The Curvature of Space and the Expanding Universe

Summary

The idea of curved space and the curvature of our own universe may take some time to fully appreciate. We will begin by looking at some examples from other "universes" first, and then how we might apply that knowledge to our own. Using a two-dimensional analogy we will explore the expansion of the Universe.

From observations of galaxies, we find their distances and their velocities, and discover that the more distant the galaxy, the faster it is moving away from us. This relation, the Hubble Law, tells us that our Universe is expanding and allows us to estimate the age of our Universe. If the Universe has always been expanding at the same rate, then we can determine the time that distant galaxies have been traveling to reach their present locations.

PART 1: THE CURVATURE OF SPACE

Introduction

Imagining a 3-dimensional space that's curved into a fourth dimension can be challenging, so we will use as an analogy 2-dimensional spaces that are curved into a third dimension.

Imagine yourself in Flatland, a 2-dimensional flat space, like an infinite piece of paper. If a sphere floated down through the plane of Flatland, a Flatlander would first see a point, which would grow to a circle, reach a maximum size, shrink to a point again and disappear.

We can determine the curvature by utilizing the following facts:

1) On any surface, of any curvature, the sum of the angles at any point is equal to 360 degrees.
2) The sum of the interior angles of a triangle will be:
    a) exactly 180 degrees for a **flat** surface
    b) less than 180 degrees for a **negatively curved** surface (*hyperbolic geometry*)
    c) more than 180 degrees for a **positively curved** one (*elliptic/spherical geometry*)

1A: ESCHER'S WORLD

In many of his works, the Dutch graphic artist M.C. Escher explored two problems. The first was the regular division of a plane into tiles; the second was the representation of 3-dimensional objects and infinities in 2 dimensions. The two works in this assignment are projections of creatures "living" in 2-dimensional spaces, which may or may not be flat, onto the page, which is definitely flat. ***The creatures are all the same size in their own world*** - the apparent change in size of the angels and devils in Circle Limit IV is an artifact of the projection onto the flat page, like the distortion of the size of Greenland on a Mercator map in an atlas.

In Escher's works, the creatures are tiled, their bodies fitted together to form regular patterns. The curvature can be found by locating a repeating triangular shape and counting how many triangles surround its vertex points. These numbers will allow the size of ***each of the interior angles of the triangle*** to be calculated.

Procedure:

In Figure 1, there are two types of birds, which are all roughly triangular (their vertices being the points where 6 birds are touching: the wing-tips and the beaks). Look at the triangle with the numbered labels marking its interior angles. Look at vertex number one. There are six birds (triangles) coming together at this vertex. The birds are all the same size in their space, so that the 6 angles at this vertex are all the same, that is:

$$360° / 6 = 60°$$

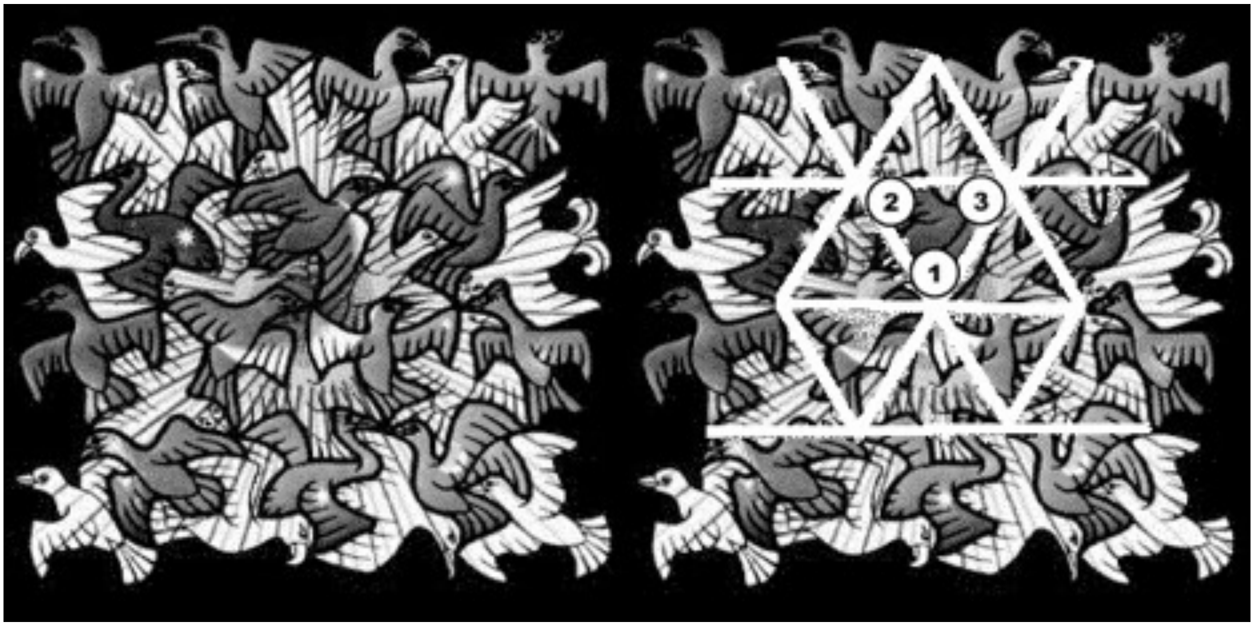

Figure 1: Sun and Moon by M.C.Escher (1948).

If we look at the other two vertices in turn we find a similar situation for each of them – six birds (triangles) surround each vertex point. So each of the interior angles (numbered 1 to 3) of each bird's triangle is also 60 degrees. The sum of the three angles of the triangle of each bird is therefore ***60° + 60° + 60° = 180°,*** so that we now know that the curvature is flat. (There should be no surprises here.)

QUESTIONS:

1. Guess what the curvature is in Figure 2 located on page 4. Give reasons for your guess.
2. Remember that the angels and devils occupy identical triangles in their own universe. Also note that the triangles are not equilateral. That is, in this picture, all three angles are not the same! Determine the curvature using the method from the example. (**Show your work!**)

1B: BALLOON WORLD

Procedure: *(Any questions in this section are just intended to direct your thinking so you will be able to more easily provide answers in the questions section.)*

Imagine that you are a 2-dimensional creature living on the surface of a sphere. Can you list three geometrical tests to determine if your universe is positively curved?

a) Blow up one of your balloons and tie it off if you wish. Establish an equator and two poles and indicate them with your marker.
b) If you draw circles parallel to the equator between the equator and a pole what do you notice about their sizes? These circles are called *small circles*.

c) If you draw circles joining the poles what do you notice about their sizes? These circles are called *great circles*. What two interesting properties do they have? What sort of circle is the equator?

d) Straight lines are usually thought to be the shortest distance between two points (more generally these lines are called *geodesics*). “Straight” lines on the surface of the balloon are curved. Using your marker locate two positions on the balloon and join them with a line. Using your string stretch it between the points. How well does it coincide with your line? Continue stretching your string and extending your line beyond the two points completely around the balloon. What do you notice about its position with respect to the centre of the sphere and its diameter?

QUESTIONS:

1. What would a 4-dimensional sphere look like if it passed it through our space? What *possible* shapes could you see as a cube passed through Flatland?
2. Name three geometrical tests to see if your universe is positively curved?
3. What did you determine about the sum of the interior angles of the triangles on Balloon world? Indicate how you did the determination. How does this sum relate the size of the triangle? Compare your result from Escher’s world.
4. How might you use two beams of light to figure out what the curvature of our universe is?
5. What dominates the curvature in the inner solar system? Is it flat, positive or negative? How do we know?

PART 2: THE EXPANSION OF THE UNIVERSE

Procedure:

Blow up your balloon a little bit. DO NOT TIE IT SHUT!

a) Draw and number ten galaxies on the balloon. Mark one of these galaxies as the reference galaxy.

b) Measure the distance between the reference galaxy and each of the numbered galaxies. The easiest way to do this is to use a piece of string. Stretch it between the two points on the balloon, then measure the string. *Record these data in a table.* Be sure to indicate the units you are using.

c) Now blow up the balloon. You can tie it shut this time if you like.

d) Measure the distance between the reference galaxy and each of the numbered galaxies. *Record these data also in the table***.**

e) Subtract the first measurement from the second measurement, *record the difference in the data table.*

f) Estimate the amount of time it took you to blow up the balloon (in seconds). Divide the distance traveled (the difference) by this time to get a velocity. *Record these calculations also in the table***.**

QUESTIONS:

1. Plot the velocity versus the second measurement to get the "Hubble Law for Balloons". Label the units on your axes.
2. Draw a 'best-fit' straight line through your points. The line should go through (0,0) on your graph. Why?
3. Find the slope. This is exactly the way that we find the value of H from Hubble's Law.
4. Find the age of your balloon universe from this slope.
5. How does this age compare to the time it took to blow up the balloon the second time?
6. What assumptions do you make about your balloon universe when you find its age by this method? Are these sensible assumptions?
7. How would your results change if you used a different reference "galaxy" on the balloon? *(If you are not sure, try it!)*

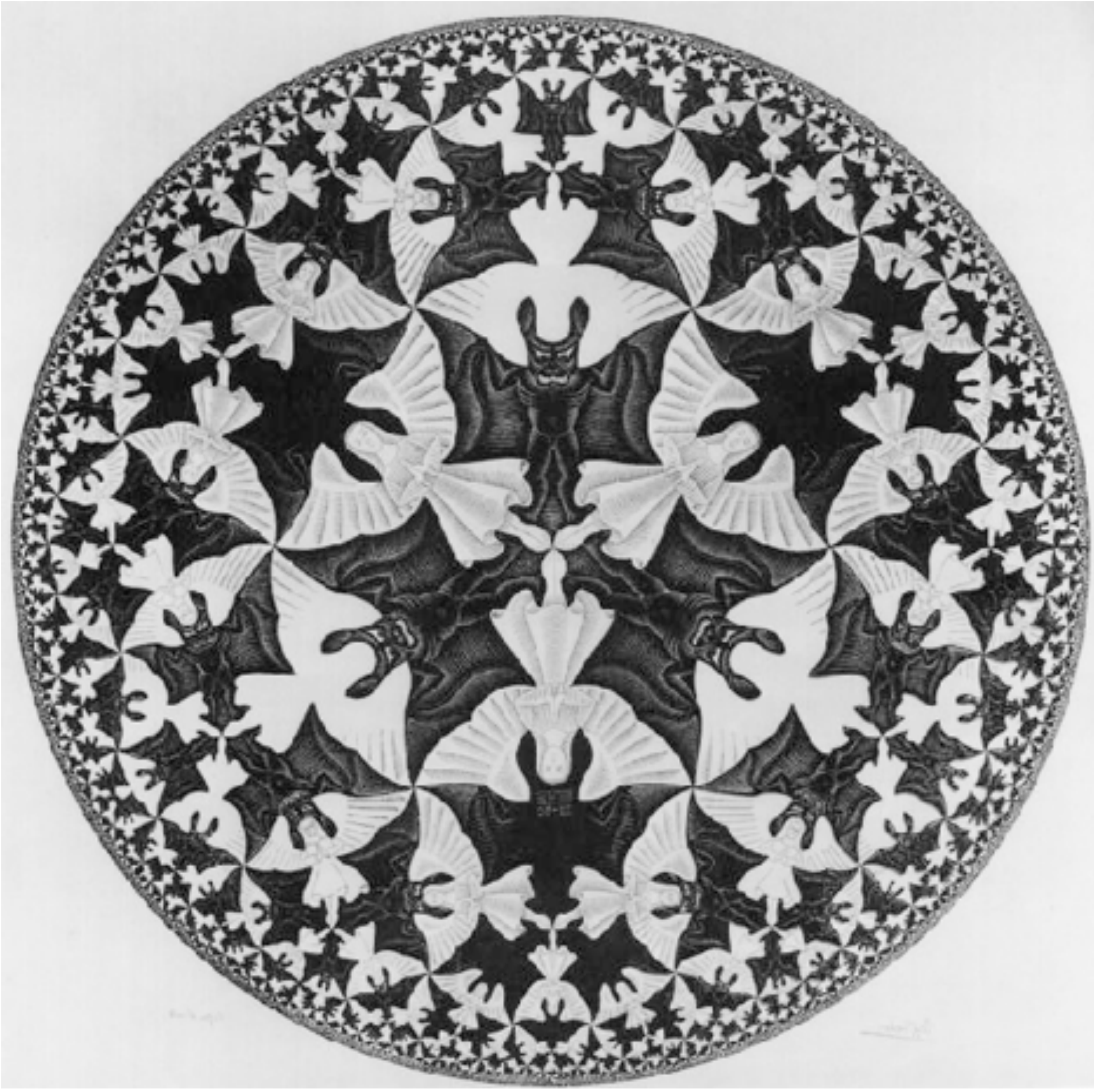

Figure 2: Circle Limit 4 -- (Heaven and Hell) by M.C.Escher (1960).

References: http://www.astro.washington.edu/labs/clearinghouse/labs/labs.html