The Hubble Law

Background

In the 1920's, Edwin P. Hubble discovered a relationship that is now known as Hubble's Law. It states that the recessional velocity of a galaxy is proportional to its distance from us

$$v = H_0 d \ ,$$

where *v* is the galaxy's velocity (in km/sec), *d* is the distance to the galaxy (in megaparsecs; 1 Mpc = 1 million parsecs), and H_o is the proportionality constant, called the *Hubble constant*. This equation is telling us that a galaxy moving away from us twice as fast as another galaxy will be twice as far away.

The velocity of a galaxy is measured using the Doppler effect. An object in motion will have its radiation (i.e. light) shifted in wavelength:

$$\frac{(\lambda - \lambda_0)}{\lambda_0} = \frac{v}{c} = z$$

where λ is the measured wavelength, λ_0 is the rest wavelength, *v* is the speed of the object, and *c* is the speed of light. Wavelengths are usually measured in Angstroms (Å). The speed of light has a constant value of **300 000 km/sec**. The quantity on the left side of the equation above is usually called the *redshift*, and is denoted by the letter **z**.

We can determine the velocity of a galaxy from its spectrum: we measure the wavelength shift of a known absorption line and solve for *v*. Example: *An absorption line with a rest wavelength of 5000Å is found at 5050Å when analyzing the spectrum of a particular galaxy. Therefore this galaxy is moving with a velocity v = (50/5000) * c = 3000 km/sec away from us.*

Determing a galaxy's distance relies on more indirect methods. The *standard ruler* assumption assumes that all galaxies are the **same physical size**, no matter where they are. In this exercise we assume that all galaxies are **22 kpc** in diameter. To determine the distance to a galaxy one would only need to measure its apparent (angular) size (i.e. the size we see in a picture), and use the following approximation for small angles

$$a = \frac{s}{d}$$

where *a* is the measured angular size (in radians!), *s* is the galaxy's true size (diameter, **22 kpc** for this lab), and *d* is the distance to the galaxy.

Measurements

Record your measurements in the table provided.

1. Measure the apparent size of the galaxy in each image. Note that the images used are negatives, so that bright objects (stars and galaxies) appear dark! Note also that there may be more than one galaxy in the image; the galaxy of interest is always the one closest to the centre. Measure the size along the longest part in mm. You will then have to convert your measurement to an angular size, in milliradians (1 mrad = 0.057 degrees). The scale of the photograph is **0.0425 mrad/mm**. You can determine the angular size by multiplying your measurement by the scale.

2. The velocity of the galaxy is determined by measuring the redshift of the Calcium H and K lines in the galaxy's spectrum. The small dark bar near the lower left corner of the spectrum indicates the *rest wavelength* of the spectral line in question. Measure the wavelength by reading off the scale at the middle of the spectral line (i.e. at the peak of emission or absorption) in the galaxy's spectrum. Record this wavelength in your table. Note that due to peculiarities of each galaxy, some spectral lines may be absent, or show up in emission instead of absorption!

Calculations

For each measured line calculate the redshift, **z** (recall $z = \frac{(\lambda - \lambda_0}{\lambda_0}$).

Take the average redshift of the measured lines for each galaxy. Use this **average redshift** to calculate the velocity, *v*, of the galaxy using the Doppler-shift formula:

$$v = c * \mathbf{z} \ .$$

Assuming that all of these galaxies are about the same size, *s*, of 22 kpc across (1 kpc = 1 kiloparsec = 1000 pc), find the distance, *d*, (in Mpc) to the galaxies:

$$d[Mpc] = \frac{s[kpc]}{a[mrad]}$$

where *a* is the angular size you calculated in part 2 above.

Galaxy name	Apparent size (mm)	Angular size, *a* (mrad)	Calcium K line (λ_0 = 3933.7 Å)			Calcium H line (λ_0 = 3968.5 Å)			average redshift z	*v* (km/s) (=c*z)	*d* (Mpc) (*s*/*a*)
			λ	λ-λ_0	z	λ	λ-λ_0	z			
NGC1357											
NGC3034											
NGC3147											
NGC3368											
NGC5548											
NGC7469											

Make a graph with the distance on the x-axis (horizontal), and velocity on the y-axis (vertical). Draw a (straight!) line that best fits the points on the graph; remember that this line must pass through the origin! (the 0,0 point). (**Why?**)

1. Measure the slope of this line (rise/run) − this is the value of the Hubble constant, $\mathbf{H_0}$, in the units of km/sec/Mpc. Your graph probably does not make a perfect line, and you will notice that you had to make a guess as to where to draw your line. One simple way to estimate the uncertainty in the value of $\mathbf{H_0}$ is to draw the steepest reasonable line and the shallowest reasonable line on the graph, and measure the slopes of those lines. Half of the difference between these two slopes is your uncertainty in $\mathbf{H_0}$.

   If the universe has been expanding since its beginning at a constant speed, the universe's age would simply be 1/$\mathbf{H_0}$.

2. Convert $\mathbf{H_0}$ to units of inverse-seconds (1/sec) by canceling out the distance units: 1Mpc = 3.09 x 10^{19} km.

3. The "expansion age" of the universe is $t = \frac{1}{H_0}$. Find the age of the universe, in seconds, using your value for H_0. Convert this to an age in years (First figure out how many seconds there are in a year?).

4. The above is a very simple model for the expansion of the universe. Another model might account for the deceleration caused by gravity. Models like this predict the age of the universe to be: $t = \dfrac{2}{3H_0}$. Redo the age using this relation (both in seconds and in years). *Actual evidence suggests an accelerating universe.*

5. How does your value compare to the current best estimate of the age of the universe (13.75 ± 0.11 GYr according to recent results from WMAP – see http://map.gsfc.nasa.gov/). How does your value compare to the age of the Sun?

Further Questions

Theoretically, your plot should be a straight line, but it probably isn't. Think carefully about the following sources of error and answer the questions below.

1. One obvious source of error is the assumption we made that all spiral galaxies have the same diameter. How would an over-estimate of a galaxy's diameter affect your estimate of the distance to it? How about an under-estimate? Explain.

2. Another consideration is the fact that galaxies are found in groups. The motion of these galaxies through space as they revolve around their common centre is called peculiar motion. How does this peculiar motion affect your velocity measurement?

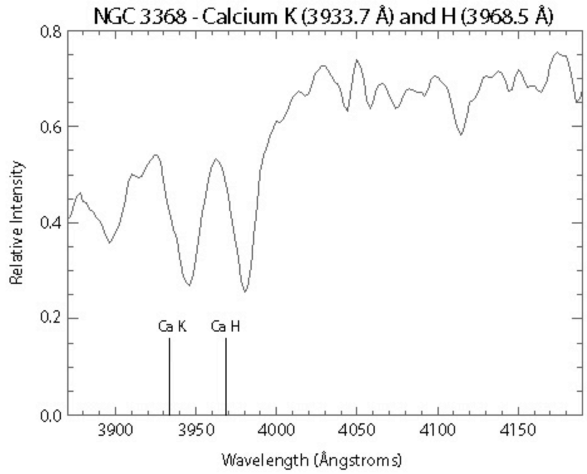
3. A third source of error is the error in the measurement you make.

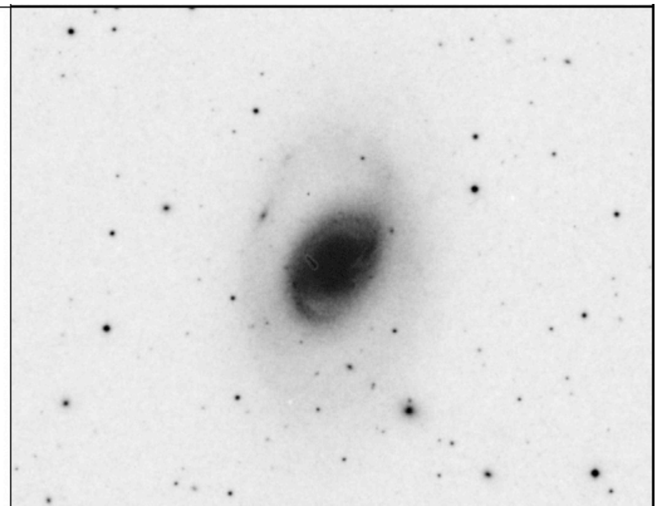
   These three sources of error can be categorized as either "random" or "systematic" errors. Categorize each of them and explain your reasoning. (Note that some of the aforementioned sources of error can be both systematic and random.)

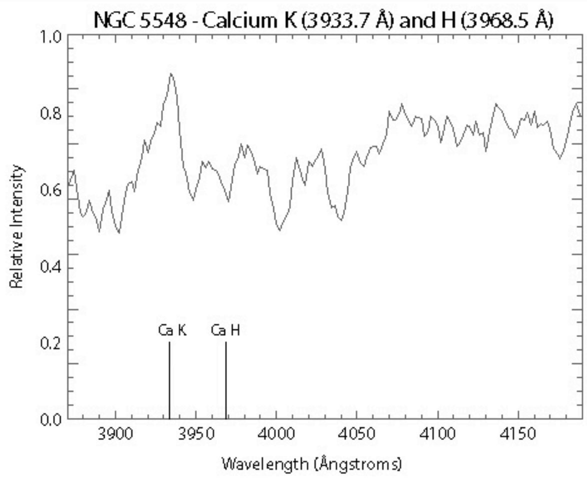
References

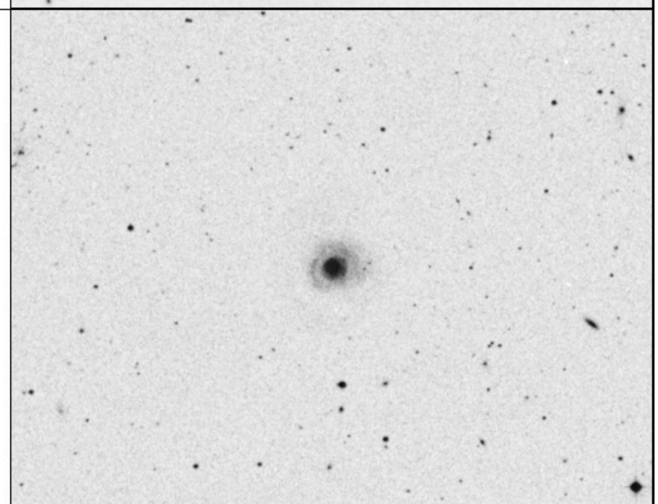
Spectra from: University of Washington, Astronomy Department, Introductory Astronomy Clearinghouse
http://www.astro.washington.edu/labs/clearinghouse/labs/Spectclass/spectralclassweb.html

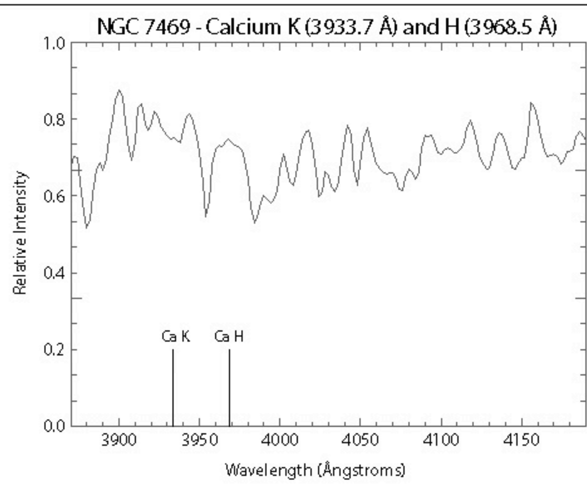
Images from: STScI Digitized Sky Survey
http://archive.stsci.edu/cgi-bin/dss_form

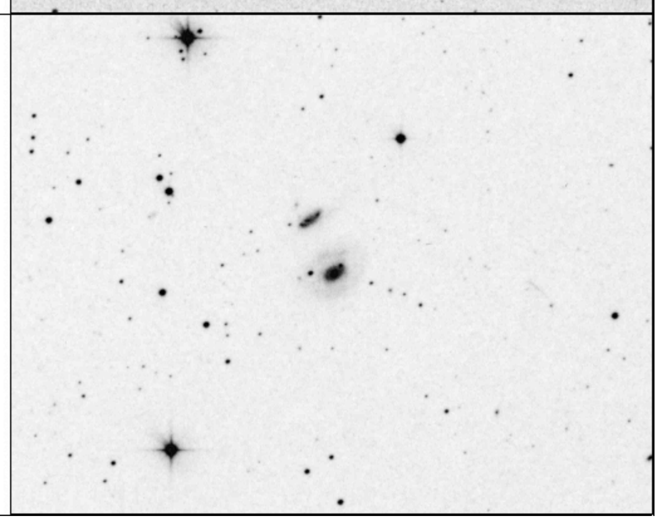

NGC 3368 - Calcium K (3933.7 Å) and H (3968.5 Å)
Relative Intensity
Wavelength (Ångstroms)
Ca K
Ca H
0.0
0.2
0.4
0.6
0.8
3900
3950
4000
4050
4100
4150

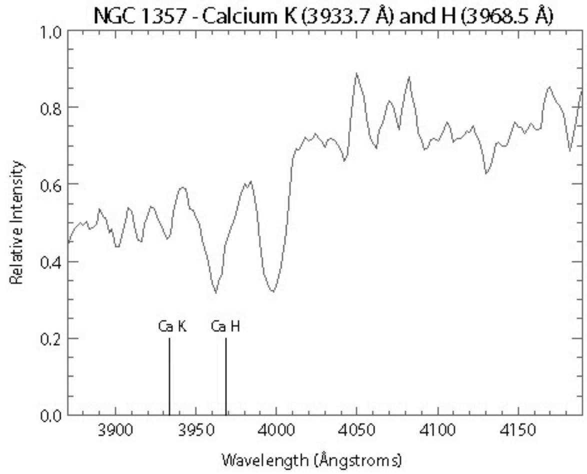


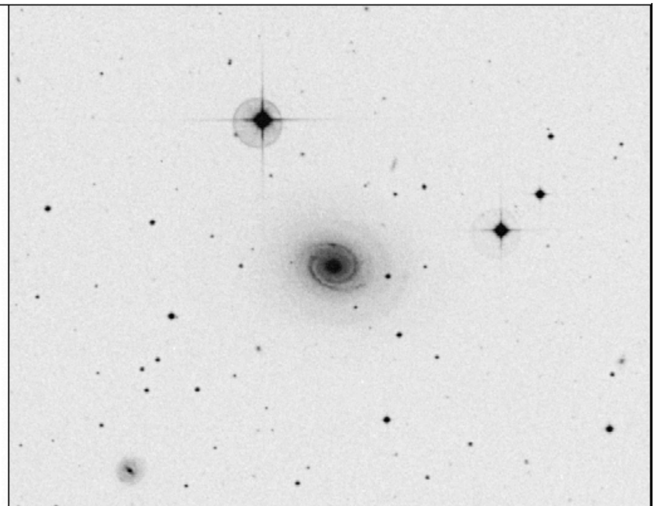

NGC 5548 - Calcium K (3933.7 Å) and H (3968.5 Å)
Relative Intensity
Wavelength (Ångstroms)
Ca K
Ca H
0.0
0.2
0.4
0.6
0.8
1.0
3900
3950
4000
4050
4100
4150

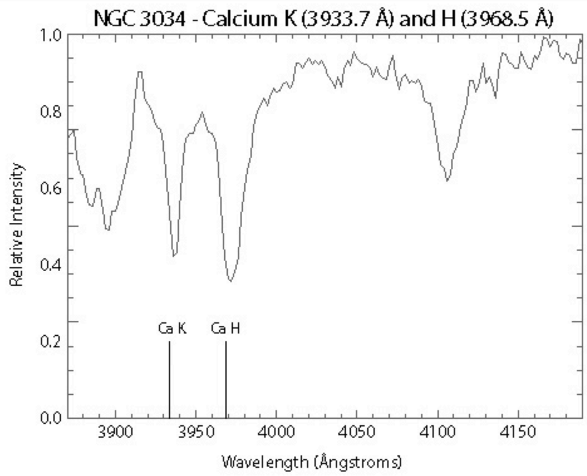


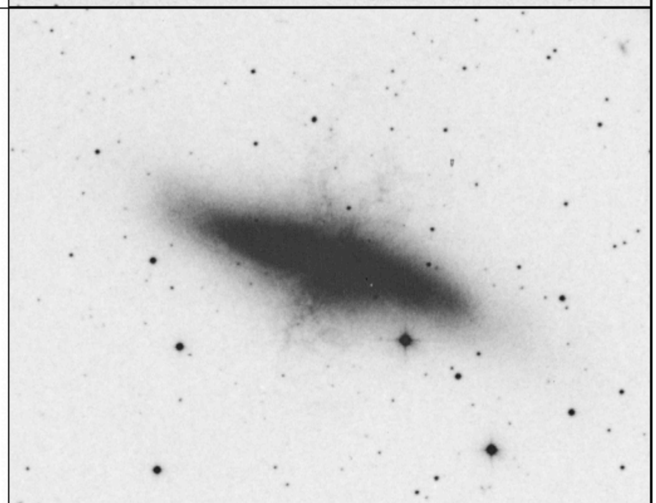

NGC 7469 - Calcium K (3933.7 Å) and H (3968.5 Å)
Relative Intensity
Wavelength (Ångstroms)
Ca K
Ca H
0.0
0.2
0.4
0.6
0.8
1.0
3900
3950
4000
4050
4100
4150

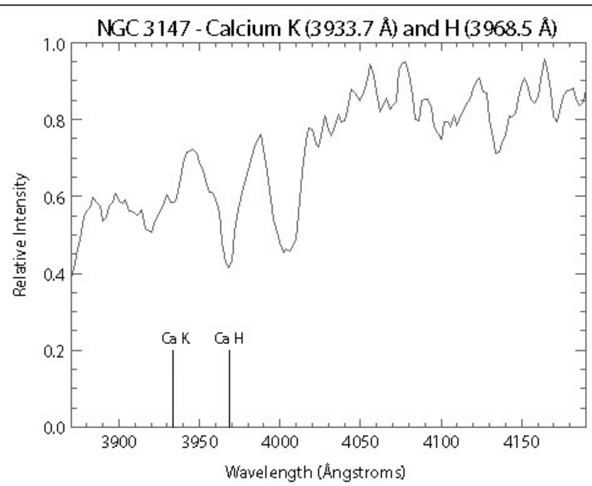



NGC 1357 - Calcium K (3933.7 Å) and H (3968.5 Å)
Relative Intensity
Wavelength (Ångstroms)
Ca K
Ca H
0.0
0.2
0.4
0.6
0.8
1.0
3900
3950
4000
4050
4100
4150




NGC 3034 - Calcium K (3933.7 Å) and H (3968.5 Å)
Relative Intensity
Wavelength (Ångstroms)
Ca K
Ca H
0.0
0.2
0.4
0.6
0.8
1.0
3900
3950
4000
4050
4100
4150




NGC 3147 - Calcium K (3933.7 Å) and H (3968.5 Å)
Relative Intensity
Wavelength (Ångstroms)
Ca K
Ca H
0.0
0.2
0.4
0.6
0.8
1.0
3900
3950
4000
4050
4100
4150