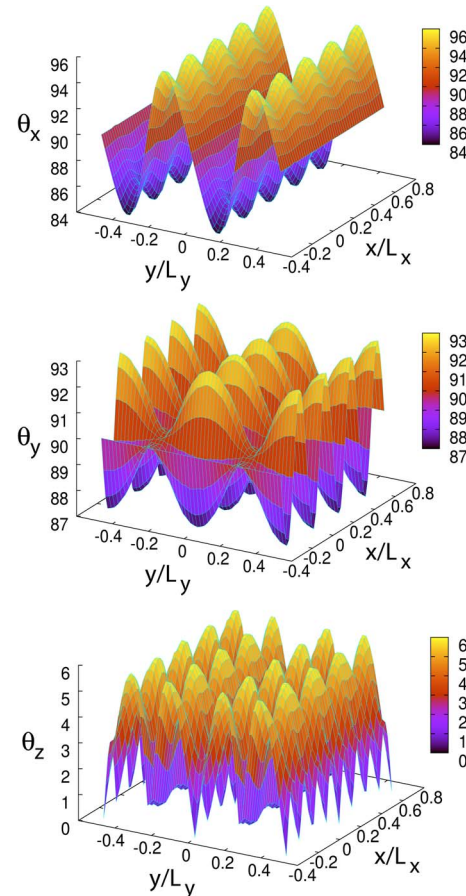

FIG. 2. (Color online) Variation across the supercell of the angles between $\langle S \rangle$ and the three principal axes, for $\alpha=40$.
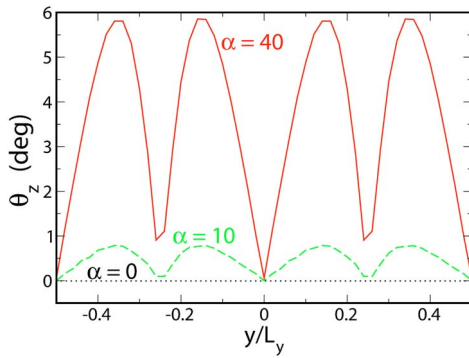
FIG. 3. (Color online) Angle between $\langle \mathbf{S} \rangle$ and the z axis as a function of the position along the y axis, for $\alpha=0$ (dotted line), 10 (dashed line), 40 nm meV (solid line) and $B=1$ T.

advantages over that of conventional 2DEG systems: (i) no need for spin injection; (ii) no uncontrolled spin decoherence (scattering) effects. (i) Although efficient spin injection is fundamental for spintronic devices, it has been elusive so far.[15] The $\nu=1/3$ FQH state, being *naturally* spin polarized would remove the need for spin injection altogether. In fact, in the FQH state complete spin polarization is achieved via electron correlations *without* any assistance from the Zeeman term.[5,14] The initial phase of the spin (usually fixed at the interface with the injector) is zero (i.e., the spin is parallel to B) at the channel edge. This can be understood from Fig. 1 by noticing that for $y/L_y \sim \pm 0.25$ the spin in-plane component is zero and, unlike anywhere else in the supercell, is the same for all values of x. The position at $y/L_y \sim \pm 0.25$ is due to the periodic boundary conditions used in the calculations. (ii) As is the case for QH states with (odd) integer filling factors, there is no spin scattering through the $\nu=1/3$ state, resulting in a long spin lifetime.

In InAs structures, together with low spin scattering rates in the FQH state (yet to be observed in conventional InAs 2DEGs), there is the added advantage of a high g factor. The only source of spin "decoherence" in such a FQH system is therefore introduced only by the electric-field-driven Bychkov–Rashba field, that leads to a (position dependent) spin precession. It follows that if the (ferromagnetic) drain contact has a spin polarization $\mathbf{P}_D$, an electron will be able to leave the channel (FQH state) only if its spin at the end of the channel $\langle \mathbf{S} \rangle$ is aligned with $\mathbf{P}_D$. In our system we found that

$$\langle \mathbf{S} \rangle = \begin{pmatrix} S_x(x,y,j,\alpha) \\ S_y(x,y,j,\alpha) \\ S_z(x,y,j,\alpha) \end{pmatrix},$$

where x and y are the coordinates in the 2DEG plane, j is related to k via $k_y=2\pi j/L_y$, L_y is the supercell size along y and α is the Bychkov–Rashba coupling strength (the variation of the angles that $\langle \mathbf{S} \rangle$ forms with the different axes, as it precesses across the supercell, is shown in Fig. 2). By choosing, for example, $\mathbf{P}_D \| z$ we found that the angle between $\langle \mathbf{S} \rangle$ and $\mathbf{P}_D$ is

$$\theta_z = \arccos(S_z(x,y,j,\alpha)/|\langle \mathbf{S} \rangle|).$$

This angle is plotted in Fig. 3 for three values of α.

A tunable device is obtained by varying the value of the Bychkov–Rashba coupling constant α, via the applied electric field. As shown in Fig. 3, this will induce a variation in

θ_z: its value will change from 0° for $\alpha=0$ (i.e., $\langle \mathbf{S} \rangle \| \mathbf{P}_D$), up to about 6° for $\alpha=40$ along the 2DEG edge. This value might seem rather small for device applications, however it can be increased by optimizing the quantum well (QW) configuration. This can be done by taking advantage of the intrinsic anisotropies of the system.

The presence of the intrinsic bulk inversion asymmetry (BIA), caused by the underlying crystal structure, can be used to enhance the effect of the Rashba field and increase the amplitude of the precession angle. The direction of the effective field due to BIA depends on the crystallographic orientation of the QW. In a (110)-oriented QW, the BIA effective field is along the external magnetic field and its direction is antiparallel to B for $k_x=0$ and large positive k_y (i.e., $k_y \| [\bar{1}10]$ direction). In this configuration, its effect is therefore to partially counterbalance the Zeeman term (i.e., equivalent to increasing the g factor to an effective value $g^* > g$), making it easier for the spin to bend under the SO field. This would result in an increase of θ_z compared to the values shown in Fig. 3. Low carrier concentrations and high mobilities have been achieved in 2DEGs in [110]-grown InAs very close to the surface.[16] Using spin-polarized scanning tunneling microscopy[17] on these systems it should be possible to image the spin configurations displayed in Figs. 1–3.

In conclusion, we presented a many-body approach to the effects of SO coupling on the spin configuration in the $\nu=1/3$ FQH state, including electron-electron interactions. Possible device applications of such a system are suggested together with experimental techniques to image the calculated spin distribution.

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